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PROBABILITY

See also A12079, A12080, A12092, A12201, A12317, A12447, 12675, 12791, 12857, 12858, 12867, 13326, 13339, 13348, 13350.

12546:

Gnedenko, B. V.; Khinchin, A. Ya. ★An elementary introduction to the theory of probability. Translated by W. R. Stahl; edited by J. B. Roberts. A Series of Undergraduate Books in Mathematics. W. H. Freeman and Co., San Francisco-London, 1961. iv + 139 pp. $1.75.

A translation of *Elementarnoe vvedenie v teoriyu veroyatnosteĭ* from the fourth Russian edition [Gosudarstv. Izdat. Tehn.-Teor. Lit., Moscow, 1957; MR **19**, 889] which does not differ appreciably from the earlier editions. There exist translations into German [Deutscher Verlag der Wissenschaften, Berlin, 1955; MR **16**, 838] and French [Dunod, Paris, 1960; MR **22** #240].

12547:

Špaček, Antonín. Probability measures in infinite Cartesian products. Illinois J. Math. **4** (1960), 210–220.

Author's summary: "The main purpose of this paper is to give a general scheme for the constructing of probability measures in infinite Cartesian products of measurable spaces by using an appropriate imbedding of this Cartesian product into a standard random process of function space type." The paper includes two particular applications. The first application is the construction of probability measures in infinite Cartesian products of separable metric spaces. The second one establishes a theory concerning the construction of probability measures in infinite Cartesian products of sets of Schwartz distributions. *Mark Mahowald* (Syracuse, N.Y.)

12548:

Mosteller, Frederick; Rourke, Robert E. K.; Thomas, George B., Jr. ★Probability: A first course. Addison-Wesley Science and Mathematics Education Series. Addison-Wesley Publishing Co., Inc., Reading, Mass.-London, 1961. xv + 319 pp. $5.00.

This is a very elementary introduction to probability theory (mathematical prerequisite: a second course in high-school algebra). In the first four chapters the theory of probability for finite sample spaces is developed. The remaining three chapters deal with random variables, the binomial distribution, and statistical applications (statistical inference from binomial experiments). Chebyshev's inequality is derived and used. The normal distribution and the central limit theorem do not appear. (The references to them on pp. 184 and 207 have evidently crept in inadvertently from the longer version of the book reviewed below [#12549].) Statistical problems are treated both in the now standard non-Bayesian way and in the manner of the personalistic school. The skillful presentation makes the book well suited to arouse the student's interest and to introduce him to the basic ideas of probability and its manifold applications. *W. Hoeffding* (Chapel Hill, N.C.)

12549:

Mosteller, Frederick; Rourke, Robert E. K.; Thomas, George B., Jr. ★Probability with statistical applications. Addison-Wesley Series in Statistics. Addison-Wesley Publishing Co., Inc., Reading, Mass.-London, 1961. xv + 478 pp. $6.50.

This is an extended version of the book reviewed above [#12548], written on the same elementary level. In addicion to the seven chapters of the latter (some of which are here expanded), it contains chapters on joint distributions and continuous distributions; theory of sampling; and least squares, curve-fitting and regression. Continuous distributions are treated informally, in terms of "area probability graphs". In a book of this size and level only a few topics in statistics can be covered. The chapter headings above and the preceding review roughly indicate the topics chosen by the authors.

W. Hoeffding (Chapel Hill, N.C.)

12550:

Greenberg, Irwin. The evaluation of certain probability integrals. Math. Comp. **14** (1960), 376–378.

The integral expressing the probability that, of n independent stochastic processes, the ith yields the highest output, is evaluated by the author in case $n=2$ or 3, and the output of each process is normally distributed. The evaluation is done by reducing each integral—by means of manipulation of the integrand and changes in the order of integration—to a well-known tabulated form.

S. Haber (Washington, D.C.)

12551:

Rybarz, J. Elementarer Beweis zweier Sätze über die momenterzeugende Funktion. Skand. Aktuarietidskr. **1959**, 148–158 (1960).

The author claims to give short and very elementary proofs for the uniqueness theorem for characteristic functions and for the continuity theorem using, however, moment-generating functions instead of characteristic functions. Unfortunately, these proofs contain an error. The transitions from formula (19) to formula (22) and from formula (42) to formula (48) are incorrect.

L. Schmetterer (Hamburg)

12552:

van Eeden, Constance; Bloemena, A. R. On probability distributions arising from points on a lattice. Math. Centrum Amsterdam Rap. No. S 257 (1959), 25 pp.

A rectangular lattice of $N=km$ points (k rows, m columns) is wrapped around a torus. n points are chosen at random. The distribution of x, the number of adjoining chosen points, is obtained for $k=2$, $n=2, 3, 4$; for $k\geq 3$ and $n=2, 3$; for $k=m=n=4$. A number of other results are also obtained, including the first three moments of x and its asymptotic distribution as $N\to\infty$, $nN^{-1}\to 0$.

P. A. P. Moran (Oxford)

12553:

Medgyessy, Pál. ★Decomposition of superpositions of distribution functions. Akadémiai Kiadó, Budapest, 1961. 227 pp. $7.50.

The present monograph deals with the analysis of finite mixtures $G(x)=\sum_{k=1}^N p_k F_k(x)$ $[p_k\geq 0, \sum_{k=1}^N p_k=1]$ or superpositions $[p_k\geq 0]$ of non-degenerate distribution functions $F_k(x)$ belonging to some parametric families. It is desired to decompose the mixture (or superposition, if $\sum_{k=1}^N p_k\neq 1$), that is, to determine N, the weights $p_1, \cdots, p_N$, as well as the parameters of the distributions $F_k(x)$. The same problem is also treated for mixtures (or superpositions) of frequency functions. The basic idea is illustrated by considering normal distribution functions $v(x; \alpha_k, \beta_k)=\Phi((x-\alpha_k)/\sqrt{(2\beta_k)})$, where

$$\Phi(y)=(2\pi)^{-1/2}\int_{-\infty}^{y} e^{-y^2/2}\,dy$$

is the standardized normal distribution. Let $0<\beta_1<\cdots<\beta_N$, and let $G(x)=\sum_{k=1}^N p_k v(x; \alpha_k, \beta_k)$ be the mixture (superposition) considered. A parameter λ $(0\leq\lambda<\beta_1)$ and a second mixture (superposition)

$$G(x;\lambda) = \sum_{k=1}^{N} p_k v(x; \alpha_k, \beta_k-\lambda)$$

are introduced. If $\lambda\to\beta_1$, then $G(x;\lambda)$ tends to a distribution function which has a saltus p_1 at the point α_1 but is continuous at all other points. Let $g(t;\lambda)$ and $g(t)$ be the Fourier transforms of $G(x;\lambda)$ and $G(x)$ respectively; then $g(x;\lambda)=g(x)e^{\lambda t^2}$. Using this relation, it is possible to determine $G(x;\lambda)$ either numerically or graphically and hence also the parameters p_1, α_1 and β_1. The procedure can be iterated to find the other parameters.

The author summarizes the results of his earlier work [Magyar Tud. Akad. Alkalm. Mat. Int. Közl. **2** (1953), 165–177; **3** (1954), 139–153, 155–169, 331–341, 321–329; MR **16**, 270; **17**, 862, 863]. He also extends and generalizes his method to cover a much wider class of distribution functions.

The decomposition of distribution functions has undoubtedly many important practical applications. However, the author does not emphasize this aspect but strives instead at the greatest possible generality. This reviewer regrets that the treatment of such a basically practical problem is complicated by the buildup of a general theory to such an extent that the usefulness of the book for scientists working in experimental fields is greatly reduced. The language of the book is stilted and complicated, and reading it is an unpleasant chore. Part of the more difficult mathematical discussion is given in appendices. Appendix 5 and Appendix 6 are particularly useful since they make some of the interesting results of the author easily accessible. Appendix 5 deals with partial differential equations for stable density functions and is essentially an English version of the author's earlier paper [Magyar Tud. Akad. Mat. Kutató Int. Közl. **1** (1956), 489–518; MR **20** #1345]. Appendix 6 treats integro-differential equations for stable density functions and reproduces results published in Publ. Math. Debrecen **5** (1958), 288–293 [MR **21** #354].

E. Lukacs (Washington, D.C.)

12554:

Ruegg, Alain. Sur la continuité d'une transformation intégrale des fonctions caractéristiques. C. R. Acad. Sci. Paris **249** (1959), 2000–2002.

Etant donnée une suite de fonctions caractéristiques tendant au sens de la convergence simple vers une limite qui est elle-même une fonction caractéristique, l'auteur donne des conditions suffisantes dans lesquelles la suite des intégrales de ces fonctions caractéristiques tend vers un limite. [Cf. la Note précédente, mêmes C. R. **249** (1959), 494–495; MR **21** #7554.]

A. Fuchs (Strasbourg)

12555:

Teicher, Henry. On the mixture of distributions. Ann. Math. Statist. **31** (1960), 55–73.

From the author's summary: "If $\mathscr{F}=\{F\}$ is a family of distribution functions, and μ is a measure on a Borel field of subsets of $\mathscr{F}$ with $\mu(\mathscr{F})=1$, then $\int F(\cdot)d\mu(F)$ is again a distribution function, which is called a μ-mixture of $\mathscr{F}$. Convergence when either F_n or μ_k (or both) tend to limits is dealt with in the case where $\mathscr{F}$ is indexed by a finite number of parameters. Mixtures of additively closed families are considered and the class of such μ-mixtures is shown to be closed under convolution. A sufficient and some necessary conditions are given for a μ-mixture of normal distributions to be normal. The problem of mixtures of Poisson distributions is linked to a moment problem."

M. Rosenblatt (Providence, R.I.)

12556:

Laha, R. G. On a class of unimodal distributions. Proc. Amer. Math. Soc. **12** (1961), 181–184.

Following the work of Yu. V. Linnik [Ukrain. Mat. Ž. **5** (1953), 247–290; MR **15**, 724] the author shows that $f(t)=1/(1+|t|^\alpha)$ $(-\infty<t<\infty)$ is the characteristic function of a symmetric unimodal absolutely continuous distribution when $0<\alpha\leq 2$. He then gives a simple proof of A. Wintner's theorem [*Asymptotic distributions and infinite convolutions*, Lecture Notes, Institute for Advanced Study, Princeton, 1938] that each symmetric stable distribution is unimodal, by writing $\exp(-|t|^\alpha)$ as $\lim_{n\to\infty}(1+|t|^\alpha n^{-1})^{-n}$.

D. G. Kendall (Oxford)

12557:

Franckx, Edouard. Sur la convergence forte. Mitt. Verein. Schweiz. Versich.-Math. **59** (1959), 293–296. (German. English and Italian summaries)

Generalizing a classical result of A. N. Kolmogorov for the independent case, the author proves: Let x_n $(n=1, 2, \cdots)$ be a sequence of uniformly bounded random variables (with $Ex_n=0$) and put $S_n=E((x_1+\cdots+x_n)^2-$

$x_1+\cdots+x_{n-1})^2$); then $\sum_{n=1}^{\infty} S_n/n^2<\infty$ implies that the sequence (x_n) obeys the strong law of large numbers.

A. Dvoretzky (Jerusalem)

12558:

Furstenberg, H.; Kesten, H. Products of random matrices. Ann. Math. Statist. **31** (1960), 457–469.

Let $X_1, X_2, \cdots, X_n$ be a sequence of independent random matrices with given distribution function. The problem of determining the asymptotic behavior of $E(\log y_{ij}^{(n)})$, where $(y_{ij}^{(n)})=Y_n=X_nX_{n-1}\cdots X_2X_1$, was first considered by the reviewer [Duke Math. J. **21** (1954), 491–500; MR **15**, 969] and some partial results obtained. Using difficult and ingenious analysis, the authors establish under certain assumptions the existence of the limit of $n^{-1}E(\log y_{ij}^{(n)})$ as $n\to\infty$ and the normality of the quantity $n^{-1/2}(\log y_{ij}^{(n)}-E(\log y_{ij}^{(n)}))$. Examples are given to show that this need not always be the case.

R. E. Bellman (Santa Monica, Calif.)

12559:

Spitzer, Frank. The Wiener-Hopf equation whose kernel is a probability density. II. Duke Math. J. **27** (1960), 363–372.

Continuing the work of his previous paper [same J. **24** (1957), 327–343; MR **19**, 890], the author studies finer properties of the nondecreasing solution F of $F(x)=\int_0^\infty k(x-y)F(y)dy$, $x>0$, satisfying $F(0+)=1$, where k is an even probability density function. If $\sigma^2=\int x^2k(x)dx<\infty$, it is natural to write $F(x)=2^{1/2}[x+G(x)]/\sigma$, since $F(x)\sim 2^{1/2}x/\sigma$ as $x\to\infty$ by a result of the above cited paper. The main result of the present paper is that

$$\lim_{x\to\infty}[G(x+h)-G(x)]=0$$

for every $h>0$, and that $\lim_{x\to\infty}G(x)=+\infty$ or

$$(2\pi)^{-1}\int_{-\infty}^{\infty}\lambda^{-2}\log\{\lambda^2\sigma^2/2[1-\phi(\lambda)]\}\,d\lambda<\infty,$$

according as whether or not $\int|x|^3k(x)dx=\infty$; here $\phi(\lambda)=\int e^{ix\lambda}k(x)dx$. The proof is based on a renewal-theory argument concerning $\sum_{k=1}^{\infty}P_r\{Z_1+\cdots+Z_k<x\}=F(x)-1$, where X_i are independent with common density k, $S_n=X_1+\cdots+X_n$, $Z_0=0$, $Z_i=$ first $S_k>Z_{i-1}$, and on a computation of EZ_1^2. This also yields an iterative procedure for obtaining F. The discrete (Toeplitz matrix) analogue is also stated.

J. Kiefer (Ithaca, N.Y.)

12560:

Cantelli, Francesco Paolo. Su qualche applicazione della legge uniforme dei grandi numeri per la deduzione delle leggi di frequenza da considerazioni di probabilità. Scritti matematici in onore di Filippo Sibirani, pp. 41–48. Cesare Zuffi, Bologna, 1957.

Betrachtung einiger Verteilungen, die sich nach der Methode von Maxwell-Boltzmann behandeln lassen.

W. Saxer (Zbl **77**, 127)

12561:

Lévy, Paul. Définitions faibles et définitions complètes des fonctions aléatoires. Application aux processus stochastiques laplaciens. Bull. Sci. Math. (2) **84** (1960), 47–64.

A stochastic process $X(t)$ is said to be "weakly defined" if the joint distribution function of $\{X(t_1), \cdots, X(t_n)\}$ is known for every finite subset $(t_1, \cdots, t_n)$. The author's point is that while different processes may have the same weak definition—e.g., one may modify the usual Wiener process by having it ± 1 on a countable dense t-set almost surely, yet not alter its weak definition (and hence the necessity to consider "completely defined" processes)—an effective (non-transfinite) "construction" of $X(t)$ involves only its weak definition.

Without ever invoking the notion of separability, the author discusses this point and gives several interesting examples. If $\{U_\nu\}$ is a (possibly uncountable) family of mutually independent random variables each uniformly distributed over (0, 1), $X(t)$ can be completely defined as some (non-random) function of the U_ν. If Φ is a measurable functional of $X(t)$, it is proved that there is a Φ^* which is a function of at most countably many of the U_ν and such that $\Phi=\Phi^*$ with probability one.

The paper closes with three weak and corresponding complete definitions of a Gaussian process, and a comparison of them with the weakly defined Gaussian processes of Doob and Fréchet.

D. A. Darling (Ann Arbor, Mich.)

12562:

Zítek, František. Sur un type spécial d'équations différentielles stochastiques. Czechoslovak Math. J. **9 (84)** (1959), 452–458. (Russian summary)

Dans le cadre de ses définitions [voir même J. **8 (83)** (1958), 465–472, 473–482; MR **21** #2303, 2304], l'auteur considère l'équation $\delta X(t)\sim g(dt)\,_{\circ}Y$, dont la solution est

$${}_{2}\psi x(t;s)=tg'(0+)\int_{-\infty}^{\infty}(e^{isx}-1)\,dF(x)$$

où F est la fonction de répartition de Y.

G. Marinescu (Bucharest)

12563:

Jurkat, W. B. On semi-groups of positive matrices. I, II. Scripta Math. **24** (1959), 123–131, 207–218.

$P(t)$ is a matrix of non-negative real-valued functions $p_{ij}(t)$, $i, j=1, 2, \cdots$, $t\geqq 0$, with $\lim_{t\to 0}p_{ij}(t)=p_{ij}(0)=\delta_{ij}$ for each i, j and $\sum_k p_{ij}(s)p_{jk}(t)=p_{ij}(s+t)$. Then, many results known for stochastic matrices still hold: continuity of the $p_{ij}(t)$, existence of $\lim_{t\to 0}(p_{ij}(t)-\delta_{ij})/t$ (perhaps infinite when $i=j$), existence and continuity of $p_{ij}'(t)$ for $t>0$ provided the derivative of p_{ij} at 0 is finite, necessary and sufficient conditions for the forward or backward Kolmogorov differential equations to hold, etc. Feller's construction of a "minimal" solution to the Kolmogorov equations is generalized to this setting and discussed.

J. Feldman (Berkeley, Calif.)

12564:

Prékopa, A. On secondary processes generated by random point distributions. Ann. Univ. Sci. Budapest. Eötvös. Sect. Math. **2** (1959), 139–146.

This paper is a generalization of an earlier one in same Ann. **1** (1958), 153–170 [MR **22** #10009]. Let T and Y be abstract spaces. Let $\{t_n\}$ be a sequence of random points having a "mixed Poisson distribution" in T. (A Poisson point distribution with random density is said to be a mixed Poisson point distribution.) Let $\{y_n\}$ be a sequence

of identically distributed, mutually independent random variables taking values in Y. The author proves that if $\{t_n\}$ and $\{y_n\}$ are independent sequences, then the point pairs $\{t_n, y_n\}$ are distributed in the product space $T \times Y$ in accordance with the mixed Poisson law. Several examples are given. *L. Takács* (New York)

12565:

Chow, Y. S. Martingales in a σ-finite measure space indexed by directed sets. Trans. Amer. Math. Soc. **97** (1960), 254–285.

Assume a fixed σ-finite complete measure space $(W, \mathscr{F}, \mu)$. An extended real-valued function on W is said to be σ-integrable on $\mathscr{F}_1 \subset \mathscr{F}$ if either its positive or its negative part is a Radon-Nikodym derivative, with respect to μ, of a σ-finite measure over $\mathscr{F}_1$. For a directed set Δ, let $(\mathscr{F}_\delta, \Delta)$ be a net of sub-σ-algebras of $\mathscr{F}$ with $\mathscr{F}_{\delta'} \subset \mathscr{F}_\delta$ whenever $\delta' < \delta$. The triple $(x_\delta, \mathscr{F}_\delta, \Delta)$ is called a semi-martingale stochastic process if each x_δ is σ-integrable on every $\mathscr{F}_{\delta'}$, and if for each δ, $E(x_\delta | \mathscr{F}_{\delta'}) \geqq x_\delta$ a.e. whenever $\delta' < \delta$. It is called a martingale if $\geqq$ is replaced by $=$. A much more detailed presentation of these concepts, as is necessary, is given in sections 1 and 2 of the paper, including a definition of conditional expectation of σ-integrable functions with respect to σ-finite σ-algebras. It is shown that the usual properties of conditional expectation are shared by this generalized concept, including the analogue of Jensen's inequality. It is in section 3 that the author defines semi-martingales and stopping and sampling variables and proves several basic closure properties, system theorems and inequalities, including generalizations of Theorems 1.1, 2.2, and 3.2 of J. L. Doob, *Stochastic processes* [Wiley, New York, 1953; MR **15**, 445].

The pointwise convergence of martingales is studied in section 4. Let $\bar{x} = \lim\sup x_\delta$ and $\bar{x}^* = \inf_\delta^* \sup_{\delta' \geq \delta}^* x_\delta$ where inf* (sup*) denotes essential infimum (supremum). Let $\tilde{x}$ and $\tilde{x}^*$ denote the corresponding quantities for lower limits. Several Vitali covering assumptions are introduced, under which theorems proving the a.e. equality of $\bar{x}$ and $\tilde{x}$ and of $\bar{x}^*$ and $\tilde{x}^*$ are given. (The Vitali condition $V_1{}^*$ of Theorem 4.2 is undefined, but should be understood as being related to V_1 as $V_0{}^*$ is related to V_0 in Definition 4.1.) The author then proves convergence results for the special cases in which the index set is a subset of the real line, the negative integers and the positive integers. In section 5, the classical Lebesgue differential theorem for additive functions of bounded variation on finite-dimensional Euclidean spaces is derived as an application of the main convergence theorem.

A semi-martingale is called regular if, for every sampling process, the resulting process is also a semi-martingale. When the index set is restricted to be a subset of the rationals, it is shown that regularity is equivalent to the condition of quasi-regularity, namely, that for every individual sampling variable s, x_s is σ-integrable and $E(x_s | \mathscr{F}_r) \geqq x_r$ a.e. on the set $[s \geqq r]$. Quasi-regularity, under the name of regularity, is the condition used by Snell [Trans. Amer. Math. Soc. **73** (1952), 293–312; MR **14**, 295]. After generalizing Andersen and Jessen's result [Danske Vid. Selsk. Mat.-Fys. Medd. **25** (1948), no. 5, 1–8; MR **10**, 437] concerning convergence a.e. of Radon-Nikodym derivatives of the restrictions of an absolutely continuous countably additive set function to members of a nondecreasing sequence of σ-algebras, in the case of a finitely additive set function, the author gives some characterizations of regularity when Δ is the set of positive integers.

There are at least nine examples given throughout the paper to illustrate the degree of necessity for the various assumptions. Earlier work to which this paper is closely related, in addition to the references given above, is that of Helms [Trans. Amer. Math. Soc. **87** (1958), 439–446; MR **20** #1350], of Jerison [Proc. Amer. Math. Soc. **10** (1959), 531–539; MR **22** #8556] and of Krickeberg [Trans. Amer. Math. Soc. **83** (1956), 313–337; MR **19**, 947]. *R. Pyke* (Seattle, Wash.)

12566:

Fuchs, A. Quelques problèmes concernant les chaînes de Markoff dans le cas dénombrable. Bull. Soc. Math. Belg. **11** (1959), 9–21.

Some remarks concerning the calculation of mean first-passage times for Markov chains, chiefly in the case when the number of states is finite. *D. G. Kendall* (Oxford)

12567:

Anselone, Philip M. Ergodic theory for discrete semi-Markov chains. Duke Math. J. **27** (1960), 33–40.

Let T be the non-negative integers and $\{\phi_k, k \in T\}$ be a Markov chain with a stationary transition matrix $\{p(i, j)\}$ taking values in T. Let $\{\theta_k, k = 1, 2, 3, \cdots\}$ be a sequence of random variables also taking values in T and such that

$$P\{\theta_k = j \,|\, \phi_h = i_h,\ h \leqq k+1;\ \theta_h = j_h,\ h < k\} = a(i_k, j),$$

independent of k. Also assume

$$P\{\phi_{k+1} = m \,|\, \phi_h = i_h,\ \theta_h = j_h,\ h \leqq k\} = p(i_k, m).$$

Let $t_0 = 0$ and for $n > 0$, let $t_n = \sum_1^n \theta_k$; define $x_t = \phi_n$ when $t_n \leqq t < t_{n+1}$; then $\{x_t, t \in T\}$ is the semi-Markov process in discrete time which the author studies. He defines $y_t = t_{n+1} - t$ when $t_n < t \leqq t_{n+1}$ and explains that $\{(x_t, y_t), t \in T\}$ is a Markov chain to which the classical theory applies. In this way he derives certain limit theorems concerning the semi-Markov chain $\{x_t, t \in T\}$. *W. L. Smith* (Chapel Hill, N.C.)

12568:

Bílý, Josef. Eine Markoffsche Kette, die zu einer Faltung zweier binomischen Verteilungen und zu kumulierten binomischen Verteilungen führt. Časopis Pěst. Mat. **84** (1959), 327–334. (Czech. Russian and German summaries)

The author considers Makov chains with finitely many states, finding the limit distribution in several special cases. *J. L. Doob* (Urbana, Ill.)

12569:

Chung, Kai-Lai. Continuous parameter Markov chains. Proc. Internat. Congress Math. 1958, pp. 510–517. Cambridge Univ. Press, New York, 1960.

In this paper, the author gives a brief survey of the theory of continuous Markov chains as it stood in mid-1958, with particular emphasis upon the rôle played by the strong Markov property. Several open problems are also described, including the problems of properly com-

pactifying the state-space so as to be able to extend the strong Markov property to situations in which the process at the optional stopping time may be at "infinity", of the differentiability of all transition functions, and of finding relations between a continuous Markov chain and certain discrete Markov chains formed by sampling from it at equidistant time points (the latter processes are called the discrete skeletons of the continuous process). Much work has been done on the theory of continuous Markov chains (including a complete solution by D. Ornstein [Bull. Amer. Math. Soc. **66** (1960), 36–39; MR **22** #10021] of the second problem mentioned above) in the three years since this discussion was presented. In particular, the author's recent comprehensive book on this subject has appeared, *Markov chains with stationary transition probabilities* [Springer, Berlin, 1960; MR **22** #7176] to which the reader is referred. *R. Pyke* (Seattle, Wash.)

12570:

Tweedie, M. C. K. Generalizations of Wald's fundamental identity of sequential analysis to Markov chains. Proc. Cambridge Philos. Soc. **56** (1960), 205–214.

Bellman [same Proc. **53** (1957), 258–260; MR **18**, 681] has indicated an identity for Markov chains related to Wald's fundamental identity for sequential analysis. In this paper the author extends Bellman's idea to more general Markov chains. *J. L. Doob* (Urbana, Ill.)

12571:

Mandelbrot, Benoit. Processus stochastiques à loi stable positive, permanents, markoviens, stationnaires (non additifs). C. R. Acad. Sci. Paris **250** (1960), 451–453.

Given $\alpha \in (0, 1) \cup (1, 2)$, let l_α have the stable distribution with exponent α and maximal skewness, let $a \equiv b$ mean that a and b are identical in law, and consider a stochastic process $u(t): t \geq 0$ such that: (1) $u(t) \equiv l_\alpha$;

$$(2) \qquad (u(t), u(t+s)) \equiv \int_0^{\pi/2} [d_\theta D(\theta, s)]^{1/\alpha} l_\alpha(\theta);$$

where $d_\theta D(\theta, s) \geq 0$,

$$\int_0^{\pi/2} d_\theta D(\theta, s)(\cos\theta)^\alpha = \int_0^{\pi/2} d_\theta D(\theta, s)(\sin\theta)^\alpha = 1,$$

$l_\alpha(\theta) \equiv l_\alpha \times e^{i\theta}$, and (in P. Lévy's language) the $l_\alpha(\theta)$ $(0 \leq \theta \leq \pi/2)$ are all independent; (3) $(u(t), u(t+s))$; $t \geq 0$ is Markov for each $s > 0$. The discussion {most of it obscure to the reviewer} centers about the idea of divisibility. An additional note contains indications that, if $1 < \alpha < 2$, then the density $p(u)$ of the law of l_α satisfies

$$\lg|\lg p(u)| \sim \frac{\alpha}{\alpha - 1} \lg|u|, \quad u \downarrow -\infty;$$

for additional information on this point, see the author, same C. R. **249** (1959), 2153–2155 [MR **22** #625] and the following review.

H. P. McKean, Jr. (Cambridge, Mass.)

12572:

Mandelbrot, Benoit. Variables et processus stochastiques de Pareto-Lévy, et la répartition des revenus. C. R. Acad. Sci. Paris **249** (1959), 613–615.

A considerable amount of literature already exists on the problem of generalizing Pareto's law of the income distribution so as to fit the facts also for low and moderate incomes. The present article is an addition to this literature. *T. Haavelmo* (Oslo)

12573:

Babkin, V. I.; Belyaev, P. F.; Maksimov, Yu. I. Some remarks on Gončarov's paper "From the domain of combinatorics". Teor. Veroyatnost. i Primenen. **4** (1959), 445–450. (Russian. English summary)

Authors' summary: "This note contains some results on the asymptotic distribution of the random vector $(x_1, x_2, \cdots, x_k)$, where $x_1, x_2, \cdots, x_k$ are the numbers of A-series of lengths $1, 2, \cdots, k-1$ greater than or equal to k, respectively, in the simple homogeneous Markov chain with two states A and B. The asymptotic distribution of the above-mentioned vector (when appropriately formed) is shown to be multivariate normal with the parameters of the distribution calculated. Possible extensions for a number of states greater than two are also discussed."

[The Gončarov paper of the title is Izv. Akad. Nauk SSSR Ser. Mat. **8** (1944), 3–48; MR **6**, 88.]

H. P. Edmundson (Pacific Palisades, Calif.)

12574:

Karlin, Samuel; McGregor, James. Classical diffusion processes and total positivity. J. Math. Anal. Appl. **1** (1960), 163–183.

The authors continue their work on coincidence probabilities and the total positivity of the transition functions of Markov processes [Pacific J. Math. **9** (1959), 1109–1140, 1141–1164; MR **22** #5071, 5072]. They consider examples of diffusion processes whose transition function has a representation of the type

$$p(t; x, y) = \sum_{n=0}^{\infty} e^{-\lambda_n t}\phi_n(x)\phi_n(y),$$

whose terms are classical orthogonal functions, such as Hermite, Laguerre, Jacobi polynomials and Bessel functions. Many of these processes are realized by projecting n-dimensional Brownian motion constrained to a surface of revolution onto the axis of revolution. Such processes are known to have the continuity of paths and strong Markov property which, by the coincidence probability theorem in the second paper cited above, imply that their transition function is totally positive. For other processes, including the Ornstein-Uhlenbeck process and certain diffusions related to problems in population growth, total positivity is established by analytic arguments or by virtue of their being a limiting case of birth and death processes. *F. L. Spitzer* (Princeton, N.J.)

12575:

Sarmanov, O. V. Characteristic correlation functions and their applications in the theory of stationary Markov processes. Dokl. Akad. Nauk SSSR **132** (1960), 769–772 (Russian); translated as Soviet Math. Dokl. **1**, 651–654.

The author calls the function $R_1(t)$ the maximal correlation function of the real process $x_t[\mathscr{M}(x_t)=0;$ $\mathscr{M}(x_t^2)=1]$, when putting $x=x_{t_1}$, $y=x_{t_1+t}$,

$$R_1(t) = \sup_{f,g} |\mathscr{M}[f(x)g(y)]|, \qquad t > 0,$$

where the functions f, g satisfy the conditions $\mathscr{M}[f(x)] = \mathscr{M}[g(y)] = 0$, $\mathscr{M}[f^2(x)] = \mathscr{M}[g^2(y)] = 1$, and, for $t = 0$,

$R_1(0)=1$. This notion is particularly applicable when x and y have a symmetrical joint distribution $\rho(t, x, y)$ satisfying the condition

$$\int \frac{\rho^2(t, x, y)}{\rho(x)\rho(y)}\, dx\, dy < \infty.$$

By means of these distributions one may give the necessary and sufficient conditions in order to generate a Markov process. One also may define the stationarity and other important characteristics of those processes.

O. Onicescu (Bucharest)

12576:

Sarmanov, O. V. Pseudonormal correlation and its various generalizations. Dokl. Akad. Nauk SSSR **132** (1960), 299–302 (Russian); translated as Soviet Math. Dokl. **1**, 564–567.

The author calls pseudonormal the bidimensional distribution with density

$$\rho = \frac{1}{4\pi(1-\lambda^2)^{1/2}}\left\{\exp\frac{x^2+y^2-2\lambda xy}{2(1-\lambda^2)} + \exp\frac{x^2+y^2+2\lambda xy}{2(1-\lambda^2)}\right\},$$

where $|\lambda|<1$. The maximal correlation coefficient [see preceding review #12575] has the value λ^2 and shows that the correlation between x and y is weak with small λ. The paper deals with a more general distribution expressed by means of generalized Laguerre polynomials, and contains the one mentioned above as a particular case, which appears in the study of the stationary Markov processes. Its very simple characteristic function is

$$\varphi(\tau_1, \tau_2) = \frac{1}{[1-i\tau_1-i\tau_2-(1-\lambda^2)\tau_1\tau_2]^{1+\alpha}},$$

by means of which the investigation of various properties of the distributions is quite easily carried out.

O. Onicescu (Bucharest)

12577:

Volkonskiĭ, V. A. Continuous one-dimensional Markov processes and additive functionals derived from them. Teor. Veroyatnost. i Primenen. **4** (1959), 208–211. (Russian. English summary)

Let X be a continuous uniform Feller process on a phase space E, which is regular in the interval (r_1, r_2). Here E is defined as the interval (r_1, r_2) together with the accessible boundaries r_i of the process. In this paper a description is given of all continuous one-dimensional Feller processes which are regular in (r_1, r_2), and their infinitesimal generators determined (Theorem 4). In particular, it is shown that the description of all processes of the above type amounts to a description of all multiplicative or additive functionals corresponding to regular Feller subprocesses of nonterminating processes. The notation and terminology employed throughout is that of E. B. Dynkin [Teor. Veroyatnost. i Primenen. **1** (1956), 38–60; MR **19**, 691–692; cf. also *Osnovaniya teorii markovskih processov*, Gosudarstv. Izdat. Fiz.-Mat. Lit., Moscow, 1959].

From Theorem 2 it follows that a Feller process regular in $[r_1, r_2]$ can be characterized by three nondecreasing functions $u(x)$, $v(x)$ and $m(x)$ on $[r_1, r_2]$, and two nonnegative constants $\alpha(r_1)$ and $\alpha(r_2)$. Such a process is denoted by $X=(u, v, m, \alpha)$. The question of the boundary behavior of the process is answered by Theorem 3: (1) The boundary r_i is accessible if and only if the following quantities are finite:

$$\lim_{x\to r_i} u(x),\quad \int_y^{r_i} v(x)\,du(x),\quad \int_y^{r_i} m(x)\,du(x),\quad y\in(r_1, r_2).$$

(2) An accessible boundary r_i is passable into the interval if and only if $m(r_i)$ and $v(r_i)$ are finite. The main result is given by Theorem 4: An infinitesimal generator of the process $X=(u, v, m, \alpha)$ is a restriction of the operator $\mathscr{A}$:

$$\mathscr{A}f(x) = D_v[D_u f(x) - \int f(x)\,dm(x)],$$

supplemented by the following boundary conditions: (i) If the boundary r_i is nonaccessible, then no supplementary boundary conditions are imposed. (ii) If r_i is accessible and passable into the interval (r_1, r_2), then

$$\lim_{x\to r_i}\{D_u f(x)+[v(r_i)-v(x)]\mathscr{A}f(x)+[m(r_i)-m(x)]f(x)\} = 0.$$

(iii) If r_i is accessible, but not passable into (r_1, r_2), then $\mathscr{A}f(r_i)+\alpha(r_i)f(r_i)=0$. (A boundary r_i is called passable into the interval (r_1, r_2) if the system starting from r_i is able to reach an interior point of (r_1, r_2) in finite time with positive probability.)

A. T. Bharucha-Reid (Eugene, Ore.)

12578:

Volkonskiĭ, V. A. Continuous one-dimensional Markov processes and additive functionals derived from them. Theor. Probability Appl. **4** (1959), 198–200.

Translation of original Russian [Teor. Veroyatnost i Primenen. **4** (1959), 208–211; #12577].

12579:

Rényi, A.; Révész, P. On mixing sequences of random variables. Acta Math. Acad. Sci. Hungar. **9** (1958), 389–393.

Using results obtained by Rényi [same Acta **9** (1958), 215–228; MR **20** #4623] the authors extend and apply these results to prove the following theorem: A sequence of random variables $\{\zeta_n\}$ defined on (Ω, A, P) has the mixing property [cf. above reference] if and only if for every $k=0, 1, 2, \cdots$, there exists a set E_k such that $P(\zeta_k\in E_k)=1$ and such that if $y\in E_k$ then the distributions $P\{\rho_n<x\,|\,\rho_k=y\}$ converge weakly as $n\to\infty$ to a distribution function $F(x)$ independent of y.

Several examples of Markov processes are considered and are shown to have the mixing property. Among these examples is the following: Let the sequence of random variables be $\{q_n(t)\}$, where $q_n(t)$ is an integer not less than 2, and let t be expanded in the Engel series

$$t = \frac{1}{q_1}+\frac{1}{q_1q_2}+\cdots+\frac{1}{q_1q_2\cdots q_n}+\cdots;$$

then it follows from the above result and the paper quoted above that

$$\lim_{n\to\infty} Q\left\{t\,\middle|\,\frac{\log q_n-n}{\sqrt{n}}<x\right\} = \frac{1}{2\pi}\int_{-\infty}^{x} e^{-u^2/2}\,du,$$

where Q is any probability measure in $(0, 1)$ which is absolutely continuous with respect to the Lebesgue measure.

Y. N. Dowker (London)

12580:
Dynkin, E. B. One-dimensional continuous strong Markov processes. Theor. Probability Appl. **4** (1959), 1–52.
English translation of Teor. Veroyatnost. i Primenen. **4** (1959), 3–54 [MR **21** #2309].

12581:
Linnik, Yu. V. General theorems on the factorization of infinitely divisible laws. II. Sufficient conditions (case of a finite Poisson spectrum). Theor. Probability Appl. **4** (1959), 53–82.
English translation of Teor. Veroyatnost. i Primenen. **4** (1959), 55–85 [MR **21** #2300].

12582:
Linnik, Yu. V. General theorems on the factorization of infinitely divisible laws. III. Sufficient conditions (countable bounded Poisson spectrum; unbounded spectrum; "stability"). Theor. Probability Appl. **4** (1959), 142–163.
Translation of original Russian [Teor. Veroyatnost. i Primenen. **4** (1959), 150–171; MR **22** #1935].

12583:
Chistyakov, V. P. Generalization of a theorem for branching processes. Theor. Probability Appl. **4** (1959), 103–106.
English translation of Teor. Veroyatnost. i Primenen. **4** (1959), 109–113 [MR **21** #926].

12584:
Sevast'yanov, B. A. Transient phenomena in branching stochastic processes. Theor. Probability Appl. **4** (1959), 113–128.
Translation of Russian original [Teor. Veroyatnost. i Primenen. **4** (1959), 121–135; MR **21** #3922].

12585:
Venttsel', A. D. On boundary conditions for multidimensional diffusion processes. Theor. Probability Appl. **4** (1959), 164–177.
Translation of original Russian [Teor. Veroyatnost. i Primenen. **4** (1959), 172–185; MR **21** #5246].

12586:
Volkonskiĭ, V. A.; Rozanov, Yu. A. Some limit theorems for random functions. I. Theor. Probability Appl. **4** (1959), 178–197.
Translation of original Russian [Teor. Veroyatnost. i Primenen. **4** (1959), 186–207; MR **21** #4477].

12587:
Prokhorov, Yu. V. An extremal problem in probability theory. Theor. Probability Appl. **4** (1959), 201–203.
Translation of original Russian [Teor. Veroyatnost. i Primenen. **4** (1959), 211–214; MR **21** #4473].

12588:
Prokhorov, Yu. V. Some remarks on the strong law of large numbers. Theor. Probability Appl. **4** (1959), 204–208.
Translation of original Russian [Teor. Veroyatnost. i Primenen. **4** (1959), 215–220; MR **21** #4476].

12589:
Petrov, V. V. Asymptotic expansions for distributions of sums of independent random variables. Theor. Probability Appl. **4** (1959), 208–211.
Translation of original Russian [Teor. Veroyatnost. i Primenen. **4** (1959), 220–224; MR **21** #5234a].

12590:
Petrov, V. V. On a class of limit theorems for independent random variables. Theor. Probability Appl. **4** (1959), 211–214.
Translation of original Russian [Teor. Veroyatnost. i Primenen. **4** (1959), 224–228; MR **21** #5234b].

12591:
Sirazhdinov, S. Kh. On an exact estimate for a local theorem. Theor. Probability Appl. **4** (1959), 215–218.
Translation of original Russian [Teor. Veroyatnost. i Primenen. **4** (1959), 229–232; MR **21** #4474].

12592:
Bílý, Josef. Zusammengesetzte Poissonsche Verteilungen. Časopis Pěst. Mat. **84** (1959), 424–432. (Czech. Russian and German summaries)
A branching process involving only one kind of individual starts with one; each individual of the rth generation is replaced by k in the next generation with probability $[\exp(-\alpha_r)]\alpha_r^k/k!$. The author studies the generating function of the distribution of the number of individuals in the rth generation, with special attention to the case $r=2$. *J. L. Doob* (Urbana, Ill.)

12593a:
Blom, Gunnar. Hierarchical birth and death processes. I. Theory. Biometrika **47** (1960), 235–244.

12593b:
Blom, Gunnar. Hierarchical birth and death processes. II. Applications. Biometrika **47** (1960), 245–251.

The definition of one type of hierarchical birth and death process can be motivated by the following application. There are n machines, serviced by one operator. If the ith machine is operating at time t, the probability is $\lambda_i\,dt$ that it requires service before $t+dt$. If a machine is being serviced at t, the probability is $\mu\,dt$ (same for any machine) that service ends before $t+dt$. Let $E_{ij\cdots y}$ be the state that the ith machine is being attended to and that the jth, $\cdots$, yth machines are queueing for service in this order. Then there is a transition to the state $E_{j\cdots y}$ if service is completed on the ith machine before a new machine needs service, and a transition to $E_{ij\cdots yz}$ if the

zth machine requires service before the ith is repaired. In general a hierarchical process is a Markov process with states $E_{ij\cdots y}$, where the number of indices is 0, 1, 2, $\cdots$, and $i, j, \cdots, y$ take the values 1, 2, $\cdots$, N. The state $E_{ij\cdots y}$ can change only to a state having one additional index z inserted in any position, or to a state with one of the original indices deleted. Thus $E_{ij\cdots y}$ goes to $E_{zij\cdots y}$ with intensity $a_{m-1,1}p_z\,dt$, to $E_{izj\cdots y}$ with intensity $a_{m-1,2}p_z\,dt$, etc.; and to $E_{j\cdots y}$ with intensity $b_{m,1}q_i\,dt$, to $E_{i\cdots y}$ with intensity $b_{m,2}q_j\,dt$, etc. Certain of the quantities a, b, p, and q may be 0 for particular processes. The author determines conditions for the existence of equilibrium probabilities, finds expressions for these probabilities, and gives applications to service and telephone trunking problems.

T. E. Harris (Santa Monica, Calif.)

12594:

Gardner, J. W. The characteristic-functional method in cascade theory. Nuovo Cimento (10) **17** (1960), 205–218. (Italian summary)

The author uses the method of characteristic functionals to treat a general Markoff cascade. *H. Messel* (Sydney)

12595:

Kawata, Tatsuo. The Fourier series of some stochastic processes. Japan. J. Math. **29** (1959), 16–25.

Let $\{X(t),\ -\infty<t<\infty\}$ be a stochastic process which is stationary in the wide sense and has zero means. Let $S_n(t)$ be the nth partial sum of the Fourier series, over the finite interval $[0, T]$, of a sample function. It is proved that $E\{|S_n(t)-X(t)|^2\}$ and the integral of this expectation over $[0, T]$ both go to zero with $1/n$. The distributions of the Fourier coefficients of the sample functions of a process obtained by summing over Poissonian events are also investigated. *J. L. Doob* (Urbana, Ill.)

12596:

Rosenblatt, M. Some comments on narrow band-pass filters. Quart. Appl. Math. 18 (1960/61), 387–393.

Let $\{x(t),\ -\infty<t<\infty\}$ be a strictly stationary stochastic process. Under specified conditions on the process and on the weighting function it is shown that the weighted average $\int_0^T w_T(t)x(t)\,dt$ is asymptotically normal as $T\to\infty$. This result corresponds to what the author calls the 'engineering folklore' that a narrow band-pass filter applied to a stationary random input yields an output that is approximately normally distributed. The result is based on a previous paper by the author [Proc. Nat. Acad. Sci. U.S.A. **42** (1956), 43–47; MR **17**, 635].

J. L. Doob (Urbana, Ill.)

12597:

Baxter, Glen. On the measure of Hilbert neighborhoods for processes with stationary, independent increments. Proc. Amer. Math. Soc. **10** (1959), 690–695.

Suppose that $\{x(t),\ 0\leq t<\infty\}$ is a separable stochastic process with symmetric stationary and independent increments, $x(0)=0$. If

$$\varphi(\xi) = \int_0^\infty e^{-st}E\{\exp[-u\int_0^t x^2(\tau)\,d\tau+i\xi x(t)]\}\,dt,$$

then it is proved that

$$\varphi(\xi) = \int_{-\infty}^\infty \tilde\varphi(x, \xi)\,dx,$$

where $\tilde\varphi(x)\equiv\tilde\varphi(x, \xi)$ is the unique solution of the system: $u\tilde\varphi''-[s+\psi(x)]\tilde\varphi=0$, $\tilde\varphi\to0$ as $x\to\pm\infty$, $\tilde\varphi$ is continuous for all x, $\tilde\varphi'$ is continuous except at $x=\xi$, $\lim_{x\to\xi^-}\tilde\varphi'-\lim_{x\to\xi^+}\tilde\varphi'=1/u$; here $\psi(\xi)$ is the function which appears in the Lévy-Hinčin representation of the characteristic function $E\{e^{i\xi x(t)}\}=e^{-t\psi(\xi)}$ of $x(t)$. Moreover, if $E\{x^2(t)\}$ exists, $0\leq t<\infty$, and has a finite Laplace transform, then $u\varphi''(\xi)-[s+\psi(\xi)]\varphi(\xi)=-1$. Finally, as examples, the Cauchy and Poisson processes are considered. *R. Theodorescu* (Bucharest)

12598:

Jaglom, A. M. Einführung in die Theorie der stationären Zufallsfunktionen. Deutsche Übersetzung unter wissenschaftlicher Redaktion von Dr. Herbert Goering. Deutsch Akad. Wiss. Berlin. Schr. Forschungsinst. Math. **6** (1959), viii+177 pp.

The Russian original of this expository paper [Uspehi Mat. Nauk **7** (1952), no. 5 (51), 3–168] was reviewed in MR **14**, 485. A list of 14 references has been added to the translation.

12599:

Erdös, P.; Taylor, S. J. Some problems concerning the structure of random walk paths. Acta Math. Acad. Sci. Hungar. **11** (1960), 137–162. (Russian summary, unbound insert)

The authors study the ordinary random walk in d-dimensional space, continuing and greatly extending the work of A. Dvoretzky and P. Erdös [Proc. Second Berekeley Sympos. in Math. Stat. and Prob. 1950, pp. 353–367, Univ. of Calif. Press, Berkeley, Calif., 1951; MR **13**, 852]. They first study the number of returns to the origin and prove that if R_n denotes the number of returns to the origin of the plane random walk in the first n steps, then $P\{R_n<x \log n\}\to-e^{-\pi x}$ as $n\to\infty$ for $0\leq x\leq(\log n)^{3/4}$. From this they deduce various results of the precise iterated logarithm type. They proceed to study the distribution of returns to the origin. From the results of this section the following is typical. Let $f(n)$ increase monotonically to $+\infty$ and denote by E_n the event that the two-dimensional random walk does not return to the origin between n and $f(n)$ steps. Then the probability that infinitely many E_n occur is 0 or 1, according as the series $\sum 1/f(2^{2^k})$ converges or diverges. (Applying results of K. L. Chung and G. A. Hunt [Ann. of Math. (2) **50** (1949), 385–400; MR **10**, 613] a similar theorem is proved in the one-dimensional case.) Next they consider the distance $\rho_d(n)$ from the origin after n steps and prove, among other things, the existence of positive constants λ_d such that, as $n\to\infty$,

$$\text{(i)}\qquad \sum_{n=1}^N n^{-1/2}(1+\rho_1(n))^{-1} \sim \lambda_1 \log N;$$

$$\text{(ii)}\qquad \sum_{n=1}^N (1+\{\rho_2(n)\}^2)^{-1} \sim \lambda_2 \log^2 N;$$

$$\text{(iii)}\qquad \sum_{n=1}^N (1+\{\rho_d(n)\}^2)^{-1} \sim \lambda_d \log N$$

for $d \geqq 3$. Finally, some results on the multiplicity of points are obtained.

{Remark: Lemma 2 on p. 154 holds even with the $o(1)$ omitted and is due not to the reviewer but to S. Kakutani [see, for $d=3$, Proc. Imp. Acad. Tokyo **20** (1944), 648–652; MR **7**, 315].} *A. Dvoretzky* (Jerusalem)

12600:

Schwinger, Julian. Brownian motion of a quantum oscillator. J. Mathematical Phys. **2** (1961), 407–432.

An action principle is used to evaluate expectation values for the case of an harmonic oscillator acted upon by a random force. *D. ter Haar* (Oxford)

12601:

Kinney, John R. First passage times of a generalized random walk. Ann. Math. Statist. **32** (1961), 235–243.

Let $X(t)$, $t=1, 2, \cdots$, be integer-valued independent identically distributed random variables with $\Pr(X(t)=i)=p(i)$, where $p(-m)>0$, $p(i)=0$ for $i<-m$. Let $P(z)=E(z^{X(t)})$ be the generating function of $X(t)$. Let $S(0)$ be a random variable with non-negative integer values having the generating function $K(z)$. Let $S(t)=S(0)+\sum_{i=1}^{t} X(i)$; define $S^*(t)$ by $S^*(0)=S(0)$, $S^*(t)=\max[S^*(t-1), 0]+X(t)$; let $Z(t)=\max[S^*(t), 0]$. The author derives generating functions of the distributions of $S^*(t)$ and $Z(t)$, and finds the form of the generating function

$$\sum_{i,j=0}^{\infty} \Pr[S(j)=i, \quad \min_{0<k<j} S(k) \geqq 0] z^i w .$$

The latter quantity and the distribution of $Z(t)$ have applications to the theory of queues. The results are not easily stated in summary form. The author states that the results could be deduced from results of Spitzer [Trans. Amer. Math. Soc. **82** (1956), 323–339; MR **18**, 156], but his methods are different.

T. E. Harris (Santa Monica, Calif.)

12602:

Chung, K. L.; Erdös, P.; Sirao, T. On the Lipschitz's condition for Brownian motion. J. Math. Soc. Japan **11** (1959), 263–274.

Let ψ throughout be a non-negative monotone, non-decreasing function on some (varying) interval (T, ∞). ψ is said to belong to the "upper class" if, for almost every Brownian path $X(t)$ in $[0, 1]$, there exists $\varepsilon>0$ such that $|s-t| \leqq \varepsilon$ implies $|X(s)-X(t)| \leqq \psi(|s-t|^{-1})|s-t|^{1/2}$. If, on the other hand, there is, for almost every Brownian path, no such ε, then ψ is said to belong to the "lower class". The following theorem had been stated without proof by P. Levy in his book *Le mouvement brownien*, Gauthier-Villars, Paris, 1954 [MR **16**, 601], and is proven in the present paper. Theorem 1: ψ is upper or lower, according as $\int_T^\infty \psi^3(t) \exp(-\frac{1}{2}\psi^2(t))\,dt$ is finite or infinite. As a corollary of this, the following result sharpens previously proven special cases of Levy and Sirao. Theorem 2: For $t>T$, let $\psi(t)=\sqrt{(2 \log t+5 \log^{(2)} t+2 \log^{(3)} t+\cdots+2 \log^{(n)} t+c \log^{(n+1)} t)}$. (Here $\log^{(n)} t$ means $\log(\log(\cdots(\log t)\cdots))$.) Then ψ is upper or lower according as $c>2$ or $c \leqq 2$.

J. Feldman (Berkeley, Calif.)

12603:

Henze, E. Zur Theorie der diskreten unsymmetrischen Irrfahrt. Z. Angew. Math. Mech. **41** (1961), 1–9. (English and Russian summaries)

A random walk in a plane square grid is considered under the assumption that each step leads to one of the four nearest grid neighbors with four constant and positive, but otherwise arbitrary, transition probabilities. Setting up the appropriate difference equations and solving them explicitly by definite integrals, various probabilities and expectations connected with this random walk are calculated. Among them are the probability of being at a given point at the nth step, the generating function for the probability of returning after n steps, the mean duration and the probability of absorption. The regions in which the walks are to take place are either the whole plane or the upper half-plane.

W. Wasow (Madison, Wis.)

12604:

Nakamura, Gisaku. On stochastic representations for incoming telephone calls. Rev. Elec. Comm. Lab. **8** (1960), 270–279.

A study is made of incoming calls at a telephone exchange under the single assumption of independence. Particular attention is paid to "impulsive processes" in which calls can only occur impulsively, i.e., all calls occur at the same instant. The author investigates the properties of such processes and then investigates how any independent process can be decomposed into its independent impulsive processes and a residual process containing no impulsive processes. It is shown that there are a countable number of impulsive processes which are a part of any independent process. A generating-function representation is developed for the residual process, and values for the parameters in the function are given to allow for several representative distributions. Finally, it is shown that the residual function can be decomposed into a countable set of generalized Poisson processes (i.e., multiples of a fixed number of calls) with variable parameters.

H. M. Gurk (Princeton, N.J.)

12605:

Capello, Franco; Sanneris, Antonio. Congestione nelle centrali automatiche. Alta Frequenza **29** (1960), 243–250.

Authors' summary: "Scopo dell'articolo è analizzare un metodo di campionatura per la determinazione del grado di perdita globale nelle centrali automatiche — che si effettua eseguendo chiamate di prova con frequenza e distribuzione determinate, per un certo numero di ore di punta — proponendo nello stesso tempo un'interpretazione dei risultati ottenuti. È il metodo di misura previsto dalle Norme del Comitato Elettrotecnico Italiano."

12606:

Gani, J.; Pyke, R. The content of a dam as the supremum of an infinitely divisible process. J. Math. Mech. **9** (1960), 639–651.

Authors' summary: "This paper discusses the relation between the content of a dam and the supremum of a certain infinitely divisible process.

"It is pointed out that the distribution functions of the supremum over time $0 \leqq t \leqq T$ of a Poisson process with shift and the content at time T of a dam with Poisson inputs and a constant release rate have identical forms.

The backward Kolmogorov equation of the first process is shown to be identical with the forward equation of the second.

"A closer relation between the two processes is proved to exist, and a new definition for dam processes in discrete time is given. This is generalized to dam processes in continuous time whose net inputs are from a wide class of infinitely divisible processes.

"The distribution function of the total time during which such dams are empty and non-empty is found. Lastly, using a result of Kendall [J. Roy. Statist. Soc. Ser. B **19** (1957), 207–212; MR **19**, 1092] in the theory of dams, a simplification of the double Laplace transform due to Baxter and Donsker [Trans. Amer. Math. Soc. **85** (1957), 73–87; MR **18**, 944] for the distribution function of the supremum over $0 \leq t \leq T$ of certain infinitely divisible processes with positive jumps is obtained."

P. A. P. Moran (Oxford)

12607:

Yeo, G. F. The time-dependent solution for a dam with geometric inputs. Austral. J. Appl. Sci. **11** (1960), 434–442.

Let $\{X_n\}$ be a sequence of independent, identically and geometrically distributed variables: $P\{X_n=i\}=ab^i$, $0<a<1$, $a+b=1$, $i=0, 1, 2, \cdots$. Suppose Z_0 given and, for $n=1, 2, \cdots$, define

$$Z_n = Z_{n-1}+X_{n-1}-\min(Z_{n-1}+X_{n-1}, 1).$$

The sequence $\{Z_n\}$ is proposed as a model for the content of a reservoir of infinite capacity. The author obtains, by a combinatorial argument, an explicit formula for $P\{Z_n=0 | Z_0=u\}$. He then uses this formula to obtain explicit expressions (rather too complicated to be given here) for $P\{Z_n=i | Z_0=u\}$ when $i>0$. The limiting behavior of these transition probabilities are also studied.

W. L. Smith (Chapel Hill, N.C.)

12608:

Beneš, V. E. Transition probabilities for telephone traffic. Bell System Tech. J. **39** (1960), 1297–1320.

The author considers a set of N service facilities with no provision for delays. The intervals between arrivals of successive customers form a sequence of independent and identically distributed random variables. If a customer on arrival does not find a service facility available, then he is lost to the system; otherwise, he occupies an available facility for a service period having a fixed negative exponential distribution. Service periods and inter-arrival intervals are assumed to be mutually independent. At time t the number of occupied facilities is $N(t)$. The transition probabilities of $N(t)$ are determined, and their limiting behavior is studied. These transition probabilities are stated to have practical value in making estimates of sampling error in traffic measurements. There are three appendices which treat related mathematical questions, including an approach to the problem by using a certain Markov process and an alternative approach via renewal theory and regenerative processes.

W. L. Smith (Chapel Hill, N.C.)

12609:

Haight, Frank A. Queueing with balking. II. Biometrika **47** (1960), 285–296.

As in an earlier paper [Biometrika **44** (1957), 360–369; MR **19**, 692], balking is a refusal to join a waiting line if it is too long; if the line is of length x, the probability of balking is a prescribed function $F(x)$. The queueing system consists of a single server with exponential service time distribution and Poisson arrivals; service is in order of arrival and there are no defections from waiting. The earlier paper considered queue length only. Now the stationary distribution of virtual waiting time is added; this is determined first for an arbitrary customer, then for one who balks, then for one who does not balk. Finally, the stationary distribution function for the interval between an arrival and the first of its predecessors which did not balk, and the corresponding function when the arrival itself does not balk, are determined. Two tables are given comparing results from queue simulation with numerical calculations from the author's formulas for a Poisson balking rule.

J. Riordan (New York)

12610:

Blumenthal, R. M.; Getoor, R. K. A dimension theorem for sample functions of stable processes. Illinois J. Math. **4** (1960), 370–375.

The authors prove for general stable processes a result they had previously obtained [Trans. Amer. Math. Soc. **95** (1960), 263–273; MR **22** #10013] for the symmetric stable processes. The details of the method used in the present paper are in some ways simpler than those used previously. They again use the connection between α-capacity and β-measure {this connection is not quite accurately stated: line (−7) on p. 371 should read $\Lambda^\beta(E)>0 \Rightarrow C_\alpha(E)>0$ for $\alpha<\beta$; however, only a very minor modification is required to correct the argument of the following pages} to obtain the following general theorem. If E is a Borel subset of [0, 1] with dimension γ, and α is the index of a stable process $X(t)$ taking values in R_n, then, with probability 1, the dimension of the image set $X(E)$ is $\min(1, \alpha\gamma)$.

S. J. Taylor (Ithaca, N.Y.)

12611:

Widom, Harold. Stable processes and integral equations. Trans. Amer. Math. Soc. **98** (1961), 430–449.

Let $x(t)$, $t \geq 0$, be the symmetric stable process with exponent α, $0<\alpha\leq 2$, normalized so that the paths are continuous on the right and start at 0. For a point x_0 with $-1<x_0<1$ set

$$q(x_0, x, t) = \frac{d}{dx}\Pr\{\max_{\tau\leq t}|x_0+x(\tau)|<1, x_0+x(t) \leq x\}.$$

The first main result of the author is that if $K(x, y)$ is defined for $-1\leq x, y\leq 1$ by the formula

$$K(x, y) = \frac{\sec \alpha\pi/2}{2\Gamma(\alpha)}|x-y|^{\alpha-1} - \frac{\tan \alpha\pi/2}{2\pi\Gamma(\alpha)}(1-y^2)^{\alpha/2}\int_{-1}^{1}\frac{(1-xt)^{\alpha-1}}{(1-t^2)^{\alpha/2}(1-yt)}\,dt,$$

and if $\lambda_1\geq\lambda_2\geq\cdots$ and $\varphi_1, \varphi_2, \cdots$ are the eigenvalues and normalized eigenfunctions of the integral equation on $(-1, 1)$ with the symmetric kernel $K(x, y)$, then

$$(*)\qquad q(x_0, x, t) = \sum_1^\infty e^{-t/\lambda_j}\phi_j(x_0)\phi_j(x).$$

This is well known for the Wiener process, and was proved for $\alpha=1$ by Kac and Pollard [Canad. J. Math. **2** (1950), 375–384; MR **12**, 114]. If T is the first passage time of the process $x_0+x(t)$ to the exterior of the open interval $(-1, 1)$, then the joint distribution of T and the place of first passage can be expressed using $q(x_0, x, t)$.

Let $k(x)$ be an even probability density function such that for some α, $0<\alpha\leqq 2$, and some c, $0<c<\infty$,

$$(**)\qquad \lim_{\xi\to 0}\,[1-k\hat{}(\xi)]/|\xi|^\alpha = c.$$

Denote by $\mu_{1,A}\geqq\mu_{2,A}\geqq\cdots$ and $\psi_{1,A}, \psi_{2,A}, \cdots$ the positive eigenvalues and the corresponding normalized eigenfunctions of the integral equation

$$A\int_{-1}^{1} k(A(x-y))\psi(y)\,dy = \mu\psi(x), \quad -1\leqq x\leqq 1.$$

It is proved that $\mu_{j,A}=1-c\lambda_j^{-1}A^{-\alpha}+o(A^{-\alpha})$ as $A\to\infty$, and that in a certain sense the $\psi_{j,A}$'s converge to the ϕ_j's. The argument is based on the result described above and a theorem of Kimme [Trans. Amer. Math. Soc. **84** (1957), 208–229; MR **18**, 770]. This result, suitably modified, asserts the following. (It is assumed here that $c=1$.) Let $X_1, X_2, \cdots$ be independent random variables having the same density function k satisfying (**). Let m be a positive integer and $-1<x_0, y<1$; then

$$\lim_{A\to\infty}\Pr\{\max_{i\leq[A^\alpha]m}|x_0+A^{-1}S_i|<1, x_0+A^{-1}S_{[A^\alpha]m}\leqq y\}=$$
$$\Pr\{\max_{t\leq m}|x_0+x(t)|<1, x_0+x(m)\leqq y\}.$$

Here $S_i=X_1+\cdots+X_i$, and $[A^\alpha]$ is the largest integer not exceeding A^α. *I. I. Hirschmann, Jr.* (Erlenbach)

12612:

Kastenbaum, Marvin A. The separation of molecular compounds by countercurrent dialysis: a stochastic process. Biometrika **47** (1960), 69–77.

In this paper the author formulates a stochastic model for a m-stage dialysis system. Each dialysis cell is termed a stage, and each dialysis period is called a cycle. Let p be the probability that, at any stage, a particle inside the sac will not penetrate to the outside volume, and $(1-p)$ the probability that it will. It is assumed that, once outside, the particle cannot return inside, except by manual transfer. The problem is to determine the probability with which a particle of the isolate will be at a specified stage in the system after a given number of cycles have been carried out. The matrix of diffusion probabilities for the m stages, denoted by D, is a square matrix of order $2m$, with zeros everywhere except on the diagonal; and these elements are of the form

$$D_j=\begin{bmatrix} p & (1-p)\\ 0 & 1\end{bmatrix}\quad (j=1, 2, \cdots, m).$$

Another square matrix of order $2m$ is introduced, called the transfer matrix, and denoted by T, the elements of which are either zero or one. The value of these elements is determined as follows. At the end of each cycle: (i) a particle on the inside at stage 1 will remain on the inside of stage 1 with probability 1; (ii) a particle on the inside at stage j ($j=2, 3, \cdots, m$) will be transferred to the inside of stage $(j-1)$ with probability 1; (iii) a particle on the outside of stage j ($j=1, 2, \cdots, m-1$) will be transferred to the inside of stage $(j+1)$ with probability 1; and (iv) a particle m on the outside at stage m will remain on the outside of stage m with probability 1.

The product DT is a stochastic matrix, the elements of which are the conditional probabilities that a particle will make a transition from one of the m stages to another. It is shown that the matrix DT can be reduced to a matrix **DT** of order $m+1$. The Markov chain involved can be considered as a random-walk process on the integers $1, 2, \cdots, m+1$, with transition probabilities $p_{11}=p$, $p_{12}=1-p$; $p_{i,i-1}=p$, $p_{i,i+1}=1-p$ for $i=2, \cdots, m$; and $p_{m+1,m+1}=1$. Hence the first stage is a reflecting barrier, and $(m+1)$st state is an absorbing barrier. The main part of the paper is concerned with the algebraic reduction of the matrix $(\mathbf{DT})^n$, firstly for a 3-stage system, and then for the general m-stage system.

A. T. Bharucha-Reid (Eugene, Ore.)

12613:

Lyon, Richard H. Response of a nonlinear string to random excitation. J. Acoust. Soc. Amer. **32** (1960), 953–960.

12614:

Ganguly, S. The application of the saddle point method of integration to shower problems of cosmic ray physics and the accuracy of the method. Proc. 4th Congress Theoret. Appl. Mech. 1958, pp. 299–303. Indian Soc. Theoret. Appl. Mech., Kharagpur.

Author's summary: "The solution for the cascade functions in cosmic-ray showers is of the general form

$$I=(2\pi i)^{-1}\int_{\sigma-i\infty}^{\sigma+i\infty} e^{F(s)}\,ds.$$

The contour integration in a closed form by method of residues is not in general practicable and so the integrals are evaluated approximately by the saddle-point method to yield values of the type

$$I=\frac{e^{F(s_0)}}{\sqrt{(2\pi F''(s_0))}},$$

where s_0 is the saddle point. In general, the accuracy of the approximation is more than 90 percent, and this has been demonstrated by evaluating the Bessel function integral

$$J_\nu(iz)=\frac{(\frac{1}{2}iz)^\nu}{2\pi i}\int_{c-i\infty}^{c+i\infty} t^{-\nu-1}\exp\left\{t+\frac{z^2}{4t}\right\}dt$$

by the saddle-point method and comparing the results with tabulated values." *H. Messel* (Sydney)

12615:

Gardner, J. W. Markoff cascades with general source terms. Nuovo Cimento (10) **16** (1960), 977–990. (Italian summary)

The author extends the solution of the well-known integro-differential equations appearing in cosmic-ray shower-theory for a point source to the case of an extended source. *H. Messel* (Sydney)

12616:

Cucconi, Odoardo. L'aggiotaggio delle quote del totalizzatore. I, II. Archimede **12** (1960), 81–85, 135–141.

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See also 12564, 12694, 12752, 13276.

12617:

Weatherburn, C. E. ★A first course in mathematical statistics. Cambridge University Press, New York, 1961. xv+277 pp. $2.75.

This is a reprint of the 2nd edition (1949) which differed from the 1st edition [1946; reviewed in MR 8, 392] only in the correction of a few errors and misprints, the addition of new references and of a section on the distribution of the range of a sample.

12618:

Blyth, Colin R.; Hutchinson, David W. Table of Neyman-shortest unbiased confidence intervals for the binomial parameter. Biometrika **47** (1960), 381–391.

The paper gives the tables of Neyman-shortest unbiased confidence intervals for the binomial parameter for the conventional values of $\alpha=0.95$ and $\alpha=0.99$. The values of n are from 2 to 24 and then from 26 to 50 at the interval of two.

The tables are based on Eudey's results [Tech. Report No. 13, Stat. Lab., Univ. of California, Berkeley (1949)]. Comparison with existing tables is given and the method of constructing the tables is also explained. Similar work for the Poisson parameter is announced.

V. S. Huzurbazar (Poona)

12619:

Silverman, Richard A. The fluctuation rate of the chi process. IRE Trans. **IT-4** (1958), 30–34.

Author's summary: "The chi process is defined as a natural generalization of the chi distribution of statistical theory. A formula is derived for the expected number of level crossings per second of the chi process. The formula contains as a special case the familiar expression for the fluctuation rate of the envelope of Gaussian noise."

12620:

Cohen, A. Clifford, Jr. Estimation in the truncated Poisson distribution when zeros and some ones are missing. J. Amer. Statist. Assoc. **55** (1960), 342–348.

Author's summary: "This paper is concerned with maximum likelihood estimation of the Poisson parameter λ when the zero class has been truncated and when errors of observation have resulted in eliminating some though not necessarily all of the one class. Estimators are derived both for λ and the proportion θ of ones eliminated as a consequence of faulty observation. A table and a graph of the estimation function involved are given. Asymptotic variances and covariances of the estimators are derived, and a table of the variance function applicable in calculating $V(\hat{\lambda})$ is given. An illustrative example is included."

L. E. Moses (Stanford, Calif.)

12621a:

Lazarsfeld, Paul F. The algebra of dichotomous systems. Studies in item analysis and prediction, pp. 111–157. Stanford Univ. Press, Stanford, Calif., 1961.

12621b:

Bahadur, R. R. A representation of the joint distribution of responses to n dichotomous items. Studies in item analysis and prediction, pp. 158–168. Stanford Univ. Press, Stanford, Calif., 1961.

12621c:

Bahadur, R. R. On classification based on responses to n dichotomous items. Studies in item analysis and prediction, pp. 169–176. Stanford Univ. Press, Stanford, Calif., 1961.

Let $X=(X_1, \cdots, X_n)$ be a random vector, each component being either 0 or 1. Then 2^n-1 parameters are required to specify the distribution of X. For purposes of describing data Lazarsfeld considers a set of "symmetric" parameters which are slightly modified for analytic purposes by Bahadur. He uses $a_i=EX_i$, $r_{ij\cdots m}=EZ_iZ_j\cdots Z_m$, where $i<j<\cdots<m$ and

$$Z_i = (X_i-a_i)/\sqrt{(a_i(1-a_i))}.$$

As important applications Lazarsfeld considers latent-structure analysis and Bahadur the classification problem.

Bahadur uses $(1, Z_1, \cdots, Z_n, Z_{12}, \cdots, Z_{n-1\ n}, \cdots, Z_{n\ n-1\cdots 1})$ as a basis in the space of real-valued functions on the values of X. The r's then serve as coefficients in the expansion of the distribution function of X. A necessary condition for a set of numbers to be these coefficients is given. When the distribution of X is symmetric, e.g., the probability of X is the same as the probability of X', where X' is X with some permutation of its coordinates, then all r's with the same number of subscripts are equal, the a's are equal, there are n parameters, and $t=\sum_{i=1}^n X_i$ is a sufficient statistic.

I. R. Savage (Minneapolis, Minn.)

12622:

Lindley, D. V.; East, D. A.; Hamilton, P. A. Tables for making inferences about the variance of a normal distribution. Biometrika **47** (1960), 433–437.

It is pointed out that both the Neyman-Pearson theory and the Bayesian theory of statistics sometimes call for numerical solution of the following problem: For a given integer n and a small, positive fraction α, find v_1 and v_2 such that the probability of the interval (v_1, v_2) under the χ^2-distribution with n degrees of freedom is $1-\alpha$ and such the density of the χ^2-distribution with $n+2$ degrees of freedom is the same at v_1 as at v_2. The solution is tabulated for $n=1(1)100$ and $\alpha=5\%$, 1%, and 0.1%. Discrepancies between this and some earlier tables are mentioned: p. 223 of *Advanced statistical methods in biometric research* by C. R. Rao [Wiley, New York, 1952; MR **14**, 388] and the article by R. F. Tate and G. W. Klett in J. Amer. Statist. Assoc. **5** (1959), 674–682 [MR **21** #6648].

L. J. Savage (Ann Arbor, Mich.)

12623:

Roy, A. D. Some notes on pistimetric inference. J. Roy. Statist. Soc. Ser. B **22** (1960), 338–347.

The notions of "pistimetric inference" are closely related to those of fiducial inference. In particular, the author discusses "pistimetric distributions" of population parameters in a variety of specific situations. These distributions are obtained by the use of pivotal quantities in

much the same way that fiducial distributions are obtained. Posterior probability distributions of parameters for given values of certain random variables (corresponding to experimental results) are obtained by the use of Bayes' theorem. The ideas of pistimetric inference are extended to several two-stage sample situations. No attempt is made to develop a general theory apart from the particular examples considered. *S. S. Wilks* (Princeton, N.J.)

12624:

Hájek, Jaroslav. Optimum strategy and other problems in probability sampling. Časopis Pěst. Mat. **84** (1959), 387–423. (Czech and Russian summaries)

To the ith element of a finite population S of size N there is associated an unknown number y_i, and the problem is to estimate $Y=\sum_{i=1}^{N} y_i$ by means of a sample $s\subset S$. A sampling scheme assigns to each $s\subset S$ a probability $\mathrm{P}(s)$, and the author considers only linear estimators of the form $\sum y_i w_i(s)$, where the summation is taken over all $i\in s$. He gives a formula for obtaining the estimated variance of these estimators and shows how these estimators can be improved by means of conditioning with sufficient statistics. For a given cost of sampling and a given sampling budget, "best" linear, unbiased estimators are derived—"best" in the sense of minimizing the expected sampling variance where the expectation is taken with respect to prior distributions on the y_i's. He considers two prior distributions: (1) the y_i's are non-correlated random variables; (2) the y_i's have a stationary convex correlation function and stationary coefficients of variations. *H. Raiffa* (Cambridge, Mass.)

12625:

Robson, D. S.; Vithayasai, C. Unbiased componentwise ratio estimation. J. Amer. Statist. Assoc. **56** (1961), 350–358.

The Hartely-Ross unbiased ratio estimator of the population total Y [H. O. Hartely and A. Ross, Nature **174** (1954), 270–271] is given by $Y'=X\bar{r}+n(N-1)\times(\bar{y}-\bar{x}\bar{r})/(n-1)$, where $\bar{r}$ is the mean value of y/x in a random sample of size n from a population of size N and X is the population total for x. If there are k strata, the authors propose the estimator

$$Y' = \sum Y_i' = \sum X_i\bar{r}_i+n(N-1)(\bar{y}_i-\bar{x}_i\bar{r}_i)/(n-1),$$

where summation is over the strata. The variance of Y' is given by $\sum \operatorname{var}(Y_i')+2\sum \operatorname{cov}(Y_i', Y_j')$ and its computation is indicated. Example: Component-wise ratio estimation of corn plot total dry weight.

G. Tintner (Ames, Iowa)

12626:

Cohen, A. Clifford, Jr. Estimating the parameter in a conditional Poisson distribution. Biometrics **16** (1960), 203–211.

Tables and graphs are given for the calculation of the maximum likelihood estimate, and an estimate of the asymptotic variance of λ, the parameter of a Poisson distribution truncated away from zero. The author compares his work with that of Hartley [Biometrics **14** (1958), 174–194] and Irwin [ibid. **15** (1959), 324–326].

R. F. Tate (Seattle, Wash.)

12627:

Cohen, A. Clifford, Jr. An extension of a truncated Poisson distribution. Biometrics **16** (1960), 446–450.

The author proposes a model for the distribution of organisms among colony sites with no migration between colonies. He lets ϑ be the probability that a particular colony site is chosen; once a site is chosen, the number of organisms, x, to be found there obeys a Poisson frequency law with the zero class missing. He obtains the likelihood equations for the estimation of ϑ and the Poisson parameter λ from a sample of N observations of x which are readily solved by means of a table and chart in a previous paper of the author [#12626]. The variance of the two estimates is found from the asymptotic variance-covariance matrix. A numerical example is given in which the proposed model is very successful, particularly in comparison with other models which had been tried with the same experimental data. *C. C. Craig* (Ann Arbor, Mich.)

12628:

Gjeddebaek, N. F. Contribution to the study of grouped observations. V. Three-class grouping of normal observations. Skand. Aktuarietidskr. **1959**, 194–207 (1960).

[For part IV, see Biometrics **15** (1959), 433–439; MR **21** #6652.] This paper is concerned with estimates of the mean and standard deviation of a normal population based on grouped observations when there are only the three cells $(-\infty, D_1)$, (D_1, D_2), and (D_2, ∞). Computations indicating the asymptotic efficiency of m.l. estimates of the mean based on the grouped data are compared with distributions, based on samples of size 40, of (a) the error in the m.l. estimate of the mean and of (b) the error in the m.l. estimate of the mean divided by the m.l. estimate of the standard deviation. The latter distributions fit their asymptotic approximations very well. The efficiency is surprisingly high, being 80% if D_1 and D_2 are both 0.6 standard deviations away from the mean.

H. Chernoff (Stanford, Calif.)

12629:

Levin, Morris J. Optimum estimation of impulse response in the presence of noise. IRE Trans. **CT-7** (1960), 50–56.

In physical problems one rarely has a complete statistical description of the noise. A practical solution to the problem of estimating parameters of a digital system corrupted by noise is generally restricted to a consideration of linear estimates. The author discusses the optimum estimation of impulsive responses by linear estimates which are claimed to be well-suited to digital computers or desk calculators. *K. S. Miller* (New York)

12630:

Ogawa, Junjiro. Determination of optimum spacings for the estimation of the scale parameter of an exponential distribution based on sample quantiles. Ann. Inst. Statist. Math. **12** (1960), 135–141.

A formula is obtained for computing the maximum likelihood estimate of the scale parameter σ of the exponential distribution, $g(x)=(1/\sigma)e^{-x/\sigma}$, $x>0$, $g(x)=0$ otherwise, based on a fixed number k of quantiles from a large sample. A table is given of numerical values of the quantiles and the corresponding coefficients, for $k=1(1)15$.

The method is applied to data on the time intervals between fatal mine explosions.

E. S. Keeping (Edmonton, Alta.)

12631:

Birnbaum, Allan. Confidence curves: an omnibus technique for estimation and testing statistical hypotheses. J. Amer. Statist. Assoc. 56 (1961), 246–249.

This note describes, advocates and illustrates the use of confidence curves as an "omnibus technique" for problems of statistical estimation and testing hypotheses. A more extensive presentation appears in Ann. Math. Statist. **32** (1961), 112–135.

H. Teicher (New York)

12632:

Aivazian, S. A. Comparison of the optimal properties of the Wald and Neyman-Pearson criteria. Theor. Probability Appl. **4** (1959), 83–89.

English translation of Teor. Veroyatnost. i Primenen. **4** (1959), 86–93 [MR **21** #942].

12633:

Goodman, Leo A. A note on Stepanov's tests for Markov chains. Theor. Probability Appl. **4** (1959), 89–92.

Republication of Teor. Veroyatnost. i Primenen. **4** (1959), 93–96 [MR **21** #1682]. The summary is translated into English.

12634:

Vere-Jones, D.; Kendall, David G. A commutativity problem in the theory of Markov chains. Theor. Probability Appl. **4** (1959), 92–95.

Republication of Teor. Veroyatnost. i Primenen. **4** (1959), 97–100 [MR **21** #1642]. The summary is translated into English.

12635:

Kovalenko, I. N. On a class of optimal decision functions for a binomial family of distributions. Theor. Probability Appl. **4** (1959), 95–99.

English translation of Teor. Veroyatnost. i Primenen. **4** (1959), 101–105 [MR **21** #407].

12636:

Cochran, William G.; Hopkins, Carl E. Some classification problems with multivariate qualitative data. Biometrics **17** (1961), 10–32.

The authors discuss the problem of assigning specimens to one or two more universes when the measurements on each specimen are qualitative rather than quantitative. An optimum rule for classifying the specimens is first presented: (1) the effect of the initial sample sizes on the performance of the proposed classification rule; (2) the relative discriminating power of qualitative and continuous variates; (3) use of classification experience for improvement of the rule.

P. S. Dwyer (Ann Arbor, Mich.)

12637:

Bennett, B. M. On a certain multivariate non-normal distribution. Proc. Cambridge Philos. Soc. **57** (1961), 434–436.

The general class of multivariate distributions having density functions given by

$$f_{n,p}(\mathbf{x}, \boldsymbol{\theta}, \Lambda) = K\left\{1+\frac{1}{n}(\mathbf{x}-\boldsymbol{\theta})'\Lambda(\mathbf{x}-\boldsymbol{\theta})\right\}^{-(n+p)/2}$$

is introduced and certain regression and estimation properties of these distributions are discussed.

R. Pyke (Seattle, Wash.)

12638:

Elfving, Gustav; Sitgreaves, Rosedith; Solomon, Herbert. Item-selection procedures for item variables with a known factor structure. Studies in item analysis and prediction, pp. 64–80. Stanford Univ. Press, Stanford, Calif., 1961.

Except for its appendix, this paper has already been published [Psychometrika **24** (1959), 189–205; MR **21** #6054]. The appendix merely points out that the theory underlying the methods of the present paper is more fully developed in the paper mentioned.

S. S. Wilks (Princeton, N.J.)

12639:

Freund, Rudolf J.; Vail, Richard W.; Clunies-Ross, C. W. Residual analysis. J. Amer. Statist. Assoc. **56** (1961), 98–104.

The residuals resulting from the application of a least squares regression equation can themselves be analyzed as originally observed quantities. If this analysis is also a regression analysis, the process can be described as a two-stage regression. The authors show that such a two-stage regression is not the equivalent of a one-stage regression in which all the predictors are used at once. General matrix formulas are provided which show the amount of bias, due to the residual analysis, of the regression coefficients, of the sum of squares due to regression, and of the residual variance. Considerable simplification of the results appears when only one variable is used in the second state. This is the situation studied by Goldberger and Jochems in a paper [#12640] which was published in the same issue of J. Amer. Statist. Assoc. with an editors' note to the effect that there are sufficient differences of emphasis and exposition to warrant publishing both papers.

P. S. Dwyer (Ann Arbor, Mich.)

12640:

Goldberger, Arthur S.; Jochems, D. B. Note on stepwise least squares. J. Amer. Statist. Assoc. **56** (1961), 105–110.

The residuals $\tilde{y}$ which result from the least-squares linear regression of y on x_1 are used as the values of a new linear least-squares regression of $\tilde{y}$ on x_2. The relation between these results and the results of the least-squares linear regression of y on x_1 and x_2 is studied. It is shown that the stepwise procedure underestimates both the regression coefficient of x_2 and the marginal contribution of x_2 to the explanation of y. Explicit formulas feature $1-r_{12}^2$, as does a formula for the significance requirement that b_2 be different from zero. A numerical illustration is presented. The results are generalized to the case of the contribution of an explanatory variable x_{k+1} in which the formulas feature $1-R^2$, where R is the multiple correlation of x_k with the other x's. The problem of this paper is a

special case of the problem studied by Freund, Vail, and Clunies-Ross in a paper [#12639] which was published in the same issue of J. Amer. Statist. Assoc. with an editors' note to the effect that there are sufficient differences of emphasis and exposition to warrant publishing both papers. *P. S. Dwyer* (Ann Arbor, Mich.)

12641:

Ludwig, O. Über Erwartungswerte und Varianzen von Ranggrössen in kleinen Stichproben. Metrika **3** (1960), 218–233. (English summary)

A distribution-free upper limit for the expectation of the differences of order statistics in terms of the population standard deviation, the sample size n and ranks is obtained from the Schwarz inequality. The exact values given up to $n=5$ for the normal logistic, exponential distributions and several cases of the function $F(x)=x^n$ $(0<x<1)$ differ only by small percentages from the upper limit. The expectations of the distributions between consecutive-order statistics and Plackett's formula [Biometrika **34** (1947), 120–122; MR **8**, 395] are special cases of this inequality. It is shown that the expectation of any order statistic in a sample of size n can be computed from the expectation of the smallest value in samples up to n. This is illustrated by the use of the Gamma distribution. *E. J. Gumbel* (New York)

12642:

Savage, I. Richard. Contributions to the theory of rank order statistics: computation rules for probabilities of rank orders. Ann. Math. Statist. **31** (1960), 519–520.

This paper gives two rules for computing the probabilities of rank orders. The first rule permits the computation, for sample s of size n, from the results of samples of size $n+1$. The second rule permits the computation for samples of size m and n from the results of samples of $m+1$ and n, or m and $n+1$. When probabilities of rank orders are computed analytically, these rules may have use for checking the numerical work. When the probabilities are obtained by sampling, these rules may be used to extend the results. *D. Teichroew* (Stanford, Calif.)

12643:

Tamura, Ryoji. On the nonparametric tests based on certain U-statistics. Bull. Math. Statist. **9** (1960), no. 2/3, 61–67.

If X and X' [Y and Y'] are a pair of observations from the first [second] population, score 1 if $\min(Y, Y')<\min(X, X')$ and $\max(X, X')<\max(Y, Y')$, e.g., the X's are between the Y's, and score 0 otherwise. A test statistic is formed by adding the scores of all possible pairs where the observations have been adjusted by subtracting off either the known values of the medians or the observed values of the medians. The statistic is used for testing the hypothesis that two populations have the same scale parameters versus the alternative that their scale parameters differ. The procedures are quite efficient; see Ansari and Bradley, Ann. Math. Statist. **31** (1960), 1174–1189 [MR **22** #8609]. *I. R. Savage* (Cambridge, Mass.)

12644:

Sukhatme, Balkrishna V. Power of some two-sample non-parametric tests. Biometrika **47** (1960), 355–362.

The tests, described by Rosenbaum [Ann. Math. Statist. **24** (1953), 663–668; MR **15**, 450] and Kamat [Biometrika **43** (1956), 377–387; MR **18**, 774], are for common scale given common location. Exact distributions and numerical results (normal alternatives) are given for small samples. *I. R. Savage* (Cambridge, Mass.)

12645:

Fisz, M. On a result by M. Rosenblatt concerning the von Mises-Smirnov test. Ann. Math. Statist. **31** (1960), 427–429.

Let x_k $(k=1, \cdots, n)$ and y_j $(j=1, \cdots, m)$ be two independent random samples from two populations with the same continuous distribution functions $F(t)$. Let $S_1(t)$ and $S_2(t)$ be the corresponding empirical distribution functions. The reviewer [same Ann. **23** (1952), 617–623; MR **14**, 665] proved that

$$(mn/(m+n)) \int_{-\infty}^{\infty} [S_1(t)-S_2(t)]^2 d[(nS_1(t)+mS_2(t))/(n+m)]$$

has the same limiting distribution when $m, n\to\infty$, $m/n\to\lambda>0$ as the von Mises-Smirnov statistic $n\times \int_{-\infty}^{\infty} [S(t)-F(t)]^2 dF(t)$. The author points out an incorrect step in the reviewer's derivation and indicates how it can be corrected. *M. Rosenblatt* (Providence, R.I.)

12646:

Walsh, John E. Probabilities for Cramér-von Mises-Smirnov test using grouped data. Ann. Inst. Statist. Math. **12** (1960), 143–145.

12647:

Paulson, Edward. A non-parametric solution for the k-sample slippage problem. Studies in item analysis and prediction, pp. 233–238. Stanford Univ. Press, Stanford, Calif., 1961.

The author's problem [Ann. Math. Statist. **23** (1952), 610–616; MR **14**, 569] is modified to the nonparametric case. The solution involves the theory of locally most powerful rank procedures, particularly the case of normal alternatives described by Terry [ibid., 346–366; MR **14**, 190]. *I. R. Savage* (Cambridge, Mass.)

12648:

Johns, Milton Vernon. An empirical Bayes approach to non-parametric two-way classification. Studies in item analysis and prediction, pp. 221–232. Stanford Univ. Press, Stanford, Calif., 1961.

An individual is to be put into one of two categories on the basis of an observed value X. If he is correctly classified in terms of a future observable Y, there is no loss, and, if he is misclassified, the loss is $L_i(Y)$, where $i=1(2)$ if he is wrongly put into class 1(2). Starting with no information about the joint distribution, rules based on accumulating information are devised so that the expected risk per decision converges in probability to the expected risk with the best rule, given complete knowledge of the joint distribution of X and Y. Interesting but involved rules arise when the Y values are not observed for individuals placed in one of the categories.

I. R. Savage (Cambridge, Mass.)

12649:

Laha, R. G.; Lukacs, E. On a problem connected with quadratic regression. Biometrika **47** (1960), 335–343.

Let $X_1, X_2, \cdots, X_n$ be a sample of n independently and identically distributed random variables. Let $\Lambda = \sum_{i=1}^n X_i$ and $Q = \sum_{i=1}^n \sum_{j=1}^n a_{ij}X_iX_j + \sum_{i=1}^n b_iX_i$, where the a's and b's are given constants. The authors study all the populations for which the regression of Q on Λ is quadratic, i.e., the conditional expectation of Q, given Λ, has the form $\beta_0+\beta_1\Lambda+\beta_2\Lambda^2$. They distinguish several cases which are defined in terms of relations between the coefficients a_{ij}, b_i, β_0, β_1 and β_2. In each case they show that the population is characterised by the property they mention. A typical result is the following.

Suppose that the population has finite variance σ^2 and that

$$A = n \sum_{i=1}^n a_{ii} - \sum_{i=1}^n \sum_{j=1}^n a_{ij} \neq 0,$$

$$\beta_1 = n^{-1} \sum_{i=1}^n b_i, \quad \beta_2 = n^{-2} \sum_{i=1}^n \sum_{j=1}^n a_{ij}.$$

Then the relation

$$E(Q|\Lambda) = \beta_0+\beta_1\Lambda+\beta_2\Lambda^2$$

holds almost everywhere if and only if $\beta_0 = n^{-1}\sigma^2 A$, and the population is normal. *J. Durbin* (London)

12650:

Harter, H. Leon. On the analysis of split-plot experiments. Biometrics **17** (1961), 144–149.

This is quite a refreshing study of appropriate tests of significance in split-plot experiments of the randomised block type under different conditions regarding the randomness or fixedness of the factors of interest. The almost automatic procedure of pooling sum of squares for the interaction, sub-plots × replication, and that for the interaction whole-plot × subplot × replication has been critically examined, and definite procedures have been laid down for guidance as to whether such pooling is justified or not. For this purpose a distinction has been made as to whether the whole-plot effect and the subplot effect are fixed or random variables in the replications. All the four cases thus arising have been dealt with. It has been assumed that the experimental errors of pairs of plots have a correlation ρ or 0, according as they belong to the same block or to different blocks. The replication effect (the significance of which is not usually a subject of testing) has been assumed to be a random factor.

The problem has been clearly stated and neatly handled, and tables of expected mean squares have been appended for ready reference of practical workers. A useful list of reference has also been appended. Indications are given for extension to more factors, where the factors are introduced one by one in the ensuing splits. This paper will prove very suitable for theoretical and practical instruction of advanced students in statistics.

Q. M. Husain (Dacca)

12651:

Turner, Malcolm E.; Monroe, Robert J.; Lucas, Henry L., Jr. Generalized asymptotic regression and non-linear path analysis. Biometrics **17** (1961), 120–143.

12652:

Raghavarao, Damaraju. A generalization of group divisible designs. Ann. Math. Statist. **31** (1960), 756–771.

The author extends the discussion of group-divisible incomplete block designs to m-associate classes, using a definition which, in spite of formal similarity with Bose and Nair's definition of a P.B.I.B. design, is radically different from it in the conception of "associates" of a treatment. While Bose and Nair's emphasis is on the structure of the design, the author here designates varieties conventionally by means of a number of suffixes which need not necessarily correspond with the design-characteristics, such as factors or levels of factors as in factorial experiments. But this definition has been fitted nicely into certain hierarchies of the group-divisible designs.

The scheme has been well worked out and its uniqueness and consistency established, and the usual formulae for the P.B.I.B. designs are shown to be applicable here also, so that this design may be regarded as a special class of P.B.I.B. designs. Special properties of this design are worked out and previous results extended by the application of matrices, determinants, Legendre symbol, Hilbert's norm residue symbol and Hasse-Minkowsky invariants.

Finally, ten examples are given in the appendix. These involve three and four associate classes, and help in acquiring familiarity with the nature of the designs.

{The only weak point noticed is a loose sentence under Def. 2.1 (ii) which reads, "Each treatment occurs once in each of the r blocks." This is rather ambiguous and may raise questions like, "Which r blocks are meant?" or, "How can each (=every) treatment occur in any block?", and so on. The reviewer therefore suggests the substitution of the following sentence: "Each treatment is replicated r times." This is quite sufficient, as earlier it has been stated that no treatment can occur more than once in a block.}

Q. M. Husain (Dacca)

12653: [de Witte, P.]

De Witte, P.; Vandewiele, L. Maximum likelihood estimate of a straight line when both coordinates are subject to gaussian errors. Bull. Soc. Math. Belg. **11** (1959), 116–122. (Dutch)

The authors discuss the problem of estimating a straight line from a number of measurements (independent from each other):

$$x_1, y_1, \quad x_2, y_2, \quad \cdots, \quad x_n, y_n,$$

when both variables are from normal distributions and apply the principle of maximum likelihood.

This method gives the same equations as the more geometrical method of P. A. Wayman [Nature **184** (1959), 77–78] and the generalized least squares method of W. E. Deming [*Statistical adjustment of data*, Wiley, New York, 1943; MR **5**, 208; pp. 178–182].

Hian Liang Ang (Bandung)

12654:

Glenn, W. A. A comparison of the effectiveness of tournaments. Biometrika **47** (1960), 253–262.

Six tournament schemes are investigated for their effectiveness in selecting the best one of four players on the basis of games between two contestants (round robin and five types of elimination tournament, with playoffs in case of ties). The probability that a specified player wins, and the expected number of games played, are evaluated

in terms of the probabilities $\pi_{ij}(=1-\pi_{ji})$ that player i defeats player j, on the assumption that the outcomes of games are statistically independent. A series of numerical examples is given, and special consideration is given to the case: $\pi_{1j}=\pi>\frac{1}{2}$ $(j=2,\ 3,\ 4)$, $\pi_{23}=\pi_{24}=\pi_{34}=\frac{1}{2}$. The results may be interpreted in terms of the design of paired-comparison experiments.

J. R. Rosenblatt (Arlington, Va.)

12655:

Folks, John Leroy; Kempthorne, Oscar. The efficiency of blocking in incomplete block designs. Biometrika **47** (1960), 273–283.

Using a randomization model [H. Scheffé, *The analysis of variance*, Wiley, New York, 1959; MR **22** #7217] the variance of the estimate of a given treatment difference in a randomized block design may be compared with the variance of the similar estimate made from a completely randomized design. This type of comparison is here extended to general incomplete block designs and is distinguished from the loss of information due to confounding. (The symbol E is given three different meanings in the paper.)

H. L. Seal (New Haven, Conn.)

12656:

Corsten, L. C. A. Proper spaces related to triangular partially balanced incomplete block designs. Ann. Math. Statist. **31** (1960), 498–501.

This paper considers the properties of incidence matrices and other matrices arising in connection with the study of the structure of combinatorial designs, particularly of the triangular type. The $v\times v$ symmetrical matrix arising from the multiplication of the incidence matrix with its transpose and certain submatrices have been studied as sets of vectors; and their proper values and spaces have been investigated, leading to certain necessary conditions to be satisfied by the parameters of an actual design. {This is a neat and interesting paper.}

Q. M. Husain (Dacca)

12657:

Graybill, Franklin A.; Seshadri, V. On the unbiasedness of Yates' method of estimation using interblock information. Ann. Math. Statist. **31** (1960), 786–787.

This paper may be considered as a supplement to Graybill and Weeks, same Ann. **30** (1959), 799–805 [MR **21** #5260]. It shows, using the notation of op. cit. and an argument of similar character, that Yates' combined estimate of a treatment contrast in a balanced incomplete block design (with normal errors for both blocks and plots) is unbiased.

L. E. Moses (Stanford, Calif.)

12658:

Schwarz, Gideon. A class of factorial designs with unequal cell-frequencies. Ann. Math. Statist. **31** (1960), 749–755.

A class of "almost" symmetrical factorial experiments is considered. For a two-factor $(I\times J)$ experiment, the (i,j) factor combination is used on either n or $n+1$ experimental units, depending on whether the (i,j) element in a certain matrix, $A(I\times J)$, is 0 or 1. It is assumed that both I and J are divisible by a given integer, d. We start with a $(d\times d)$ unit matrix and replace each diagonal entry by an $(I/d\times J/d)$ matrix of "ones" and each non-diagonal entry by a similar matrix of zeros; the final $(I\times J)$ matrix is A. The least squares estimates of the main effects, assuming a no-interaction model, are derived. The reader should be cautioned on notation, because U is used for the unit matrix and because I is used for the number of levels of one of the factors. Unfortunately, the author does not present formulas for the variances of differences between estimated effects. Similar results are obtained for $n+1$ replaced by $n=1$ above.

An extension is made to the case of q factors when the number of levels is the same for all factors.

R. L. Anderson (Raleigh, N.C.)

12659:

Kishen, K.; Srivastava, J. N. Mathematical theory of confounding in asymmetrical and symmetrical factorial designs. J. Indian Soc. Agric. Statist. **11** (1959), 73–110.

From the authors' summary: "The method of finite geometries has been extended to the construction of balanced confounded asymmetrical factorial designs by using curvilinear spaces or hypersurfaces and truncating the $EG(m,s)$ suitably. A more general method, using vectors in Galois fields, has also been introduced and a unified theory for the construction of both symmetrical and asymmetrical factorial designs developed. Symmetrical confounded factorial designs s^m, where s is not a prime number or a prime power, as also most all types of asymmetrical factorial designs, can be constructed. Methods of deriving symmetrical and asymmetrical factorial designs, using the balanced incomplete block property, have also been given, besides methods of reducing the number of replications required for balancing in asymmetrical designs and of deriving balanced designs of the type $a_1s_1\times a_2s_2\times\cdots\times a_ms_m$ from a given $s_1\times s_2\times\cdots\times s_m$ design. Finally, two methods of analysis designs have been briefly discussed."

J. Kiefer (Ithaca, N.Y.)

12660:

Bankier, J. D. An operational approach to the r-way crossed classification. Ann. Math. Statist. **31** (1960), 16–22.

An operational method based on Mann's notation is used to obtain known formulas for the expected values of the mean squares and the variances of estimates of variance components obtained from the analysis of variance of an r-way crossed classification with equal replication of all classes. Results are presented for the so-called fixed effects model (Model I), the random model (Model II) and the finite population model (Model III). The results are independent of normality assumptions, but the simplifications under these assumptions are indicated. {This reviewer would like to encourage the development of these or similar methods for non-balanced situations.}

R. L. Anderson (Raleigh, N.C.)

12661:

Raiffa, Howard. Statistical decision theory approach to item selection for dichotomous test and criterion variables. Studies in item analysis and prediction, pp. 187–220. Stanford Univ. Press, Stanford, Calif., 1961.

A detailed exposition of the two-state, two-action

problem with finite sample space is presented. Comparison of experiments, sequential sub-optimal procedures, randomization, and varying costs of observations are considered. *I. R. Savage* (Cambridge, Mass.)

12662:

Weiss, Lionel. Statistical decision theory in engineering. Information and decision processes, pp. 170–177. McGraw-Hill, New York, 1960.

Elementary statement of statistical decision theory, with applications to industrial engineering problems.

M. M. Flood (Ann Arbor, Mich.)

12663:

Налимов, В. В. [Nalimov, V. V.]. ★Применение математической статистики при анализе вещества [Application of mathematical statistics to analysis of matter]. Fiziko-Matematičeskaya Biblioteka Inženera. Gosudarstv. Izdat. Fiz.-Mat. Lit., Moscow, 1960. 430 pp. 12.95 r.

The book presents a systematic and generalized presentation of works on the subject of elementary statistical analysis useful in the experimental laboratory works. The basic premises of mathematical statistics are not proven, but rather explained using examples from the works connected with the analysis of matter.

R. M. Evan-Iwanowski (Syracuse, N.Y.)

12664:

Ewan, W. D.; Kemp, K. W. Sampling inspection of continuous processes with no autocorrelation between successive results. Biometrika **47** (1960), 363–380.

The authors consider the problem of quality control in a continuous manufacturing process. They define the average run length (when the quality remains constant, the average run length of an inspection scheme is the expected number of samples obtained before action is taken) and show the formula. Using the idea of Wald sequential schemes with horizontal boundaries distance h ($h>0$) apart, they give the equations to be solved concerning with the average run length $L(Z)$, in the case where the first cumulation starts at a distance z from the lower boundary but where subsequent cumulations start on this boundary. For convenience they take the lower line as zero. Let k be a constant which is called the reference value and is substracted from each sample value. When x is a discrete value which only takes integer values, the equations to be solved can be formulated when z, h and k also take integer values. If $f(x)$ represents the probability of obtaining the value x and $F(\alpha)=\sum_{x=0}^{\alpha} f(x)$, then $L(Z)=1+L(0)F(k-z)+\sum_{y=1}^{y=h-1} L(y)f(y+k-z)$, which they solve. For the continuous case, they solve the above equations and furthermore derive the equation of the run length distribution. For the case where x is normally distributed, the authors give the nomogram for the average run length, run length distribution, the method of control of standard deviation and simultaneous control of the process mean and standard deviation. Furthermore the case of Poisson variate is treated.

C. Hayashi (Tokyo)

12665:

Morrison, Donald F.; David, H. A. The life distribution and reliability of a system with spare components. Ann. Math. Statist. **31** (1960), 1084–1094.

Suppose "components" have independent, identically distributed, operating lives. A "system" consists of n components, which start their operation simultaneously at time 0. When a component dies, it is immediately replaced by a spare one, and $L(n, k)$ is the time at which the $(k+1)$th spare is pressed into service. The authors first discuss the distribution of $L(2, k)$ in some detail, especially when $f(x)$, the probability density function of the lifetimes of components, is of gamma form; explicit formulae are given. They then turn their attention to $L(n, k)$ for $n>2$ but their work here, owing to the intractability of the mathematics, is largely confined to the calculation of $EL(n, 1)$ and $EL(n, 2)$ for a few very special forms of $f(x)$. A number of tables is given.

W. L. Smith (Chapel Hill, N.C.)

12666:

Thionet, P. La théorie de l'estimation et les sondages. Estadística **17** (1959), 702–715. (Spanish summary)

12667:

Matschinski, Matthias. Estimation des observations par plusieurs moyennes. C. R. Acad. Sci. Paris **251** (1960), 2650–2652.

12668:

Hájek, Jaroslav. Some contributions to the theory of probability sampling. Bull. Inst. Internat. Statist. **36** (1958), no. 3, 127–134. (French summary)

Some results in the theory of probability sampling in case of finite population have been obtained. For the confidence interval, for the ratio of two means, it is shown that the author's method leads always to shorter confidence intervals than those given by Fieller [Quart. J. Pharm. **17** (1944), 117–123]. Moreover, under certain conditions, large-sample confidence intervals obtained by the author have greater probability of covering the true value than those of Fieller. Upper estimate of sampling error in multistage sampling is given. Poisson sampling and some of its modifications are given. When the method of sampling and estimating is predetermined except for h parameters $\nu_1, \nu_2, \cdots, \nu_h$, an optimum sampling strategy, under a suitable definition of optimality, has been obtained. The definitions of fundamental concepts have been introduced in a formal way, through the set-theoretic notions. *V. S. Huzurbazar* (Poona)

12669:

Neyman, Jerzy. On the two different aspects of the representative method: the method of stratified sampling and the method of purposive selection. Estadística **17** (1959), 587–651. (Spanish. English summary)

Translation of the article published in the J. Roy. Statist. Soc. **97** (1934), 558–625.

L. J. Savage (Ann Arbor, Mich.)

12670:

Chiang, Tse-pei. On the estimation of regression coefficients of a continuous parameter time series with a stationary residual. Teor. Veroyatnost. i Primenen. **4** (1959), 405–423. (Russian summary)

Let $y(t)=x(t)+m(t)$ be a continuous parameter process

with trigonometric regression $m(t)=\sum_{\nu=1}^{s}\gamma_\nu \exp(i\lambda_\nu t)=Ey(t)$ and residual $x(t)$, a weakly stationary process. The author considers the problem of estimating the regression coefficients γ_ν assuming the λ_ν's are known. The process $y(t)$ is observed over time interval $-T\leq t\leq T$. The author shows that the least squares estimate of the regression coefficients is asymptotically (as $T\to\infty$) as good as the Markov estimate under the following assumptions on the spectral density $f(\lambda)$ of the residual process $x(t)$ (the spectrum of $x(t)$ is assumed to be absolutely continuous): (A) $\int_{-\infty}^{\infty}|\log f(\lambda)|(1+\lambda^2)^{-1}d\lambda<+\infty$. (B) The derivative $f'(\lambda)$ of $f(\lambda)$ exists and is continuous in a certain neighborhood of $\lambda=\lambda_\nu$ with $f(\lambda_\nu)>0$ ($\nu=1,\cdots,s$). (C) There is a $\delta>0$ such that $\log(f(\lambda)/f(x))=\theta_\nu(\lambda,x)(\lambda-x)$, and $\partial\theta_\nu(\lambda,x)/\partial x$ is a continuous function of λ, x in $\lambda_\nu-\delta\leq\lambda$, $x\leq\lambda_\nu+\delta$ ($\nu=1,\cdots,s$). The method used is due to U. Grenander [Ark. Mat. **1** (1950), 195–277; MR **12**, 511].

M. Rosenblatt (Providence, R.I.)

12671:

Zaremba, S. K. A test of fit for the spectral density function of a stochastic process. Monatsh. Math. **64** (1960), 68–79.

The author proposes tests of fit of spectral (and reduced spectral) density functions for the process $x_t=\sum_{k=-\infty}^{+\infty}h_{t-k}\varepsilon_k$, $t=0,\pm1,\pm2,\cdots$, with $h_i=0$ for $i<0$, $\sum|h_i|<\infty$, and random variables ε_k. The spectral density function is $F'(\omega)=\sum_{k=-\infty}^{+\infty}R_k e^{2\pi ik\omega}$ where $\{R_k\}$ is the covariance function. Let $G'(\omega)=\sum_{k=-\infty}^{+\infty}A_k e^{2\pi ik\omega}$ with $A_{-k}=A_k$ and $\sum|A_k|<\infty$. An estimator $Z_N(A)$ of $\int_{-1/2}^{+1/2}|F'(\omega)-G'(\omega)|^2d\omega$ is introduced. In the case of independent and identically distributed ε_k, it is shown under mild assumptions that $N^{1/2}\{Z_N(A)-EZ_N(A)\}$ is asymptotically normal with a variance tending to a limit which is computed. Similarly for the reduced spectral density function. The paper is based upon the results in Z. A. Lomnicki and the author, Monatsh. Math. **61** (1957), 318–358; **63** (1959), 128–168 [MR **19**, 1090; **22** #10136].

M. Loève (Berkeley, Calif.)

12672:

Theil, H.; Mennes, L. B. M. Conception stochastique de coefficients multiplicateurs dans l'ajustement linéaire des séries temporelles. Publ. Inst. Statist. Univ. Paris **8** (1959), 211–227.

The authors consider a generalised regression model in which the dependent sequence $\{y_t\}$ is related to the non-stochastic independent sequence $\{x_t\}$ by the equation $y_t=b_tx_t+u_t$, where the u_t are NID$(0,\sigma_0^2)$, the b_t are NID(β,σ_1^2), and the $\{u_t\}$, $\{b_t\}$ sequences are mutually independent. Simple estimates are given for the parameters $\beta,\sigma_0^2,\sigma_1^2$, and an iterative procedure for improving them is indicated.

P. Whittle (Cambridge, England)

12673:

Campbell, L. Lorne. Minimum coefficient rate for stationary random processes. Information and Control **3** (1960), 360–371.

Author's summary: "Let $x_1(t), x_2(t),\cdots,x_N(t)$ be N sample functions of a stationary random process with mean zero, variance one, and spectral density function $S(f)$. For large N and T it is possible to construct a good approximation to the product $x_1(t_1)x_2(t_2)\cdots x_N(t_N)$ on the N-dimensional cube $0\leq t_i\leq T$ by using approximately $T^N\exp[-N\int_{-\infty}^{\infty}S(f)\log S(f)df]$ terms of an infinite series. This can be interpreted as saying that one needs $\exp[-\int_{-\infty}^{\infty}S(f)\log S(f)df]$ numbers per function per unit time to determine the product. Some results are also obtained on the construction of approximations to the individual functions $x_k(t)$ from the approximate product. If $S(f)$ has a constant value in the band $(-W,W)$ and vanishes outside, this approach yields the well-known result that $2W$ numbers per function per unit time are required."

12674:

Pierson, Willard J.; Tick, Leo J. Stationary random processes in meteorology and oceanography. Bull. Inst. Internat. Statist. **35** (1957), no. 2, 271–281. (French summary)

This is a survey paper discussing a variety of fields and problems where stationary random processes are reasonable and relevant models. Literature concerned with statistical inference for such models is cited. Problems in the study of turbulence, ocean waves and ship motions are considered.

M. Rosenblatt (Providence, R.I.)

12675:

Shapiro, Harold S.; Silverman, Richard A. Alias-free sampling of random noise. J. Soc. Indust. Appl. Math. **8** (1960), 225–248.

Let $x(t)$ be a wide-sense stationary continuous parameter stochastic process with zero means and spectral density function $F(w)$. Let $\{t_n\}$ be a sequence of real numbers which represent times at which the process $x(t)$ is sampled. Suppose that the distribution of $t_{m+n}-t_m$ is independent of m, and let $\varphi_n(u)$ be its characteristic function. It is easily shown that the random sequence $\{x(t_n)\}$ is wide-sense stationary with correlation function

$$(1)\qquad c(n)=E[x(t_{m+n})x(t_m)]=\int_{-\infty}^{\infty}\varphi_n(w)F(w)\,dw.$$

A sampling sequence $\{t_n\}$ is said to be alias-free if to each correlation function $\{c(n)\}$ which satisfies (1) for some spectral density function $F(w)$, there exists exactly one such spectral density function.

This paper considers three possible kinds of sampling sequences: (i) periodic sampling ($t_n=nh$ for constant h); (ii) jittered periodic sampling ($t_n=nh+\gamma_n$, where $\{\gamma_n\}$ is a sequence of independent Gaussian random variables); and (iii) additive random sampling ($t_n=t_{n-1}+\gamma_n$, where γ_n are independent identically distributed random variables). While (i) and (ii) are in general not alias-free, conditions are given under which (iii) is alias-free. In particular, it is shown that the scheme in (iii) is alias-free if $\{\gamma_n\}$ has a characteristic function $\varphi(s)$ equal to $\rho/(\rho-is)$ or $\{\rho/(\rho-is)\}^2$ while (iii) is not alias-free if $\varphi(s)=\{\rho/(\rho-is)\}^3$.

E. Parzen (Stanford, Calif.)

12676:

Albert, G. E. Statistical methods in prediction, filtering, and detection problems. J. Soc. Indust. Appl. Math. **8** (1960), 640–653.

Suppose one observes $x(t)=s(t)+e(t)$, $0\leq t\leq T$, where $s(t)=\sum_{m=1}^{k}a_mg_m(t)$, $\{a_m\}$ unknown, $\{g_m(t)\}$ known, and where $e(t)$ is a stationary second-order process. The

author reviews three types of estimates for $\{a_m\}$: (A) the "best linear unbiased estimates" $\{\hat{a}_m\}$ which minimize $E\int_0^T [\sum_{m=1}^k \hat{a}_m g_m(t) - s(t)]^2 dt$, when the spectral density of $e(t)$ is the reciprocal of a polynomial, (B) a certain approximation $\{\hat{a}_m{}^*\}$ of $\{\hat{a}_m\}$, and (C) the least-squares estimates $\{\bar{a}_m\}$ which minimize $\int_0^T [x(t) - \sum_{m=1}^k a_m g_m(t)]^2 dt$. The efficiencies are compared in some interesting simple cases. *E. Reich* (Aarhus)

12677:

Durbin, J. Estimation of parameters in time-series regression models. J. Roy. Statist. Soc. Ser. B **22** (1960), 139–153.

Consider the stochastic difference equation

$$y_t + \alpha_1 y_{t-1} + \cdots + \alpha_p y_{t-p} = \beta_1 x_{1t} + \cdots + \beta_q x_{qt} + \varepsilon_t$$

for $t = 1, 2, \cdots, n$, where the α's and β's are unknown parameters to be estimated, the ε's pure white noise and the x's are constants or conditioned stochastic variables. Optimal estimates are discussed in terms of best unbiased linear estimating equations. If β is a vector parameter and b an estimate of β obtained as the solution of the equation $T_1 b + T_2 = 0$ (T_1 non-singular a.e., $T_1^{-1}T_2$ independent of parameters) such that $E(T_1\beta + T_2) = 0$, then the equation is called an unbiased linear estimating equation. Among these equations one tries to single out the one with minimum variance. For normal ε's the method of least squares leads to optimal estimates. The non-normal case is discussed in asymptotic terms, as well as another model with autoregressive ε's. *U. Grenander* (Stockholm)

12678:

Shklov, V.; Toop, J. H. The analysis of power spectra. Comput. Data Process. Soc. Canada (Conference, Toronto, 1960), pp. 243–249.

12679:

Masani, P. The prediction theory of multivariate stochastic processes. III. Unbounded spectral densities. Acta Math. **104** (1960), 141–162.

[For parts I and II see Wiener and Masani, same Acta **98** (1957), 111–150; **99** (1958), 93–137; MR **20** #4323, 4325.] Proofs are given for theorems stated in previous papers [C. R. Acad. Sci. Paris **246** (1958), 2215–2217, 2337–2339; MR **20** #4326a, b]. *J. L. Doob* (Urbana, Ill.)

12680:

Girault, Maurice. Le calcul des probabilités au service de l'ingénieur. Les mathématiques de l'ingénieur, pp. 270–274. Mém. Publ. Soc. Sci. Arts Lett. Hainaut, vol. hors série, 1958.

12681:

Schwartz, Daniel. La méthode statistique en médecine: les enquêtes étiologiques. Publ. Inst. Statist. Univ. Paris **9** (1960), 89–119.

An expository paper describing a statistical methodology for medical research and giving especially a measure of representing the effect of a factor.

C. Hayashi (Tokyo)

12682:

van Woerkom, Adrianus J.; Brodman, Keeve. Statistics for a diagnostic model. Biometrics **17** (1961), 299–318.

After testing and discarding the diagnostic approach based on the strict application of Bayes' theorem (but using estimated probabilities), the authors and associates developed and tested on a sample of patients an empirical technique for making presumptive diagnoses.

Denote by $P_k = \Pr(A_k)$ the (estimated) probability of attribute A_k in the sample and by $P_{jk} = \Pr(A_k | D_j)$ the (estimated) probability of A_k conditioned by the presence of disease category D_j.

The expression

$$S_{jk} = \max\{[0, (P_{jk} - P_k)/2P_k^{1/2} - 1]\}$$

was adopted for use as follows: "If the attributes A_{k1}, $A_{k2}, \cdots, A_{kn}$ are present in the patient, the corresponding sums $(Sc)_j = \sum_{k1}^{kn} S_{jk}^2$ $(j = 1, 2, \cdots, 60)$ are called the scores. The scores for a patient are normalized by dividing them by the mean score '(over patients)' for the entire '(disease) category'."

In spite of the fact that S_{jk}^2 is a monotone non-decreasing function of P_{jk} for fixed P_k, the authors claim that a high score is indicative of a low probability for disease category D_j and hence that a low score for D_j is indicative of the disease category. No adequate mathematical analysis is given of the system proposed.

M. A. Woodbury (New York)

NUMERICAL METHODS

See also A12072, A12073, 12931, 12962.

12683:

Положий, Г. Н. [Položii, G. N.] (Editor). ★Математический практикум [Mathematical practicum]. Gosudarstv. Izdat. Fiz.-Mat. Lit., Moscow, 1960. 512 pp. 10 r.

According to the introduction, this text is prepared for use at the University of Kiev for the course on machine computing, and is, moreover, the first one. Its purpose is to acquaint the student with "approximate methods of mathematics" and with the use of simple computing machines. While references are made to electronic computers, the first chapter describes the abacus, the "arithmometer", and the slide rule, and concludes with a section on the use of tables.

Chapter 5 deals with ordinary differential equations, and Chapter 7 is entitled "Methods of mathematical physics connected with the solution of linear algebraic equations", which is to say, boundary-value problems and integral equations. Elsewhere, and perhaps even here, a good course in calculus should provide an adequate mathematical background for the book. The organization is uniform throughout: the theory of a method is presented in simple terms, then simple illustrative examples are given. Each chapter contains at least one set of problems as "laboratory work".

Chapter 1 begins with a discussion of computational error; Chapter 2 deals with numerical and graphical methods for the solution of algebraic and transcendental equations; Chapter 3 with interpolation; Chapter 4 with numerical differentiation and integration; Chapter 6 with "linear algebra".

Chapter 2, for example, begins with the method of Lobachevsky, properly so-called, since they do not describe Graeffe's convenient algorithm; then goes on to the method of false position, Horner's method, Newton's method, and graphical methods. In Chapter 5, for the characteristic-value problem, only the escalator method and the method of Krylov are given. Each chapter is concluded with a short list of references, mostly textbooks, a fair number non-Russian but in Russian translation.

In some instances, one can criticize the selection of methods. This is especially so for the characteristic-value problem, and the refusal to use Graeffe's algorithm seems inexcusable. Usually, however, the selection is reasonable, and even inevitable, and the presentation is lucid. Considered as a practicium and not as a treatise, it should do quite well. *A. S. Householder* (Oak Ridge, Tenn.)

12684:

Vernotte, Pierre. Exécution de calculs numériques très difficiles par l'utilisation d'une propriété de régularité. Les mathématiques de l'ingénieur, pp. 410–416. Mém. Publ. Soc. Sci. Arts Lett. Hainaut, vol. hors série, 1958.

12685:

Березин, И. С. [Berezin, I. S.]; Жидков Н. П. [Židkov, N. P.]. ★Методы вычислений [Computational methods]. 2 vols. Gosudarstv. Izdat. Fiz.-Mat. Lit., Moscow, 1959. Vol. 1: 464 pp. 11 r.; Vol. 2: 620 pp. 14.10 r.

This book is based upon lectures given at Moscow State University to students specializing in computational mathematics. However, the students are presumed not yet to have had any substantial computational experience. Consequently, the authors consider themselves limited in the extent to which they can present a broad and systematic treatment of the basic ideas underlying the most important numerical methods. Perhaps this is their apology for the fact that they, along with so many others, seem more preoccupied with methods than with ideas, and that the result is a compendium of methods, each elaborately explained and illustrated, but with little, or at least inadequate, indication of interrelations and of possible variation. On the positive side, however, the authors remark that functional analysis is becoming increasingly important in the area of numerical mathematics, and they attempt to introduce and utilize wherever possible notions and principles borrowed from functional analysis. Since functional analysis is not a prerequisite, they are forced to explain it as they go along.

The book contains essentially all that one would expect, up to and including a 50-page chapter on ordinary differential equations, and one of just over 100 pages on partial differential equations and integral equations. However, there is a chapter of over 50 pages on "uniform" (i.e., minimax, or Chebyshev) approximation, which one would not necessarily expect, and about 80 pages on "mean-square" approximation.

The first volume deals with approximation, and the second volume with the solution of equations. There are numerous illustrative examples, fully worked out, and a fair collection of exercises. A list of references is given at the end of each chapter, about 6 items, on the average, mainly text-books.

In spite of the criticisms already made, the book can well be recommended for its breadth and balance, and for the efforts of the authors to view the subject with more perspective than is usually taken. It is interesting to note that 10,000 copies of volume II were printed in November of 1959, and an additional 15,000 in September, 1960.

A. S. Householder (Oak Ridge, Tenn.)

12686:

Stiefel, E. ★Einführung in die numerische Mathematik. Leitfäden der angewandten Mathematik und Mechanik, Bd. 2. B. G. Teubner Verlagsgesellschaft, Stuttgart, 1961. 234 pp. DM 24.80.

In manual computation the choice of a method that is particularly well adapted to the problem on hand may save a great amount of time. In automatic computation, however, this potential saving may be greatly reduced or altogether eliminated by the time needed for programming a special method. Unlike the majority of introductory texts on numerical analysis, the present one reflects this interest in a small number of algorithms of broad applicability. Typical for this approach are the first three chapters on linear algebra, linear programming, and least-squares approximation, which are largely based on a single algorithm (Jordan exchange). Further chapters are devoted to non-linear algebra, eigenvalue problems, ordinary and partial differential equations, and approximations. While numerical examples of small size are frequently used to illustrate points in the body of the text, some examples of larger size are given in Appendix I. Tables concerning numerical differentiation and integration, interpolation, and harmonic analysis and synthesis, are collected in Appendix II. Throughout the book, the presentation is distinguished by its clarity. Despite the elementary character of the work, many points of current research interest are touched upon.

W. Prager (Providence, R.I.)

12687:

Goetz, Billy E. Monte Carlo solution of waiting line problems. Management technology, Monograph No. 1, 1960, pp. 2–11.

12688:

Halton, J. H. On the efficiency of certain quasi-random sequences of points in evaluating multi-dimensional integrals. Numer. Math. **2** (1960), 84–90.

This paper describes a method for the generation of a quasi-random sequence of vectors in k dimensions. The error in estimating the integral

$$\int_0^1 dx_1 \cdots \int_0^1 dx_k f(x_1, \cdots, x_k)$$

by Monte Carlo methods using this sequence is discussed, taking as an example the function $f(x_1, \cdots, x_k)=1$ $(0 \leq x_i \leq A_i)$, $f(x_1, \cdots, x_k)=0$ otherwise. It is shown that this integral may be estimated with an error which is $O(N^{-1}(\ln N)^{k-1})$, where N is the number of points taken.

C. B. Haselgrove (Manchester)

12689:

Karst, E. Approximating transcendental numbers by continued fractions. Comm. ACM **4** (1961), 171.

In this short note ($\frac{1}{2}$ page) the first few terms of the numerical continued fractions for e^2 and π are given. No references. *H. J. Maehly* (Stanford, Calif.)

12690:

Albasiny, E. L. The use of associated Legendre polynomials for interpolation. Proc. Cambridge Philos. Soc. **57** (1961), 288–303.

Three formulas are derived, analogous to the Bessel, the Everett-Bessel, and the Everett formulas, but using associated Legendre polynomials. Truncation and rounding errors are examined. The second one is recommended in particular in that, although the truncation error is slightly larger than the corresponding one using Chebyshev polynomials, it has the advantage that only rational numbers occur in it.

A. S. Householder (Oak Ridge, Tenn.)

12691:

Schwengberg, Arnd. Eine neue Näherungsformel zum Zinsfussproblem der Leibrente mittels Interpolation der verallgemeinerten Poukkaschen Zahlen. Bl. Deutsch. Ges. Versicherungsmath. **4** (1959/60), 418–426. (English summary)

In life-assurance mathematics approximations are used for annuity values at rates of interest for which no tables are worked out. Such formulae expand the annuity value after the Taylor formula. When using linear approximations only, one arrives at Poukka's formula [Skand. Aktuarietidskr. **6** (1923), 137–152]. The author develops more accurate formulae by using approximations of second or higher order. *P. Johansen* (Copenhagen)

12692:

Cheney, E. W.; Loeb, H. L. Two new algorithms for rational approximation. Numer. Math. **3** (1961), 72–75.

The authors describe two algorithms for rational Chebyshev approximation on finite point sets. Both are essentially restricted to the case that numerator and denominator depend linearly on the free coefficients, though the proof of convergence for the first method does not use this assumption. Both methods are iterative; the first is based on a suitable local linearization, the second is a method of steepest descent. Convergence is proved for both methods. Computational details or examples are not given. *H. J. Maehly* (Syracuse, N.Y.)

12693:

Nelder, J. A. The fitting of a generalization of the logistic curve. Biometrics **17** (1961), 89–110.

This paper discusses the fitting of the family of curves defined by the differential equation

$$\frac{dW}{dt} = \kappa^W \left[1-\left(\frac{W}{A}\right)^{1/\theta}\right].$$

The author points out that the family so defined includes as special cases several curves which have been used empirically for the description of growth. When $\theta > 0$, the solution may be written in the form

$$W = A/\{1+e^{-(\lambda+\kappa t)/\theta}\}^\theta,$$

which the author calls the general logistic equation. The least-squares fit of the curve is derived for the case in which the sample values of $\ln W$ are independent, unbiased, and of constant variance. Some possible sources and consequences of deviations from the assumptions underlying the fitting are discussed. Tables are provided to assist the computing of the iterative process used for estimation. The work is compared with the related work of earlier authors.

P. S. Dwyer (Ann Arbor, Mich.)

12694:

Grossmann, Walter. ★Grundzüge der Ausgleichungsrechnung nach der Methode der kleinsten Quadrate nebst Anwendung in der Geodäsie. 2te erweiterte Aufl. Springer-Verlag, Berlin-Göttingen-Heidelberg, 1961. xii + 345 pp. DM 31.50.

The first edition (1953) of this work was reviewed in MR **15**, 650. There are several additions in the new edition: a treatment of relative errors; observations with systematic and constant error contributions; a discussion of algorithms for the solution of the normal equations which are suitable for automatic computers; and several others.

The work continues to be a most readable and exhaustive account of a subject important to scientists and engineers.

12695:

Salzer, Herbert E. Formulae for complex Cartesian hyperosculatory interpolation. J. Math. and Phys. **39** (1960), 300–307.

In a previous article [Quart. J. Mech. Appl. Math. **12** (1959), 100–110; MR **20** #7383] the author gave n-point formulas of degree $3n-1$ for direct and inverse hyperosculatory interpolation for $f(x)=f(x_0+ph)$ which employed $f(x_j)$, $f'(x_j)$ and $f''(x_j)$ at n equally spaced points $x_j=x_0+jh$, along a straight line. The present article gives similar formulas based on points at the corners of a grid-square. Formulas are given for $n=2(1)7$ for direct interpolation and for $n=2(1)5$ for inverse interpolation.

A. H. Stroud (Madison, Wis.)

12696:

Harazov, D. F. On the method of steepest descent. Akad. Nauk Gruzin. SSR. Trudy Tbiliss. Mat. Inst. Razmadze **24** (1957), 111–123. (Russian)

The author studies a compact operator A which is symmetrizable by means of a strictly positive bounded operator H, i.e., $(HAx, y)=(x, HAy)$ for all x, y in a Hilbert space [M. G. Krein, Akad. Nauk Ukrain. RSR. Zbirnik Prac. Inst. Mat. **9** (1947), 104–129; P. Lax, Comm. Pure Appl. Math. **7** (1954), 633–647; MR **16**, 832]. Introducing (Hx, y) as a new scalar product, the author derives a new Hilbert space in which A becomes self-adjoint and compact. Using the method of steepest descent [e.g., Kantorovič, Uspehi Mat. Nauk **3** (1948), no. 6 (28), 89–185; English translation: Nat. Bur. Standards Rep. **1509** (1952); MR **10**, 380; **14**, 766] on this case, the author derives the convergence of approximations given by corresponding formulae for symmetrizable operators. This is applied to ordinary differential boundary operators. *František Wolf* (Berkeley, Calif.)

12697:

Golomb, Michael; Weinberger, Hans F. ★Optimal approximation and error bounds. On numerical approximation. Proceedings of a Symposium, Madison, April 21–23, 1958, pp. 117–190. Edited by R. E. Langer. Publication No. 1 of the Mathematics Research Center, U.S. Army, the University of Wisconsin. The University of Wisconsin Press, Madison, Wis., 1959. x+462 pp. (1 insert) $4.50.

Given a linear vector space V, and $n+1$ linear functionals $F, F_1, \cdots, F_n$ which may, without loss of generality, be assumed linearly independent. If we approximate $F(u)$, $u \in V$, given $F_i(u)=f_i$, $i=1, 2, \cdots, n$, we cannot, in general, bound the error in the approximation without a non-linear constraint on u. Let $S \subset V$ be given such that $\{v \in V | av \in S \text{ for some } a>0\}$, and let $\rho(v)=\inf_{a>0;\, av\in S} a^{-1}$. Assume $\rho(av)=a\rho(v)$, $a \geqq 0$, and $|\rho(v)-\rho(w)| \leqq b\rho(v-w)$ for some $b>0$. A key assumption is the existence of a constant c such that $|F(v)| \leqq c\rho(v)$ whenever $F_i(v)=0$, $i=1, 2, \cdots, n$. Under these conditions, if σ is the set of values of $F(v)$ for $v \in S$ which satisfy $F_i(v)=f_i$, $i=1, 2, \cdots, n$, σ is unchanged if S is redefined to include its boundary. It is also proved that, without loss of generality, any F unbounded relative to ρ may be removed, i.e., it may be assumed $|F(v)|+\sum_1^n |F_i(v)| \leqq c\rho(v)$ for some c. If there is a non-empty null space $V_0=\{q \in v | \rho(q)=F(q)=F_i(q)=0\}$, it is shown that the approximation problem may be handled on V/V_0.

If S is given by a single quadratic inequality as $S=\{v|[v, v] \leqq r^2\}$, $[v, w]$ a symmetric bilinear form, and if V/V_0 is of dimension $l<n$, then V/V_0 may be completed to a Hilbert space H with the bilinear form $(v, w)=[v, w]+\sum_{i=1}^l F_i(v)F_i(w)$, where F_i are linearly independent on V/V_0. In this case σ can be identified explicitly as an interval centered on $F(\bar{u})$ and of length $2F(\bar{y}) \times (r^2-(\bar{u}, \bar{u}))^{1/2}$, where $\bar{u}$ minimizes (v, v) over all v satisfying $F_i(v)=f_i$, and $\bar{y}$ maximizes $|F(v)|$ over all v such that $(v, v)=1$ and $F_i(v)=0$. The technique can be developed further if a Riesz representation for the linear functionals F_i, or a reproducing kernel in H, can be found.

The general theory is applied throughout the paper to a great variety of examples from many fields of analysis, and yield many results both new and classical on optimal approximation. Some examples: For the problem of approximating $\int_a^b u(x)dx$, knowing $u(x_i)=f_i$ at $a=x_1<x_2<\cdots<x_n=b$, and knowing that $\int_a^b [u^2(x)+\alpha^2 u^2(x)]dx \leqq r^2$, an optimal quadrature formula and exact error bounds are given which generalize previous work of Sard [Amer. J. Math. **71** (1949), 80–91; MR **10**, 576]. Analogous results are given for the problem of approximating $a(x_0)$, knowing $u(x_i)$, $a \leqq x_1 < x_1 < \cdots < x_n \leqq b$ and a bound on $\int_a^b u^{(k)^2}(x)dx$, $k \leqq n$. The latter are extended to the case in which $u'(x_i)$ are also given. The methods are also applied, for instance, to boundary and initial value problems for ordinary differential equations and to Fredholm equations of the second kind. Many other quadrature situations are also considered, and error bounds given.

H. O. Pollak (Murray Hill, N.J.)

12698:

Pyhteev, G. N. On the valuation of certain singular integrals with a kernel of the Cauchy type. Prikl. Mat. Meh. **23** (1959), 1074–1082 (Russian); translated as J. Appl. Math. Mech. **23**, 1536–1548.

The author describes a method for evaluating principal-value integrals of the forms

$$J(x) = \frac{1}{\pi}\int_{-1}^{1} \frac{f(t)}{t-x}\,dt,$$

$$I(x) = \frac{\sqrt{(1-x^2)}}{\pi}\int_{-1}^{1} \frac{f(t)}{t-x}\,\frac{dt}{\sqrt{(1-t^2)}}.$$

It is well known that the Chebyshev polynomials $T_n(x)$ and $U_n(x)$ satisfy the equations

$$\frac{1}{\pi}\int_{-1}^{1} \frac{U_n(t)}{t-x}\,dt = -T_n(x),$$

$$\frac{\sqrt{(1-x^2)}}{\pi}\int_{-1}^{1} \frac{T_n(t)}{t-x}\cdot\frac{dt}{\sqrt{(1-t^2)}} = U_n(x).$$

In general, however, evaluation by direct expansion in Chebyshev polynomials leads to rather slowly convergent series. The author therefore modifies the method as follows. He defines the functions

$$p_1^{(s)}(x) = \sum_{n=2}^{\infty} \frac{1}{(2n-1)^s}\,T_{2n-1}(x),$$

$$p_2^{(s)}(x) = \sum_{n=2}^{\infty} \frac{1}{(2n)^s}\,T_{2n}(x),$$

$$q_1^{(s)}(x) = \sum_{n=2}^{\infty} \frac{1}{(2n-1)^s}\,U_{2n-1}(x),$$

$$q_2^{(s)}(x) = \sum_{n=2}^{\infty} \frac{1}{(2n)^s}\,U_{2n}(x),$$

and gives brief tables of a few of them. If $f(x)$ is sufficiently well behaved (in a certain precise sense), he subtracts from it an appropriate expression of the form

$$\frac{4}{\pi}\sum_{s=1}^{k} (-1)^s[\gamma_1^{(2s-1)}p_1^{(2s)}(x)-(\operatorname{sgn} x)\gamma^{(2s-2)}q_1^{(2s-1)}(x^*) \\ -\gamma_2^{(2s-1)}p_2^{(2s)}(x)-\gamma^{(2s-1)}p_2^{(2s)}(x^*)],$$

where $x^*=\sqrt{(1-x^2)}$ and the coefficients depend only on the behaviour of $f(x)$ near the points ± 1; he then expands the remainder as a series in $T_n(x)$ (or, after subtracting another similar expression in $U_n(x)$). He then uses these new expansions to evaluate the integrals. He obtains an estimate of the error produced by stopping the expansion at any stage; he also gives a numerical illustration, which indicates that four-figure accuracy is obtainable in certain cases with a comparatively small number of terms.

F. Smithies (Cambridge, England)

12699:

Salzer, Herbert E.; Kimbro, Genevieve M. Improved formulas for complete and partial summation of certain series. Math. Comp. **15** (1961), 23–39.

The formulas obtained are Lagrangian extrapolation formulas. They are derived for functions which are assumed to be even functions of x, and which are supposed known at $x=1/j, 1/(j-1), \cdots, 1/(j-m+1)$. Extrapolation is to $x=0$, or $x=1/n$, so that if the known values are the jth, $(j-1)$th, $\cdots$, $(j-m+1)$th partial sums of a series, the extrapolated value is an estimate of the sum or the nth partial sum of the series.

Tables of coefficients are given, in the case of extrapolation to zero, for $m=2(1)7$, $j=10$, and for $m=11$, $j=20$,

and, in the case of extrapolation to $1/n$, for $m=7$, $j=10$, $n=11(1)25(5)100$, 200, 500, 1000.

The authors treat a few examples of series, whose jth partial sums are even functions of j. These examples show that it is possible to obtain results which are accurate to ten decimal places or more, although, of course, one does not know in advance how good the results will be in a particular case. *T. E. Hull* (Vancouver, B.C.)

12700:

Boersma, J. Computation of Fresnel integrals. Math. Comp. **14** (1960), 380.

Table of coefficients for approximation of Fresnel integrals by finite power series. Maximum error is given as 1.6×10^{-9} for $0\leq x\leq 4$ and 0.5×10^{-9} for $x\leq 4$.

F. Stallmann (Washington, D.C.)

12701:

Mornard, S.; Adriaenssens, G.; Hirschberg, D. Deux applications industrielles de la programmation linéaire. Les mathématiques de l'ingénieur, pp. 320–323. Mém. Publ. Soc. Sci. Arts Lett. Hainaut, vol. hors série, 1958.

12702:

Elliott, David L. A note on systems of linear equations. SIAM Rev. **3** (1961), 66–69.

The method described by Semarne [same Rev. **1** (1959), 53–54; MR **20** #6780] for solving $Ax=c$ is generalized to handle the cases where there is no finite vector solution (inconsistency), and where there is an infinity of vectors with endpoints lying on a line, plane or hyperplane. The essential computation is still a Gram-Schmidt orthogonalization. *C. C. Gotlieb* (Toronto)

12703:

Wilkinson, J. H. Rounding errors in algebraic processes. Information processing, pp. 44–53. UNESCO, Paris; R. Oldenbourg, Munich; Butterworths, London; 1960. (French, German, Russian and Spanish summaries)

This paper introduces a method of evaluating the effect of the rounding errors made in extensive calculations. The paper contains a treatment of the solution of linear equations and the evaluation of determinants by Gaussian elimination and triangular decomposition and the closely related problem of calculating eigenvectors by inverse iteration.

The calculated solution x' of the linear equations $Ax=b$ can be considered to be the exact solution of the equations $(A+\delta A)x'=b+\delta b$. Upper bounds for the elements of δA and δb are obtained very easily. The improvement that can be obtained by using double-precision arithmetic in forming sums of products is demonstrated.

The results indicate a surprisingly small error for the triangular-decomposition method. The author is a staunch advocate of the use of direct methods for solving linear equations and has here presented a very strong argument in their favour. *B. A. Chartres* (Providence, R.I.)

12704:

Varga, Richard S. Factorization and normalized iterative methods. Boundary problems in differential equations, pp. 121–142. Univ. of Wisconsin Press, Madison, Wis., 1960.

The author considers iterative methods for solving systems of linear algebraic equations of the form $A\mathbf{x}=\mathbf{k}$ arising from the solution of elliptic partial differential equations by finite difference methods. The given $n\times n$ matrix A is assumed to be non-singular, and $\mathbf{x}$ and $\mathbf{k}$ are unknown and known column matrices, respectively. It is observed that most iterative methods involve the direct solution of systems of matrix equations in a few unknowns. Whereas most methods considered previously have involved the direct solution of equations with 1×1 or with tri-diagonal matrices, the author wishes to consider more general methods.

Given a non-singular matrix A, a representation of the form $A=B-C$ is called a "splitting". A splitting is said to be "regular" provided $B^{-1}\geqq0$ and $C\geqq0$. (In general, $M\geqq0$ if the matrix M has no negative elements; $M>0$ if all of the elements of M are positive.) Assuming that the direct solution of linear systems of the form $B\mathbf{x}=\mathbf{g}$ for $\mathbf{x}$ is relatively easy, then one considers the iterative method $Bx^{(m+1)}=Cx^{(m)}+\mathbf{k}$ $(m=0,1,2,\cdots)$. It is shown that if $A=B-C$ is a regular splitting and if $A^{-1}>0$, then the method converges. (Actually, the weaker condition $A^{-1}\geqq0$ would have been sufficient.) Moreover, if $A=B_1-C_1$ and $A=B_2-C_2$ are two regular splittings with $C_2\geqq C_1\geqq0$, $C_2\neq C_1$, $C_1\neq0$, and if $A^{-1}>0$, then the iterative method associated with the first splitting converges faster than the method associated with the other.

Among matrices A which admit regular splittings are those such that: $a_{i,j}\leqq0$ for $i\neq j$; A is irreducible; $\sum_{j=1}^{n}a_{i,j}\geqq0$ for all i, with strict inequality holding for some i. Matrices arising from many elliptic difference equations have the above properties. The author develops additional means for comparing the rates of convergence of methods associated with various splittings and applies his results to the successive overrelaxation method for certain splittings. For such splittings a procedure is given for estimating the optimum relaxation factor. For the five-point finite difference approximation to Laplace's equation it is shown that the rates of convergence of successive point overrelaxation (SOR), successive line overrelaxation (SLOR), and successive two-line overrelaxation (S2LOR), are approximately in the ratio $1:2^{1/2}:2$ whereas the multiplication times per iteration per point are in the ratio 5:5:6. An effective procedure for carrying out the S2LOR method is given. Applications are also given to the nine-point difference approximation to Laplace's equation, to the seven-point approximation in three dimensions, and to the thirteen-point approximation to the biharmonic equation in two dimensions. In the latter case it is shown that with suitable boundary conditions and with $\omega=1$ the S2LOR method converges, whereas it is known [Heller, #12705] that SLOR may not converge.

In a final section the author considers splittings of the form $A=T'T-C$, where T is a suitable sparse upper triangular matrix with positive diagonal elements. A procedure is given for choosing T which in many cases yields a regular splitting.

D. M. Young, Jr. (Austin, Tex.)

12705:

Heller, J. Simultaneous, successive and alternating direction iteration schemes. J. Soc. Indust. Appl. Math. **8** (1960), 150–173.

The author considers various iterative procedures for solving systems of linear algebraic equations arising from problems involving elliptic partial differential equations of order $2p$ with two independent variables. The methods of "successive block iteration", "simultaneous block iteration", and "separable line simultaneous iteration" are considered, as well as the alternating direction methods of Peaceman-Rachford, Douglas-Rachford, and generalizations thereof. Particular attention is given to problems of the form $(H+V)^pU=Z$ where U and Z are unknown and known column matrices, respectively, and where H and V are matrices obtained from finite difference representations of the differential operators

$$K(x,y)\partial^2w(x,y)/\partial x^2+L(x,y)\partial w(x,y)/\partial x+P(x,y)w(x,y),$$

$$R(x,y)\partial^2w(x,y)/\partial y^2+S(x,y)\partial w(x,y)/\partial y+Q(x,y)w(x,y),$$

respectively. The author asserts that H and V commute if and only if the given region is a rectangle, $K(x,y)$ and $L(x,y)$ depend only on x, and $R(x,y)$ and $S(x,y)$ depend only on y. That the assertion is incorrect can be seen by considering the operators $\partial^2u/\partial x^2+2(x+y)^{-1}\partial u/\partial x$ and $\partial^2u/\partial y^2+2(x+y)^{-1}\partial u/\partial y$ for which, if the region is the unit square, the corresponding matrices commute for all h such that h^{-1} is an integer. It should also be noted that, while the author initially assumes the matrix of the linear system to be symmetric and positive definite, the method of developing the difference operators may lead to non-symmetric matrices H and V as, for example, in the counterexample given above.

For the "separable line simultaneous iteration" method and for the alternating direction methods, relations are given between the eigenvalues of the associated linear transformations and the eigenvalues of H and V, as well as, in the case of the line iteration scheme, of the eigenvalues of S and R where $V=S+R$ represents a certain "splitting" of V. It is assumed that H and V commute. A more detailed analysis is given for the cases where H and V correspond to $\partial^2w/\partial x^2$ and $\partial^2w/\partial y^2$, respectively. The method converges for $p=1$ but diverges for $p>1$, although the author states that convergence could be achieved by a suitable extrapolation process. For some of the alternating direction methods sufficient conditions are given on the iteration parameters to insure convergence.

{The following corrections should be noted: In (3.16) replace $k_{i,j}$ by $K_{i,j}$; on p. 156, line 15, and p. 159, line 14, indicate [6] in the Windsor reference; the reviewer objects to the use of $(-)^p$ for $(-1)^p$ without definition; formulas (4.14), (4.2), (5.6), and (5.10) are not mutually consistent; on p. 159, line 17, replace "therfore" by "therefore".}

D. M. Young, Jr. (Austin, Tex.)

12706:

Gouarné, René. La méthode des cycles en calcul automatique. Les mathématiques de l'ingénieur, pp. 314–319. Mém. Publ. Soc. Sci. Arts Lett. Hainaut, vol. hors série, 1958.

Numerical method for the calculation of the determinant of a matrix, using cyclic permutations. Practically useful only for very sparse matrices.

F. L. Bauer (Mainz)

12707:

Stojaković, Mirko. Sur l'inversion d'une classe de matrices. Les mathématiques de l'ingénieur, pp. 188–192. Mém. Publ. Soc. Sci. Arts Lett. Hainaut, vol. hors série, 1958.

In order to determine numerically the inverse of a Vandermonde matrix V, the author uses (a) a simple algorithm for the determination of the elementary symmetric functions a_i, (b) a matrix relation which is identical with the Horner polynomial representation of the Langrange interpolation polynomials.

F. L. Bauer (Mainz)

12708:

Bruaux, A. Quelques applications du calcul matriciel. Les mathématiques de l'ingénieur, pp. 333–341. Mém. Publ. Soc. Sci. Arts Lett. Hainaut, vol. hors série, 1958.

12709:

Pietrzykowski, Tomasz. Projection method. Prace ZAM Ser. A No. 8 (1960), 9 pp.

The method is for solving linear algebraic equations, and is very nearly the same as that of Purcell [J. Math. Phys. **32** (1953), 180–183; MR **15**, 471]. Taking the equations to represent hyperplanes, $n+1$ points are selected, of which n are projected from the remaining one upon one of the hyperplanes; $n-1$ of these projected from the remaining upon another hyperplane; $\cdots$. Gaussian elimination is in fact a special case. Storage economy is cited as an advantage, since only one plane is required in fast storage at a time. Exceptional cases are considered. It is natural to start by projecting the point e_i from the origin, but there is no indication whether one should not prefer other choices at times in the interest of numerical stability.

A. S. Householder (Oak Ridge, Tenn.)

12710:

Abramow, A.; Neuhaus, M. Bemerkungen über Eigenwertprobleme von Matrizen höherer Ordnung. Les mathématiques de l'ingénieur, pp. 176–179. Mém. Publ. Soc. Sci. Arts Lett. Hainaut, vol. hors série, 1958.

The authors give a correction formula for the largest characteristic root of a hermitian matrix, if, for a principal minor of this matrix, the characteristic vector is known. This suggests an escalator-type iterative approximation, especially in those cases where, by bordering an improved matrix, approximation to a linear operator is obtained.

In order to determine some dominant roots, the authors describe an iterative method for the approximation of the invariant subspace belonging to these roots, and discuss rough convergence.

F. L. Bauer (Mainz)

12711:

Ghinea, Monique; Pham, Daniel. Une méthode nouvelle pour la détermination des valeurs et vecteurs propres d'une matrice. C. R. Acad. Sci. Paris **251** (1960), 2868–2870.

If the matrix is $A=\begin{pmatrix}a & w\\ v & B\end{pmatrix}$, then $f(\lambda)=\lambda-a+w(B-\lambda I)^{-1}v$ is equal to the characteristic polynomial of A divided by that of B. The "new method" is to form $f(\lambda)$, to find one of its zeros by use of Newton's method, then to deflate. Attention is given to the possibility that A and B have a common root. How $f(\lambda)$ and $f'(\lambda)$ are to be computed in practice we are not told.

A. S. Householder (Oak Ridge, Tenn.)

12712:

Fichera, Gaetano. Computation of eigenvalues and eigensolutions. Les mathématiques de l'ingénieur, pp. 58–69. Mém. Publ. Soc. Sci. Arts Lett. Hainaut, vol. hors série, 1958.

This is an expository paper which, for bounded, compact, one-to-one operators on Hilbert space, summarizes some methods for approximating eigenvalues and eigenfunctions. For Hermitian operators, these are the Rayleigh-Ritz and Weinstein methods; for non-Hermitian operators the author states his own method.

F. H. Brownell (Seattle, Wash.)

12713:

Mack, C. Corrigendum: "Routh test function methods for the numerical solution of polynomial equations". Quart. J. Mech. Appl. Math. **14** (1961), 128.

This corrects a statement given in the original paper [same J. **12** (1959), 365–378; MR **22** #3107] that the convergence of the methods of Frazer and Duncan [Proc. Roy. Soc. London Ser. A **125** (1929), 68–82] cannot be guaranteed. Whilst this remark is true of the simplified test functions of Frazer and Duncan, it is not true of their original test function. *F. W. J. Olver* (Washington, D.C.)

12714:

Durand, E. ★Solutions numériques des équations algébriques. Tome I: Équations du type $F(x)=0$; racines d'un polynôme. Masson et Cie, Editeurs, Paris, 1960. vii + 327 pp. 65 NF.

This well-written, lucid, and practical book is primarily a detailed presentation of apparently all known numerical methods of obtaining roots of polynomials in the real as well as in the complex domain. All methods given have been tested on desk calculators and on the IBM 650 at the University of Toulouse with some additional testing on an IBM 704.

Although many classical methods are described, there is extensive referral to the literature of the past decade, and some attention is paid in notes to the effects of such machine considerations as floating point, finite precision, and over-flow. The book's practical orientation is evident in the abundance of examples, comparison between methods and examples, and in the shortness of the theoretical considerations. A course in college algebra and one in elementary calculus are adequate mathematical preparation for an understanding of the text.

Besides presenting methods of root isolation and determination there is some treatment of related items such as evaluation of functions (as solutions of equations), operations on and with polynomials, inversion of series, rational function expansions, and solution of non-polynomial equations by solving approximating polynomial equations. Although the purpose of the book and the presentation of the material is in the main laudable, there is some inadequacy in presentation of the domain of validity of some of the methods. As one example, there is no warning that (in view of Jentzsch's theorem that every point on the circle of convergence of a power series of an analytic function is a limit point of zeros of its partial sums) the method of solving non-polynomial equations by solving approximating polynomial equations is by no means universally applicable.

The author promises a second volume dedicated to the solution of systems of equations and to the determination of eigenvalues of matrices.

M. L. Juncosa (Santa Monica, Calif.)

12715:

Lee, F. A. A note for finding complex roots of a special kind of quartic equation. J. Aerospace Sci. **27** (1960), 714–715.

12716:

Ionescu, D. V. Formules de quadrature à nœuds extérieurs. Mathematica (Cluj) **2** (**25**) (1960), no. 1, 55–142.

The author studies quadrature formulas with nodes exterior to the interval of integration, and makes a detailed study of their remainders. He makes a comparison between the upper bounds of the absolute values of the remainders of the quadrature formulas with exterior nodes and certain quadrature formulas with interior nodes and finds the latter are superior. *E. Frank* (Chicago, Ill.)

12717:

Stroud, A. H. Numerical integration formulas of degree 3 for product regions and cones. Math. Comp. **15** (1961), 143–150.

There are presented numerical integration formulas of degree three for Cartesian product regions based on formulas of degree three for the factor spaces. This approach reduces the number of points of evaluation from that which would be determined using the Cartesian product of the formulas. Thus if p and q are the respective numbers of evaluation points for such third-degree formulas in two factor regions, then $p+q+1$ is the largest number the author needs for the product region. The product formula would involve pq points.

There are also given formulas of degree three for cones when a formula of such degree is known for the base. A particular formula of interest is given for the n-simplex. It involves the vertices, the centroids of the faces and the centroid as evaluation points.

The author states, "It seems certain, although no proof is known, that the $(n+2)$-point formula of degree three for S_n (n-simplex) and the $2n$-point formula of degree three for C_n (n-cube) involve the minimal number of points for this degree." This conjecture, which may be readily settled at some future date, indicates the present tenuous state of numerical integration in higher dimensions. The minimal counts of points established so far have been independent of the region. These are the $n+1$ point formulas of degree two for arbitrary regions in n-space shown to be minimal by the author and the seven-point fifth-degree formulas for planar regions indicated as minimal by Radon. *P. C. Hammer* (Madison, Wis.)

12718:

Rothmann, Harry A. Gaussian quadrature with weight function x^n on the interval $(-1, 1)$. Math. Comp. **15** (1961), 163–168.

This paper discusses quadrature formulas of the form

$$\int_{-1}^{1} x^n f(x)\,dx = \sum_{k=1}^{m} W_k f(x_k) + E,$$

which are exact for all polynomials of degree $\leq 2m-1$. When $n=2h$, the weight function is non-negative in $[-1, 1]$, and the desired formulas are easily obtained using the theory of orthogonal polynomials for a non-negative weight function. When $n=2h+1$, formulas with an odd number of abscissas do not exist, but they do exist for an even number of abscissas. In the cases for which the formulas do exist, the corresponding orthogonal polynomials are given in closed form. Values of the W_k and x_k are given for $n=0(2)10$ for $n=2, 3, 4$ and for $n=1(2)11$ for $m=2, 4$ to 7S. The corresponding coefficients of $f^{(2m)}(\eta)$ in the representation for E are given to 2S.

A. H. Stroud (Madison, Wis.)

12719:

Fehlberg, Erwin. Neue genauere Runge-Kutta-Formeln für Differentialgleichungen n-ter Ordnung. Z. Angew. Math. Mech. **40** (1960), 449–455. (English and Russian summaries)

The differential equation $\bar{y}^{(n)}=\bar{f}(x, \bar{y}, \bar{y}', \cdots, \bar{y}^{(n-1)})$ with initial conditions $\bar{y}(x_0)=\bar{y}_0, \bar{y}'(x_0)=\bar{y}_0', \cdots, \bar{y}^{(n-1)}(x_0)=\bar{y}_0^{(n-1)}$, may be transformed by the substitution

$$\bar{y} = y+ \sum_{k=1}^{m+n} \frac{1}{k!}\,\bar{y}_0^{(k)}\cdot(x-x_0)^k+\frac{1}{n}\left(\frac{\partial \bar{f}}{\partial \bar{y}^{(n-1)}}\right)_0(x-x_0)(y-\bar{y}_0)$$

(where $m\geq 0$ is an arbitrarily chosen integer) into a new differential equation for y:

$$\begin{aligned} y^{(n)} &= f(x, y, y', \cdots, y^{(n-1)}) \\ (4) \qquad &= \left\{\bar{f}- \sum_{k=0}^{m} \frac{1}{k!}\,\bar{y}_0^{(k+n)}(x-x_0)^k \right.\\ &\quad \left. -\left(\frac{\partial \bar{f}}{\partial \bar{y}^{(n-1)}}\right)_0 y^{(n-1)}\right\}\Big/\left\{1+\frac{1}{n}\left(\frac{\partial \bar{f}}{\partial \bar{y}^{(n-1)}}\right)_0(x-x_0)\right\}\end{aligned}$$

(of course, the derivatives $\bar{y}_0^{(n+1)}, \cdots, \bar{y}_0^{(m+n)}$ entering this formula must be obtained by analytical differentiation of the given differential equation).

It turns out that, for the transformed differential equation, a modified Runge-Kutta process can be constructed for which the values of $y, y', \cdots, y^{(n-1)}$ obtained at x_0+h deviate only by $O(h^{m+6})$ from the exact values. As a consequence, the order of accuracy of the proposed method is $m+5$ (against 4 for the original Runge-Kutta method).

Thus the method can give quite high accuracy, it requires however that the differential equation can be differentiated analytically m times, and that the transformation of the differential equation is carried out for every integration step anew. Nevertheless, the given examples seem to indicate that the method allows a tremendous saving of computing time, compared with the original Runge-Kutta method.

Unfortunately, formula (4) given by the author is incorrect for $n=1$; it should be replaced by

$$y' = \left\{\bar{f} - \sum_0^m \frac{1}{k!}\,y_0^{(k+1)}(x-x_0)^k - \left(\frac{\partial \bar{f}}{\partial \bar{y}}\right)_0(y-\bar{y}_0)\right\}\Big/\left\{1+\left(\frac{\partial \bar{f}}{\partial \bar{y}}\right)_0(x-x_0)\right\}.$$

H. Rutishauser (Zürich)

12720:

Boudot, P.; Guillou, A. Une méthode matricielle pour la résolution d'un problème différentiel linéaire aux limites du second ordre. Chiffres **3** (1960), 173–180. (English, German and Russian summaries)

The authors refine the finite difference method for solving two-point boundary value problem for a second-order ordinary linear differential equation. At each interior division point the differential equation is replaced by a linear difference equation having three terms, whose truncation error is of order h^6, where h is the common length of the division interval. The linear boundary condition at each boundary point is also replaced by a difference relation which involves the values of the function at the boundary point and 3 or 4 adjacent inner points. In particular, the case of the equation with constant coefficients is studied in detail, and some numerical examples are given in comparison with the well-known usual method. These examples show the satisfactory result of the refinement.

T. Uno (Tokyo)

12721:

Bellman, Richard. Successive approximations and computer storage problems in ordinary differential equations. Comm. ACM **4** (1961), 222–223.

One effective approach to the integration of a nonlinear system of ordinary differential equations, subject to two-point boundary values, consists in using the method of successive approximations. A major difficulty is that whenever the order of the system or the number of values of the independent variable for which the values of the dependent variables are tabulated becomes large, the limited high-speed memory capabilities of modern computers must be considered as a critical factor in passing from one approximation to the next.

The author shows how the memory problem may be overcome at the expense of increased time of computation. The basic idea is to determine the kth approximation not using stored values of the $(k-1)$st approximation but recalculating all earlier approximations as solutions to initial value problems.

R. Kalaba (Santa Monica, Calif.)

12722:

Naruoka, Masao; Ōmura, Hiroshi. Digital computer analysis of influence coefficients for deflection and bending moment of orthotropic parallelogram plates. Mem. Fac. Engrg. Kyoto Univ. **21** (1959), 102–127.

The authors solve the elliptic equation

$$B_x\frac{\partial^4 w}{\partial x^4}+2H\frac{\partial^4 w}{\partial x^2 \partial y^2}+B_y\frac{\partial^4 w}{\partial y^4} = p$$

interior to a parallelogram, corresponding to a plate simply supported on two sides and free on the other two sides. They use straightforward difference techniques and a rather coarse mesh, leading to a low-order coefficient matrix which is inverted directly. Little is said about computers. There are extensive tables of w and related quantities.

M. A. Hyman (Bethesda, Md.)

12723:

Yuan', Čžao-din. Some difference schemes for the numerical solution of differential equations of parabolic type. Mat. Sb. (N.S.) **50 (92)** (1960), 391–422. (Russian)

Let $R=[0<t\leq T]\times Q$, where $Q\in E^m$, and let $S=[0<t\leq T]\times \Gamma$, where $\Gamma=\partial Q$. The author proposes and

analyzes four difference schemes for the numerical solution of the boundary-value problem: (*) $\partial u/\partial t = Lu + f(t, x)$ for $(t, x) \in R$; $u(t, x) = \psi(t, x)$ for $(t, x) \in S$; $u(0, x) = \varphi(x)$ for $x \in Q$. Here f, ψ and φ are given functions, $Lu \equiv \sum_{i,j=1}^{m} (\partial/\partial x_i)[a_{ij}(t, x)\partial u/\partial x_j] - a(t, x)u$ is uniformly elliptic, and $a_{ij} = a_{ji}$. Of the four difference schemes considered, the first is explicit and the remaining three are implicit. In each case the author proves that if (*) has a sufficiently smooth solution then the difference scheme is stable and convergent (in the L_2 mesh norm). Moreover, in each case he estimates the amount of computational labor involved.

Let $L_h u \equiv \frac{1}{2}\sum_{i,j=1}^{m} [(a_{ij}u_{x_j})_{\bar{x}_i} + (a_{ij}u_{\bar{x}_j})_{x_i}] - au$, where u_x and $u_{\bar{x}}$ denote forward and backward divided differences in x respectively. To compute $u(t+\tau, x)$ from $u(t, x)$ by the author's explicit scheme one proceeds by the following n-step routine. Let $\tau = \tau_1 + \cdots + \tau_n$ where

$$\tau_k = 2[\Lambda(1+\cos(2k-1)\pi)/2n]^{-1}$$

and Λ is an upper bound for the eigenvalues of L_h. Then, for $k = 1, 2, \cdots, n$,

$$u(t+\tau_1+\cdots+\tau_k, x) = (1+\tau_k L_h)u(t+\tau_1+\cdots+\tau_{k-1}, x) + \tau_k f(t+\tau_1+\cdots+\tau_{k-1}, x)$$

for $x \in Q$ and $u = \psi$ on Γ. An example of the type of implicit schemes considered is the following. Let

$$b_k = \tau\Lambda\left(\cos\frac{\pi}{2n} - \cos\frac{(2k-1)}{2n}\pi\right)\left(1+\cos\frac{\pi}{2n}\right)^{-1} \quad \text{for } k = 1, \cdots, n,$$

where $\tau\Lambda \leq \cot^2(\pi/4n)$. Then, for $k = 1, 2, \cdots, n$,

$$u^{(k)}(t+\tau, x) = (b_k+1)^{-1}[(b_k+\tau L_h)u^{(k-1)}(t+\tau, x) + u(t, x) + \tau f(t+\tau, x)]$$

for $x \in Q$ and $u^{(k)} = \psi$ on Γ.

The schemes considered are related to methods employed by D. Young [J. Mathematical Phys. **32** (1954), 243–255; MR **15**, 650], and by Douglas and Rachford [Trans. Amer. Math. Soc. **82** (1956), 421–439; MR **18**, 827].

D. G. Aronson (Minneapolis, Minn.)

12724:

Meltzer, B. The stability of computation of the Pierce-Cauchy problem. J. Electronics Control (1) **8** (1960), 449–453.

The author discusses the finite-difference solution of Laplace's equation for Cauchy-type boundary conditions. Such problems arise in the design of Pierce electron guns, in transonic flow, etc. The author obtains the growth rate for numerical errors during a "marching" solution, by solving Laplace's difference equation analytically. He proposes to avoid the well-known instability by calculating all values of the solution from such analytical formulas. Most of the results are contained in the reviewer's paper [Appl. Sci. Res. B **2** (1952), 325–351; MR **13**, 993] where the marching (or "stepping-ahead") solution of rather general elliptic boundary-value problems is proposed, analytical formulas derived, growth rates calculated, etc. In particular, the reviewer has shown how to control error growth so as to be able to use the very simple (and non-iterative) marching methods for elliptic boundary-value problems. In elliptic problems marching methods are unstable but convergent, in contrast to the situation for well-posed parabolic and hyperbolic problems, where stability and convergence (the sine qua non) usually go hand-in-hand.

M. A. Hyman (Bethesda, Md.)

12725:

Lees, Milton. A priori estimates for the solutions of difference approximations to parabolic partial differential equations. Duke Math. J. **27** (1960), 297–311.

The author derives in a systematic manner, by employing the energy method, a priori estimates for solutions of several difference analogues of parabolic differential equations. All the standard two-level difference equations are treated, and two simple three-level formulae are considered. The arguments are presented in detail for the heat equation and generalizations are indicated. A number of the basic difference identities and inequalities that are useful in the energy approach are collected at the beginning of the article.

J. Douglas, Jr. (Houston, Tex.)

12726:

Laasonen, Pentti. On the solution of Poisson's difference equation. J. Assoc. Comput. Mach. **5** (1958), 370–382.

The author derives bounds for the truncation error that arises if the first boundary-value problem for Poisson's differential equation in two dimensions is replaced by its simplest difference equation analog, obtained by replacing the Laplacian by the linear combination of the values at five points of a square grid. The new feature of the results is that the solution of the differential problem is not required to have bounded derivatives of order three or four. This is achieved by first studying the truncation error inherent in Green's function for the discrete problem by arguments related to those of McCrea and Whipple [Proc. Roy. Soc. Edinburgh **60** (1939/40), 281–298] and Wasow [Quart. Appl. Math. **15** (1957), 53–63; MR **19**, 582]. If the domain of the problem is a rectangle, very explicit and simple inequalities for the truncation error in terms of the mesh length and the modulus of continuity of the non-homogeneous term are given. For domains bounded by finitely many analytic arcs and with suitably defined approximating boundary conditions the order of magnitude of the truncation error is obtained.

W. Wasow (Madison, Wis.)

12727:

Gates, W. Lawrence. Note on the suppression of the oscillation (weak instability) of first-forward-then-centered time differences in numerical integration. J. Meteorol. **17** (1960), 572–574.

The linearized barotropic vorticity equation is

$$(*) \qquad \frac{\partial^3\psi}{\partial t\partial x^2} + U\frac{\partial^3\psi}{\partial x^3} + \beta\frac{\partial\psi}{\partial x} = 0,$$

where U, β are constants. In an earlier paper [same J. **16** (1959), 556–568; MR **21** #6273] the author solved exactly the difference equations arising in a conventional approach to the numerical solution of (*). He now solves exactly the difference equations which arise in a modified approach. This modified approach replaces the single forward step to Δt, by a forward step to $\Delta t/4$, followed by a "centered" step to $\Delta t/2$, and another to Δt. Numerical values of the result indicate that use of the modified starting procedure

leads to a marked reduction in the amplitude oscillations, and to a slight improvement in the phase speed, as compared to the conventional approach.
T. E. Hull (Vancouver, B.C.)

12728:
Jones, J. G. On the numerical solution of convolution integral equations and systems of such equations. Math. Comp. **15** (1961), 131–142.
The paper considers an integral equation of the Volterra type

$$ag(x)-\int_0^x W(x-\xi)g(\xi)\,d\xi = f(x),$$

and simultaneous systems of such equations. For a single equation it is shown that the standard step-by-step process, with the trapezoidal rule for the integral, converges to the true solution of the difference equation like $O(h^2)$, where h is the interval size, if the second derivatives of g and W are bounded. If a is zero, the error will usually oscillate in successive steps. In this case the resulting equation of first kind can be converted into one of second kind, and the advantages and disadvantages of this method are discussed, particularly when W and f are given numerically. A simple numerical example illustrates the argument. The latter is extended to the case of simultaneous integral equations, the constant a for the single equation being analogous to the rank of a certain matrix in the simultaneous case, and there is a similar possibility of converting to equations of the second kind by differentiations and additions. *L. Fox* (Oxford)

12729:
Vorbrodt, M. Graphs of composite functions. Wiadom. Mat. (2) **3**, 285–293 (1960). (Polish)
A number of examples of graphs of composites of algebraic functions.

12730:
Fischer, Johannes. Projektiv-verzerrte Netze. Les mathématiques de l'ingénieur, pp. 266–269. Mém. Publ. Soc. Sci. Arts Lett. Hainaut, vol. hors série, 1958.

12731:
Chelazzi, Mirko. La determinazione grafica degli angoli. Archimede **12** (1960), 102–105.
The equation of the spiral of Archimedes in polar coordinates is $\rho=k\theta$. Having the graph of such a spiral with pitch $2p$, one can determine the arc $\alpha=m\pi$ in the following way. From the pole P as center describe an arc mp. The point X of intersection of this arc with the spiral is the extreme point of the arc. The angle is then obtained from standard tables. The author gives the necessary conditions in order that the line PX be determined with sufficient graphical accuracy.
D. Mazkewitsch (Cincinnati, Ohio)

12732:
Otto, Edward. ★Nomografia [Nomography]. Państwowe Wydawnictwo Naukowe, Warsaw, 1956. 220 pp. zł. 17.40.
Introductory text.

12733:
Pepinsky, Ray; Robertson, J. M.; Speakman, J. C. (Editors). ★Computing methods and the phase problem in X-ray crystal analysis. Report of a conference held at Glasgow, August, 1960. International Tracts in Computer Science and Technology and Their Application, Vol. 4. Pergamon Press, New York-Oxford-London-Paris, 1961. viii + 326 pp. (6 plates) $9.00.
This book contains the twenty-eight contributed papers and discussion of a very successful conference which preceded by a few days the International Union of Crystallography Congress held in Britain in August, 1960. It is concerned with the methods and logistics of the use of the high-speed computer for the determination of the atomic structure of crystals by X-ray diffraction and, as such, is of primary interest to the X-ray crystallographers. These form a comparatively small international scientific group whose principal interpretative techniques were virtually liberated a few years ago by the development of the modern digital computer. The stimulus of a technical advance which reduced the period of a scientific investigation from a matter of years to a matter of weeks is reflected in the enthusiastic quality of the contributions to this book.
The computers used range from the ZEBRA, Bull-Gamma 3B-AET, IBM 650, Ferranti MARK I, PEGASUS, DEUCE, MERCURY, IBM 704, EDSAC II, SWAC, to the IBM 709 and a preview of MUSE. Since they are all concerned with attempting to solve the same scientific problems, there is contained in this volume an interesting comparison of the effectiveness of a variety of machines for the same type of scientific calculation. From the mild expressions of impatience throughout, it is apparent that they have one thing in common; with the exception of MUSE, they are all too small and too slow to deal with the more complex problems which occur in studying the structure of crystalline matter on the atomic scale.
Some of the mathematical methods, such as the Fourier summations and the least-squares refinements methods, have more general applications, and it might be of interest to the applied mathematician to discover how the crystallographer tackles these methods in circumstances where there is frequently more input or output data than the computer can conveniently handle.
G. A. Jeffrey (Pittsburgh, Pa.)

12734:
Fischer, Johannes. Neuzeitliche Methoden der Nomogrammpraxis. Les mathématiques de l'ingénieur, pp. 70–78. Mém. Publ. Soc. Sci. Arts Lett. Hainaut, vol. hors série, 1958.
A discursive address at a Congress on the advantages of nomographic representation, such as the insight a nomogram gives to the physical significance of a formula, the symmetric manner in which all variables of a formula enter into a nomogram and the use of a nomogram as a basis for a mechanical analogue representation of a formula. The mathematical content of this address is slight, one or two simple formulae being used as examples to illustrate the speaker's points.
J. G. L. Michel (Teddington)

12735:
Delval, J. Sur certains abaques d'équations trigonométriques. Les mathématiques de l'ingénieur, pp. 247–

251. Mém. Publ. Soc. Sci. Arts Lett. Hainaut, vol. hors série, 1958.

The author discusses the solution of the trigonometric equation $a \sin \alpha + b \cos \alpha = c$ (in various forms) by means of intersection nomograms, single and double alignment nomograms, and intersection nomograms with a superposable transparency. *J. G. L. Michel* (Teddington)

12736:

Журина, М. И. [Žurina, M. I.]; Кармазина, Л. Н. [Karmazina, L. N.]. ★Таблицы функций Лежандра $P_{-1/2+i\tau}(x)$. Том I [Tables of the Legendre functions $P_{-1/2+i\tau}(x)$. Vol. I]. Akad. Nauk SSSR, Vyčisl. Centr. Matematičeskie Tablicy. Izdat. Akad. Nauk SSSR, Moscow, 1960. 319 pp. 34.50 r.

These are the conical functions of potential theory; they also appear in the Mehler-Fock inversion formula. We write $\nu = -\frac{1}{2} + i\tau$. The present volume gives $P_\nu(x)$ to 7S for $x = 0.9(-0.1)-0.9$ and $\tau = 0(0.01)50$. A second volume will deal with the range $x > 1$. Note that $P_\nu(x)$ is an even function of τ.

The volumes were computed from the hypergeometric series representation

$$P_\nu(x) = F(\tfrac{1}{2} - i\tau, \tfrac{1}{2} + i\tau, 1; \tfrac{1}{2}(1-x)),$$

valid for $-1 < x < 3$, on the "STRELA". They are said to be good within a unit in the last place. The values for $\tau = 0$ were checked by comparison with tables of complete elliptic integrals, using the fact that $P_{-1/2}(\cos\theta) = (2/\pi)K(\sin\frac{1}{2}\theta)$. It was verified that the differential equation $(1-x^2)u'' - 2xu' - (1+4\tau^2)u/4 = 0$ was satisfied for $\tau = 50$. Elsewhere the tables were checked by differencing with respect to τ.

To obtain $P_\nu(x)$ for larger values of τ, there is given a table of coefficients $A_i(\theta)$ $(i = 0, 1, 2, 3)$ for use in the asymptotic expansion of Fock

$$P_\nu(\cos\theta) = I_0(\tau\theta)[A_0 + A_2\tau^{-2} - 4\theta^{-1}A_3\tau^{-4} + \cdots] + I_1(\tau\theta)[A_1\tau^{-1} - \{2\theta^{-1}A_2 - A_3\}\tau^{-3} + 8\theta^{-2}A_3\tau^{-5} + \cdots].$$

The $A_i(\theta)$ are given to 7D for $x = \cos\theta = 0.99(-0.01)-0.9$. Adequate tables of I_0 and I_1 are available. This expansion gives at least 5 correct figures for $\tau \geq 50$.

A drawing of the surface $z = P_\nu(x)$ for $-1 < x \leq 1$, $0 \leq \tau \leq 14$ is given. This shows the monotonic behavior of $P_\nu(x)$ in both variables and the approximation $P_\nu(x) \approx (-\pi^{-1}\cosh \pi\tau) \log_e \frac{1}{2}(x+1)$, as $x \to -1$.

Quadratic interpolation with respect to τ is satisfactory throughout, the resulting error not exceeding 1.6 units in the last place. Interpolation with respect to x requires at least 5-point methods to get 6 correct figures, and the order increases as $x \to -1$, in view of the singularity there.

The tables are produced in the entirely satisfactory style of this series. This is apparently the first significant tabulation of this function. *John Todd* (Pasadena, Calif.)

12737:

Harumi, Kasaburô; Katsura, Shigetoshi; Wrench, John W., Jr. Values of $\frac{2}{\pi}\int_0^\infty \left(\frac{\sin t}{t}\right)^n dt$. Math. Comp. **14** (1960), 379.

The authors remark that the integral in the title arises in certain applications, and present a table of its values, to ten decimal places, for $n = 1(1)30$.

S. Haber (Washington, D.C.)

12738:

Salzer, Herbert E.; Levine, Norman. Table of a Weierstrass continuous non-differentiable function. Math. Comp. **15** (1961), 120–130.

The famous continuous non-differentiable function of Weierstrass

$$W(a, b, x) = \sum_{n=1}^\infty a^n \cos(b^n \pi x),$$

where $0 < a < 1$, $ab > 1 + 3\pi(1-a)/2$, b an odd integer, is tabulated for $a = \frac{1}{2}$, $b = 7$, $x = 0(.001).5$ to 12 decimals. The range for x is adequate since $W(a, b, 1+x) = -W(a, b, x)$ and $W(a, b, \frac{1}{2}+x) = -W(a, b, \frac{1}{2}-x)$. There is also a polygonal graph of the function. The table is intended as a proving ground with which to test out formulas for numerical differentiation, interpolation and quadrature whose remainders are usually estimated in terms of the derivative. Some of the experiments are discussed.

D. H. Lehmer (Berkeley, Calif.)

12739:

Gloden, A. Table des solutions minima de la congruence $x^4 + 1 \equiv 0 \pmod p$ pour $4000 < p < 10000$. Chiffres **3** (1960), 171–172. (English, German and Russian summaries)

The title of this table is misprinted. The modulus should be p^2. The two solutions less than $p^2/2$ are given for each of the 168 primes p between the limits indicated for which the congruence is possible. This table is an extension of the author's previous work in Scripta Math. **21** (1955), 218 [MR **17**, 347]. *D. H. Lehmer* (Berkeley, Calif.)

12740:

Faddeyeva, V. N.; Terent'ev, N. M. ★Tables of values of the function $w(z) = e^{-z^2}(1 + 2i\pi^{-1/2}\int_0^z e^{t^2}\,dt)$ for complex argument. Edited by V. A. Fok; translated from the Russian by D. G. Fry. Mathematical Tables Series, Vol. 11. Pergamon Press, Oxford-London-New York-Paris, 1961. v+280 pp. £5; $15.00; 90NF.

The original [Gosudarstv. Izdat. Tehn.-Teor. Lit., Moscow, 1954] was reviewed in MR **16**, 960 and a translation of the introduction [Friedman, Newtonville, Mass., 1956] noted in MR **18**, 155.

COMPUTING MACHINES

See also A12023, A12060, 12722, 12866.

12741:

Grabbe, Eugene M.; Ramo, Simon; Wooldridge, Dean E. (Editors). ★Handbook of automation, computation, and control, Vol. 2: Computers and data processing. John Wiley & Sons, Inc., New York; Chapman & Hall, Ltd., London; 1959. xxiii+1070 pp. $17.50.

This volume is of great value to anyone interested in either the use or the design of modern computers. Volume I (1958) was reviewed in MR **20** #2093. The present volume has six sections: A. Computer terminology. B. Digital computer programming. C. The use of digital computers and data processors. D. Design of digital computers. E. Design and application of analog computers. F. Unusual computer systems. Of these, D and B

are the longest, and together occupy a few more than half the pages.

These six sections are subdivided into 31 chapters, written by a total of 41 authors from many different American corporations and universities. Rather than being a handbook in the sense of a cookbook or a table of integrals, this volume mainly gives a broad survey of many different aspects of computers, and is surprisingly free of overlapping and omissions, for a volume with so many authors. Its chief use will be for those who are acquainted only with certain aspects of computers, and wish to broaden their background in other areas, ranging from accounting applications or magnetic core circuits to handling of non-numerical information. It certainly belongs on the circulation shelves of libraries, as well as on the reference shelves. Despite its price, it also belongs on the desk of anyone who has a professional interest in computers.

Although most of the volume is written on a fairly uniform, satisfactory level, there are two chapters which deviate notably in opposite directions. Chapter 1, "Computer terminology and symbols", is a totally inadequate attempt at a glossary, chiefly based on two old incomplete glossaries published long ago by professional societies. It would be easy to compile a list of many hundreds of words and phrases of computer jargon, actually used in this volume, which are not defined in the glossary. Such items in this volume but omitted from the glossary include not only relatively obscure terms such as "acronym", "stretched program", or "strappable load", but also such commonly used terms as "Sheffer stroke function", "algorithm", "asynchronous", "index register", and "mnemonic". This deficiency of the glossary is somewhat alleviated by the fairly thorough combined index to the entire volume, occupying 37 pages. Most of the above-mentioned terms can be found in this index, and a sufficiently ingenious reader could define most of them from the contexts on the pages from which they are indexed.

Chapter 2, "Programming and coding", by John W. Carr III, is 270 pages long, and is not only the longest but also the most valuable chapter of the volume. The opinion had long been expressed by many of the more advanced and abstract programmers that it is impossible to write a book about programming without having it be either cryptic, watered-down, or obsolete. This book is a counterexample to that opinion, since it has surprisingly little of any of these defects. Many of these programmers have already eaten their words, found that this chapter is really the book which had long been thought to be impossible, and decided that this chapter alone is worth the price of the entire volume.

This chapter is an extension of some course notes which had been prepared by Carr at various times for use in courses on computer programming. It is remarkably thorough in its coverage. The author has done a notable job of winnowing through the tremendous amount of material which has been written about programming, both the published literature and many unpublished program descriptions, distributed partly by grapevine among the programmers, and partly by the computer users' groups. He has selected, not only the material about the most important and interesting computers, but about the most important kinds of programming. He not only condenses a large amount of data about various computers and programming languages, but also gives the background of the arguments for and against different kinds of number representation, instruction formats, debugging methods, and methods of automatic programming. His language is very typical of the modern jargon of programmers, but most of it is explained as it arises. His point of view is quite general and abstract. His explanations of the difficulties of, and the motivations for, various ways of programming give very useful insights into the way of thinking of the programmers.

Although there are occasional parts of the chapter that are obviously several years out of date, it has undergone extensive revision, and is mainly quite modern. Almost half of the chapter is devoted to automatic programming of various kinds, and covers many topics about automatic programming that are not treated in any other books. Every programmer, regardless of how amateurish or professional, will find much material of value to him in this chapter.

In view of the importance which this chapter should have for programmers, the publisher would be providing an important service if he were to issue it (or perhaps even a further revision of it, to bring it a year or more closer to being current) as a separate volume. Such a book would be more suitable as a textbook for a college course in programming for scientists or mathematicians than any currently available book, and such republication could put it within the price range that would be feasible for such use. *E. F. Moore* (Murray Hill, N.J.)

12742:

Hund, Gerhard; Möhlen, Wolfgang. Bericht über die britischen Rechenanlagen. Bl. Deutsch. Ges. Versicherungsmath. 4 (1959/60), 454–462. (3 inserts)

Detailed information on, and comparison of, digital computers built in Great Britain. *F. L. Bauer* (Mainz)

12743:

Fangmeyer, Hermann; Mertens, Rudolf. Drei Kurzbeschreibungen von Grossrechenanlagen unter besonderer Berücksichtigung der Datenverarbeitung. Bl. Deutsch. Ges. Versicherungsmath. 4 (1959/60), 463–468.

Condensed information on the computers IBM 7070, Siemens 2002, Bull Gamma 60. *F. L. Bauer* (Mainz)

12744:

Kämmerer, Wilhelm. ★Ziffernrechenautomaten. 2. unveränderte Aufl. Elektronisches Rechnen und Regeln, Bd. 1. Akademie-Verlag, Berlin, 1960. viii+303 pp. DM 29.00.

This volume, one of the first of its kind written in German, gives a broad view on the various aspects of electronic computing, covering all fields from technical details to automatic programming (but not including general data processing).

The first chapter (30 pages) is an easy readable introduction to Boolean algebra which serves at the same time as a basis for the later chapters. Special consideration is given to the transformation of logical functions into one of the normal forms and to the reduction of the latter to simpler expressions.

In the second chapter (81 pages) the principles of the circuits for executing arithmetic operations, as well as their dynamic behaviour, are described in great detail. The author uses block diagrams to explain the various kinds of circuits from half-adders, adders for 3-excess representation of decimal numbers, up to the wiring for multiplication and division. The dynamic behaviour is explained by examples. Error-detecting and error-correcting codes are also discussed.

Chapter three (48 pages) treats the general principles of automatic computers (storage, sequencing, address-systems, etc.), and for 2 specific computers, namely the relay computer OPREMA (Eastern Germany) and the M3 (Russia) detailed descriptions of the sequencing circuits and other characteristics are given. The pilot ACE and the Zuse Z22 are also described in this way, but with less details.

The fourth chapter (38 pages) deals with computer hardware, down to the actual wiring of the circuits for the logical operations "and", "or" and "not", which appear as basic elements in Chapters two and three. Also the various methods for storing information (some of which are now obsolete) are treated, with special attention given to magnetic cores.

Finally, the fifth chapter (99 pages) is devoted to programming. A large proportion of that chapter deals with the use of library subroutines, especially with the problem of feeding the subroutine with the parameters which it needs for proper functioning. The technique described for that purpose is mainly that which stores the parameters immediately following the order which transfers control to the subroutine [see also Samelson and Bauer, Z. Angew. Math. Mech. **34** (1954), 262–272; MR **16**, 526], but the author shows how this technique may be simplified further by using index-registers. Chapter five contains also a section on automatic programming, where interpretive and compiling techniques are described. It should be clear, however, that the method of representing and compiling an algebraic formula as proposed by the reviewer in 1951 is now obsolete and is described by the author only as an example. *H. Rutishauser* (Zürich)

12745:

Reitwiesner, George W. Binary arithmetic. Advances in computers, Vol. 1, pp. 231–308. Academic Press, New York, 1960.

A very thorough (sometimes lengthy) study of binary arithmetic operations including square rooting. Special consideration is given to the $\{-1, 0, 1\}$-representation of binary numbers, which, for example, economises the number of additions and subtractions needed in multiplication. The paper gives valuable help to people involved in design of arithmetical units. *F. L. Bauer* (Mainz)

12746:

Fraenkel, Aviezri S. The use of index calculus and Mersenne primes for the design of a high-speed digital multiplier. J. Assoc. Comput. Mach. 8 (1961), 87–96.

A method is described which might be used to multiply small integers by addition of their indices with respect to some base for a Mersenne prime modulus. Computing time is saved but considerable high-speed storage is required for the table of indices. *C. B. Haselgrove* (Manchester)

12747:

Reitwiesner, George W. The determination of carry propagation length for binary addition. IRE Trans. **EC-9** (1960), 35–38.

A formula is derived for the determination of the expected maximum length of zero or non-zero carry propagation in the addition of two binary numbers of n digits each. *C. B. Haselgrove* (Manchester)

12748:

Vauquois, Bernard. Calculateurs électriques et pensée logique. Rev. Questions Sci. (5) **20** (1959), 93–118.

In der vorliegenden Arbeit werden die logischen Grundlagen für Konstruktion und Anwendung digitaler Rechenautomaten behandelt. Es werden die Elemente der Booleschen Algebra gestreift. Die elementaren Operationen der Booleschen Algebra lassen sich bekanntlich leicht durch elektrische Schaltungen realisieren. Diese Schaltungen bilden die Grundbausteine digitaler Rechengeräte.

In einem weiteren Abschnitt werden die Turing-Maschinen eingeführt. Eine Turing-Maschine kann man als das theoretische Vorbild eines Digitalrechners ansehen. Mit Hilfe dieser Theorie gelang eine einleuchtende Abgrenzung der Klasse der berechenbaren Funktionen, von der sich zeigen lässt, dass sie mit der Klasse der rekursiven Funktionen übereinstimmt.

Im dritten Abschnitt geht der Verfasser dann auf die Digitalrechner selbst ein. Es wird insbesondere die Sprache der Maschinenprogramme behandelt. Etwas künstlich erscheint dem Ref. die Unterscheidung in Maschinen, bei denen das Programm in der Ordnung der Speicherplätze abläuft, und in solche, bei denen das nicht notwendig der Fall ist, bei denen also bei jedem Befehl die Adresse des Folgebefehls angegeben werden muss. Die modernen Maschinen enthalten in der Regel beide Möglichkeiten. Es werden weiterhin leichte Beispiele für Programme gegeben. Ebenso wird der Begriff des Unterprogrammes und die relative Adressierung erläutert. Schliesslich geht der Verfasser noch auf reichere Programmsprachen und Autocoder ein.

Der letzte Abschnitt enthält einige Bemerkungen über das Problem der "lernenden Maschine".

Abschliessend sei noch ein Druckfehler erwähnt: Auf Seite 106; die Zeilen 5 und 6 müssen heissen: (3) 131→intersection de 131 avec···; (2) 150→transfert en 150 de la···. *H. Kiesow* (Münster)

12749:

Gaudfernau, Claire Liliane. Le calcul automatique en recherche aéronautique. Les mathématiques de l'ingénieur, pp. 305–313. Mém. Publ. Soc. Sci. Arts Lett. Hainaut, vol. hors série, 1958.

12750:

Harker, Kenneth J. Determination of electrode shapes for axially symmetric electron guns. J. Appl. Phys. **31** (1960), 2165–2170.

The mathematical problem considered is the solution of Laplace's equation for specified boundary voltage and normal field on an open curve. Following Garabedian and

Lieberstein [J. Aero. Sci. **25** (1958), 109–118; MR **19**, 1007] the problem has been reformulated so as to be properly set. After a conformal transformation which maps the beam boundary into a coordinate axis, an analytic continuation is made of Laplace's equation and its boundary values into a fictitious complex domain. Laplace's equation, which is elliptic in the real domain is thus converted into a set of hyperbolic equations. This leads to a stable scheme of computation by finite differences. The method is applicable to any configuration where the boundary conditions are given through analytic functions, possibly implicitly, as through a set of differential equations. *J. E. Rosenthal* (Passaic, N.J.)

12751:

"INTINT": Programmazione indiretta per calcolatrici elettroniche. Consiglio Naz. Ricerche Pubbl. Ist. Appl. Calcolo No. 526. Manuali per Applicazioni Tecniche del Calcolo, III. Edizioni Cremonese, Rome, 1958. 74 pp. (1 insert) L. 1000.

This handbook contains an exposition of an interpretive program (INTINT) designed to allow persons who are not trained programmers to use the FINAC at l'Istituto Nazionale per le Applicazioni del Calcolo in Rome. The programming is carried out in terms of pseudo-instructions, each containing three addresses and a function code, together with certain indicators which specify whether the quantities involved are to be treated as scalars or vectors. Aside from input, output and book-keeping commands, the functions provide for direct call-up, for example, of square root, absolute value, scalar product, and certain trigonometric, hyperbolic, and logarithmic subroutines. A number of well-chosen examples show in detail how one uses the program. The authors point out that INTINT can be adapted to other computers and express the hope that it will be favorably accepted throughout Italy.

R. N. Goss (San Diego, Calif.)

12752:

Yates, F.; Simpson, H. R. A general program for the analysis of surveys. Comput. J. **3** (1960/61), 136–140.

The program described [F. Yates, *Sampling methods for censuses and surveys*, 3rd ed., Griffin, London, 1960] is capable of performing the varied operations commonly required in the analysis of survey material. It was possible to write this program for a small computer (the Elliott 401) by using compact machine language, and by dividing the computations into two parts. In the first part the sampling units are processed one at a time, and in the second, final tables are produced. *C. C. Gotlieb* (Toronto)

12753:

Marchetti, Y.; Berthier, P. Calcul et programmation du Logarithme Népérien sur I.B.M. 650. Chiffres **3** (1960), 143–148. (English, German and Russian summaries)

12754:

Petrich, J. Trigonometric method for conformal mapping with algebraic polynomials by the use of a repetitive differential analyser. Ann. Assoc. Internat. Calcul. Anal. **3** (1961), 11–17.

The paper describes a method for evaluation of polynomials with complex coefficients by analog computer.

F. Stallmann (Washington, D.C.)

12755:

Miller, K. S.; Walsh, J. B. Initial conditions in computer simulation. IRE Trans. **EC-10** (1961), 78–80.

Authors' summary: "A technique is developed for the straightforward simulation of the transfer function of a certain class of linear systems. This method is particularly well adapted to the analysis of systems with fixed transfer function and variable initial conditions and forcing functions. In particular, a single simulation, minimal in its use of integrators, will suffice to handle forcing functions and initial conditions on both input and output."

F. Edelman (Princeton, N.J.)

12756:

Wong, Ming S. Ionospheric ray tracing with analogue computer. Electromagnetic wave propagation, pp. 37–48. Academic Press, London, 1960.

12757:

Karplus, Walter J.; Soroka, Walter W. ★Analog methods: Computation and simulation. 2nd ed. McGraw-Hill Series in Engineering Sciences. McGraw-Hill Book Co., Inc., New York-Toronto-London, 1959. xiii+483 pp. $12.50.

Analogue computers can be considered from a variety of standpoints. There is a continuous gradation from special-purpose dynamical analogies (called "direct computers" or "simulators" by the authors), which are physical ensembles satisfying the same equations as the system under study, through more flexible collections of apparatus, with which the user is still largely conscious of the physical analogy, to abstract instruments ("indirect computers"), the purpose of which is to solve mathematical problems which may be several stages removed from the physical problem in which they arise. By the time the word computer has been attached to the apparatus it has probably developed in the direction of generality and abstractness. The requirements of readers of works on analogue computation are also likely to range from an engineering interest in hardware and the design of computers to an interest in their use and field of application. In steering a course between these wide ranges of interests the authors have firstly decided that, whilst giving a good coverage of mechanical elements, detailed treatment of mechanical differential analysers is outmoded, and they have replaced discussions on techniques for their employment in an earlier edition by extensions in various aspects of electrical and electronic differential analysers. After an introductory chapter, Part I (Chapters 2–5) of the book deals with The elements of (indirect) computers, with chapter titles: Linear electrical computing elements; Electrical multipliers and dividers; Electrical function generators; Mechanical computing elements. Part II (Chapters 6–8) covers Indirect computers, with chapter titles: Differential analyzers; Machines for simultaneous linear algebraic equations; Analog solution of nonlinear algebraic equations. Part III (Chapters 9–11) deals with Direct computers (simulators) with chapter titles: Dynamical analogies; Finite difference networks; Continuous field analogs. There are appendices: (1) Tables of

transfer functions; (2) Problems; (3) Laboratory experiments.

Whilst not ignoring physical problems tractable to analogue methods, the orientation of the book is towards design and general engineering and mathematical techniques; it is aimed at engineers rather than mathematicians, but this in no way militates against its value to mathematicians concerned with analogue devices. There are extensive bibliographies which should enable a reader with specialist interests to supplement any shortcomings. Inevitably there are some misprints (two or three were noticed in Table 4.2, p. 79) but the reviewer unhesitatingly recommends this book as a comprehensive guide to analogue methods. *J. G. L. Michel* (Teddington)

12758:

Peretz, Richard. Opérateurs mathématiques analogiques. Les mathématiques de l'ingénieur, pp. 375–381. Mém. Publ. Soc. Sci. Arts Lett. Hainaut, vol. hors série, 1958.

12759:

Johnson, Clarence L. ★Analog computer techniques. McGraw-Hill Book Co., Inc., New York-Toronto-London, 1956. xi + 264 pp. $7.00.

This book appears to be aimed mainly at engineers learning to use analogue computers, so it is perhaps pedantic of a mathematician to feel that in the very useful background mathematics there is sometimes a looseness which might be dangerous. The scope of the book is analogue computers of the differential-analyser type and digital differential analysers. The author restricts himself to commercial equipment—commercial equipment of the U.S.A. only—existing at the date of publication, 1956; further, most of the examples and techniques are written around the use of REAC. The chapter headings are as follows: (1) Introduction; (2) Linear computer components; (3) Time- and amplitude-scale factors; (4) Synthesis of servomechanism systems; (5) Multiplying and resolving servos; (6) Additional computer techniques; (7) Representation of nonlinear phenomena; (8) Multipliers and function generators; (9) Miscellaneous applications of the electronic analog computer; (10) Analog computer components and computer control; (11) The checking of computer results; (12) Repetitive analog computers; (13) The digital integrating differential analyzer. There are five appendixes giving mathematical background and techniques.

J. G. L. Michel (Teddington)

MECHANICS OF PARTICLES AND SYSTEMS

See also A12261, A12429, 12613, 12857, 12858, 13222, 13246.

12760:

Legendre, Robert. Répartition des contraintes dans une bande ou dans un rectangle. C. R. Acad. Sci. Paris **250** (1960), 3108–3110.

Author's summary: "La répartition des contraintes dans une bande est calculée. Le principe d'une généralisation au calcul de la répartition dans un rectangle est indiqué."

12761:

Kilmister, C. W.; Tupper, B. O. J. The analysis of observations. III. Quart. J. Math. Oxford Ser. (2) **10** (1959), 303–312.

[For part II, see Kilmister, same J. **6** (1955), 161–172; MR **17**, 558.] It is shown that Eddington's proof of the mass-ratio equation as given in his book, *Fundamental theory* [Cambridge Univ. Press, London, 1946; MR **11**, 144] is invalid. The authors carry through a derivation by restricting the physical systems. "The physical interpretation of this restriction is, however, far from clear."

A. H. Taub (Urbana, Ill.)

12762:

Резняков, А. Б. [Reznyakov, A. B.]. ★Метод подобия [The method of similitude]. Izdat. Akad. Nauk Kazah. SSR, Alma-Ata, 1959. 151 pp. 0.54 r.

This booklet, aimed "at giving a practical guide to the method, and not the theory of similitude", is written in the spirit of the late M. V. Kirpičev.

S. Drobot (Notre Dame, Ind.)

12763:

Goodey, W. J. On the natural modes and frequencies of a suspended chain. Quart. J. Mech. Appl. Math. **14** (1961), 118–127.

The author shows that if angular displacements of chain elements are used as independent variables then only a second-order equation is needed in place of the fourth-order equation used by Saxon and Cahn [same Quart. **6** (1953), 273–285; MR **15**, 172]. The author finds general approximate solutions (suspension points at same level) for the odd and even modes and simplified approximations for the shallow suspensions. The author shows that the equation reduces to the Bessel equation of order zero when the suspension points coincide and give brief discussion for that special case. *M. G. Scherberg* (Dayton, Ohio)

12764:

Romiti, Ario. Sopra un tipo di ingranaggi conici per assi concorrenti e sghembi. Atti Accad. Sci. Torino. Cl. Sci. Fis. Mat. Nat. **94** (1959/60), 313–332.

Author's summary: "Si sono determinate le proprietà geometriche, cinematiche e statiche di ruote dentate coniche, atte ad ingranare fra di loro con angoli e distanze fra gli assi scelti a piacere, ed adatte anche ad ingranare con ruote cilindriche a denti diritti ad evolvente."

12765:

Sagdeev, R. Z.; Šafranov, V. D. On the instability of a plasma with an anisotropic distribution of velocities in a magnetic field. Ž. Èksper. Teoret. Fiz. **39** (1960), 181–184 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 130–132.

The authors consider a plasma situated in a uniform magnetic field. They show that even a weak temperature anisotropy, $|T_\perp - T_\parallel|/T_\perp \ll 1$, is sufficient to cause instability provided the magnetic field is not too large. The

instability arises from particles in the "tail" of the velocity distribution coming into cyclotron resonance with the perturbing wave. *R. M. May* (Cambridge, Mass.)

12766:

Kucenko, A. B.; Stepanov, K. N. Instability of plasma with anisotropic distribution of ion and electron velocities. Ž. Èksper. Teoret. Fiz. **38** (1960), 1840–1846 (Russian. English summary); translated as Soviet Physics. JETP **11**, 1323–1326.

A study is made of instabilities which arise in a plasma in a homogeneous magnetic field due to anisotropy in the particles' velocity distribution function. The study is similar to, but more detailed than, that of Sagdeev and Šafranov [#12765]. Both treatments are of a kinetic nature, which leads to an enlargement of the instability region compared to that obtained from a hydrodynamic approximation. *R. M. May* (Cambridge, Mass.)

12767:

Gaĭduk, V. I. On nonlinear oscillations in a weak external field, a description by Lagrange's equations. Dokl. Akad. Nauk SSSR **133** (1960), 760–763; errata, **136** (1961), 1264 (Russian); translated as Soviet Physics. Dokl. **5** (1961), 713–717.

Approximate solutions of Lagrange's equations are examined for (1) uniform rotary motion and (ii) approximately periodic motion in a weak perturbing field. The accuracy of these solutions is not considered. *A. W. Babister* (Glasgow)

12768:

Lamb, Horace. ★Dynamics. Cambridge University Press, New York, 1960. xi+351 pp. $3.75.

This is a reprint of the second [Cambridge Univ. Press, London, 1923] edition of this standard work.

12769:

de Possel, René. Influence des roues profilées et de la loi de frottement sur le mouvement de lacet d'un véhicule de chemin de fer. Les mathématiques de l'ingénieur, pp. 32–43. Mém. Publ. Soc. Sci. Arts Lett. Hainaut, vol. hors série, 1958.

12770:

Brearley, M. N. The motion of a biased bowl with perturbing projection conditions. Proc. Cambridge Philos. Soc. **57** (1961), 131–151.

This is a more accurate theory of the motion of a bowl [Brearley and Bolt, Quart. J. Mech. Appl. Math. **11** (1958), 351–363; MR **20** #1444] in that initial wobble of the bowl is considered, and is shown to damp out. The trajectories of the paths are studied and their end points given in terms of initial conditions. Excellent agreement with experiment is found.

This paper and its predecessor are worthy of the attention of many who may never see a bowl. In the derivation of remarkably accurate approximations to the solutions of difficult nonlinear differential equations systems they are classic examples of the best applied mathematical art. *E. Pinney* (Berkeley, Calif.)

12771:

Toporova, V. A. On integration of the equations of rotation of a heavy solid body around a fixed point in the general case of Goryačev-Čaplygin. Izv. Akad. Nauk UzSSR. Ser. Fiz.-Mat. **1958**, no. 2, 77–85. (Russian. Uzbek summary)

D. N. Goryačev found a particular case of heavy-body rotation about a fixed point when differential equations can be completely integrated. Subsequently S. A. Čaplygin demonstrated that this case can be fully investigated on the basis of the transformation of hyperelliptic integrals. These methods are applied for a general case of rotation of heavy bodies about a fixed point.
R. M. Evan-Iwanowski (Syracuse, N.Y.)

12772:

Klotter, K.; Kreyszig, E. On a special class of self-sustained oscillations. J. Appl. Mech. **27** (1960), 568–574.

The paper deals with the modified van der Pol equation

$$\ddot{q}-(\operatorname{sgn}\dot{q})(\delta/2)\dot{q}^2h(q)+k^2f(q) = 0,$$

in which $f(q)$ is an odd function (positive for positive q) and $h(q)$ is an even function (positive or negative as q is small or large). With these assumptions, it is shown that there exists a unique limit cycle. A method is given for determining upper and lower bounds of this cycle (for unrestricted values of δ) in terms of tabulated functions. Simple formulae are given for small values of δ.
A. W. Babister (Glasgow)

12773:

Lasher, G. J. The dynamics of a subharmonic oscillator with linear dissipation. IBM J. Res. Develop. **5** (1961), 157–161.

A first approximation to the stable subharmonic oscillation of $\ddot{q}_i+\omega_i^2q_i+A_i(A_1q_1+A_2q_i)^2=-(\omega_i|Q_i)\dot{q}_i+e_i\sin\omega_it$ $(i=1, 2;\ \omega_2=2\omega_i)$ is exhibited and employed to discuss the exponential approach to the steady oscillation.
S. P. Diliberto (Berkeley, Calif.)

12774:

Rocard, Y. Instabilités et vitesses critiques dans les systèmes oscillants mécaniques. Les mathématiques de l'ingénieur, pp. 133–149. Mém. Publ. Soc. Sci. Arts Lett. Hainaut, vol. hors série, 1958.

12775:

Duffin, R. J. The Rayleigh-Ritz method for dissipative or gyroscopic systems. Quart. Appl. Math. **18** (1960/61), 215–221.

Rayleigh's principle that for a conservative mechanical system the natural frequencies of vibration about static equilibrium are increased by additional constraints is shown to hold also for vibration about steady motion.
R. C. T. Smith (Armidale)

12776:

de Castro Brzezicki, A. ★Mecanica no-lineal [Non-linear mechanics]. Publicaciones del Instituto de Calculo del Consejo Superior de Investigaciones Cientificas. Editorial Dossat, S. A., Madrid, 1959. 106 pp. 76 pesetas.

In this excellent book, the fundamentals of the theory of oscillations, in particular, non-linear oscillations, is

presented. It is timely since an extraordinary number of phenomena of this type constantly present themselves in science and engineering, in particular, in mechanics, electricity, elasticity, and economics. The work is based on theory and applications which the author developed at the Instituto de Cálculo del Consejo Superior de Investigaciones Científicas in Madrid. It is a sequel to the text on linear oscillations by Dario Maravall [*Ingeniería de las oscilaciones*, Edit. Dossat, Madrid, 1959]. Many problems that do not require too great a degree of precision can be solved by linear differential equations. However, in the case of the phenomena of auto-oscillation and various electrical currents, these cannot be worked out by linear methods. Consequently, the study of non-linear differential equations is necessary. Thus the present text on non-linear oscillations, together with Maravall's book on linear oscillations, serves as an excellent guide for the study of the mathematical theory of oscillations and its applications.

The contents of the book are as follows: I. Plane "autonomous" systems: singular points of equations and differential systems; applications, equilibrium and stability of autonomous systems. II. Introduction to linear and non-linear oscillations: elements of the theory of oscillations and the analogue between electricity and mechanics; conservative, dissipative, and "auto-efficient" systems; examples of relaxation oscillations. III. Study of linear oscillations: (1) equations without a second member; behavior of the solutions of a differential equation; theorem of comparison; (2) equations with a second member: behavior of the integral curves of a differential equation; comparison between the complete and the homogeneous equation. IV. Non-linear oscillations: (1) autonomous systems: Lienard equation; existence of periodic solutions, uniqueness and stability; the equation $\ddot{x}+a\dot{x}+\sin x-b=0$; (2) non-autonomous systems: the theorem of Brouwer; the theorem of Massera; equations with a second member; application to pendulum clocks; relaxation oscillations in statistical time series. V. Numerical methods for the study of non-linear oscillations: method of perturbations; numerical bounds for the limit cycle of the Lienard equation; particular case of the Lienard equation, graphical integration; method of Minorsky; equation of Duffing, iterated integration. In addition to bibliographical notes throughout the text, there is an annotated bibliography of current and related literature. *E. Frank* (Chicago, Ill.)

12777:

Klotter, K.; Cobb, P. R. On the use of nonsinusoidal approximating functions for nonlinear oscillation problems. J. Appl. Mech. **27** (1960), 579–583.

Authors' summary: "When treating nonlinear oscillation problems by the Ritz-Galerkin method, the approximating function is conventionally chosen as a sinusoid or a polynomial of sinusoids with their amplitudes as the open parameters. In this paper the advantages are pointed out that may be gained from choosing a non-sinusoidal approximating function that contains additional parameters related to the shape of the function."

R. C. T. Smith (Armidale)

12778:

Rudolf, Iglisch. Über den Begriff der Resonanz bei linearen und nichtlinearen Schwingungen. Les mathématiques de l'ingénieur, pp. 207–210. Mém. Publ. Soc. Sci. Arts Lett. Hainaut, vol. hors série, 1958.

12779:

Siry, Joseph W. Satellite launching vehicle trajectories. Proc. Sympos. Appl. Math., Vol. 9, pp. 75–144. American Mathematical Society, Providence, R.I., 1959.

Although confined to the Vanguard program, the general features of this study of satellite-launching vehicle trajectories are common to all such programs.

Beginning with one- and two-dimensional cases to illustrate the basic relations, a careful and complete derivation of the equations of three-dimensional trajectories in cartesian coordinate systems, including aerodynamic gravitational and thrust forces, is presented. The allocation of staging weights with respect to maximizing projection velocity is discussed. A numerical method for finding critical points of functions of n variables by means of local linear approximations of the components of the gradient is described. Besides projection velocity other measures of merit such as the probability of success and life-time are considered and a variety of relations are presented graphically for the cases of most interest. An interesting representation is the use of paths in $q-s$ space (apogee height q and perigee heights), which arise because of drag effects. From these paths lifetimes can be estimated. There is no bibliography. A later report by the author with additions and subtractions from the present one is included in *Space technology*, edited by H. S. Seifert [Wiley, New York, 1959; pp. 6-01 to 6-60]. It has a bibliography. *M. Weisfeld* (Menlo Park, Calif.)

12780:

Гантмахер, Ф. Р. [Gantmaher, F. R.]; Левин, Л. М. [Levin, L. M.]. ★Теория полета неуправляемых ракет [Theory of flight of unguided rockets]. Gosudarstv. Izdat. Fiz.-Mat. Lit., Moscow, 1959. 360 pp. 18.45 r.

This is a college textbook on the exterior ballistics of unguided (artillery) rockets. Although the book is dated 1959, the theory is that developed during the war. The authors remark specifically that their book does not reflect the work of Rosser, Newton and Gross in the U.S., of Rankine in England, and of Carrière in France; but it contains roughly equivalent material; so that, in his compilation of history and theory of rocketry, to those three rather independent and nearly simultaneous wartime classics the student of rocketry now should properly add this fourth one.

The chapters headings (and their contents) are: 1. Equations of rocket motion (proofs of the "principle of hardening" of the rocket, viz., of the use of Newton's law with variable mass; by Coriolis forces are properly meant what is usually called "jet-damping" forces); 2. Computation of rocket trajectory (this is the idealized, "particle" trajectory; curiously, great prominence is given to Tsiolkovsii's formula, that the ratio of velocities is the logarithm of the ratio of masses, and instead of the solution of the differential equations there is introduced a series of progressive refinements on this formula, with special functions); 3. The foundations of the theory of rocket dispersion (elementary ballistic statistics); 4. Dispersion of finned rockets (non-spinning rockets,

Cornu spiral, etc.); 5. Dispersion of slowly-spinning finned rockets (superposition); 6. Dispersion of anti-tank finned rockets (by these are meant quick-burning rockets); 7. General three-dimensional problem of rocket motion (the introduction of the complex notation of ballistics); 8. Spin-stabilized rockets; and 9. On the effect of wind on rocket's motion. There are appendices with further proofs, a glossary of the elementary ballistic aerodynamics, large number of problems, and tables of special functions.

The book is easily readable and nicely systematic; those simplifications which are based on the beliefs derived from experimental information are clearly qualified. Thus, the gustiness of the surface wind is illustrated. The basic cause of dispersion is accepted as the "inclined nozzle" of a probable error of 1.5 mils, due to gas malalignment (one seems to get an impression that proof firings in USSR are surprisingly few and conclusive). A remark on page 68 is nicely calculated to arouse the enthusiasm of Soviet students of rocketry: it is computed that in 1852 the dispersion of Russian rockets was as low as 1.25% in range, and 25 mil in azimuth, probable error. The wartime origin of the text can be noted from the omission of references to Magnus torque and of the various nonlinearities of ballistics; thus, the lack of success of very slender spin-stabilized rockets is tentatively ascribed to the difficulty of achieving sufficient spin.

This reviewer would like to express some comments which apply not to this book alone, but to the general present state of unguided rocketry. Rocketry has inherited the traditions of gun ballistics, and has been formed in the days when the numerical solution of the differential equations was difficult. One feature of this situation has been the universal resort to the description of the deviation of the rocket by its instantaneous deflection, viz., the angle between the actual and the idealized trajectory; unfortunately, at low velocities this angle is poorly defined, and this leads to a series of "artificial" indeterminacies in the very important case of zero-length launcher, particularly in a crosswind. The other feature has been the general tendency to avoid, or to defer as long as possible, the resort to numerical solution of equations. From these two features there has followed the universal custom of reducing the general equations of rocketry to those of second order with constant coefficient and with the distance as the independent variable (which do fit so well with the realities of gun ballistics), whereby the peculiarities of rocketry are "swept" into a variability of the inhomogeneous parts of the equations, but return as a series of superposition integrals; and the role of the wind as merely another initial condition is "lost". The general procedure has been to start with the complete equations of rocketry, to recognize the mathematical difficulties, and to proceed with the simplifications and transformations of the problem until the skeleton of the problem begins to appear to the student as the final accomplishment. However, today one really desires to start with some skeleton, and to refine it progressively, freely using the computing machine. From this new point the classical, Fresnel-integral, formulation of the mathematics of rocketry has outlived its usefulness; in each of these classical four texts over one-half of the mathematics can be condensed into the study of the "well-behaved" equation

$$W'' + T^2W' - TW = 0, 1, T, T^2 \text{ (at start } T = T_0 \leq 0)$$

which can readily be refined if and when needed, and the solutions of which (rather than the Fresnel integrals) should be viewed as the basic mathematical functions of the unguided rocketry.

Moreover, one cannot escape the impression that the perfectly natural didactic quality of all these texts had in fact served to distract the attention of rocketeers from the need of the more radical invention. It also seems amusing that one still knows as little as one does about the exterior ballistics of the elastic arrow, of the 1812 Congreve rocket, and of the Goddard rocket.

S. J. Zaroodny (Aberdeen, Md.)

12781:

Gorelov, Yu. A. On two classes of plane extremal motions of a rocket in vacuum. Prikl. Mat. Meh. **24** (1960), 303–308 (Russian); translated as J. Appl. Math. Mech. **24**, 434–442.

Given a rocket of constant specific impulse, moving in a vacuum, which can be controlled in such a way as to have any desired direction of thrust at each instant in time without additional expenditure of mass for this purpose. The thrust is to be chosen so as to maintain the rocket in a horizontal plane despite the action of a uniform gravitational field. Subject to given initial and final values of the velocity vector and the mass, the trajectory is to be found which will yield an extremal time of flight τ. It is found that the extremal value of τ is a maximum. If the problem is reformulated to make the mass expenditure extremal for given τ, the same extremal is found, and it minimizes this expenditure. The ratio of thrust to mass is constant along an extremal, and the direction of the thrust is approximately constant. The author considers the problem also in a vertical plane, and finds similar results; however, the vertical problem leads to the following complication. In the case that the vertical component of the final velocity is less than that of the initial velocity, denote by τ_0 the time it would take the rocket to make the prescribed change in this component under the influence of gravity alone. The author's results apply to the times of flight τ that are greater than τ_0. For $\tau<\tau_0$ and given mass expenditure, the extremals minimize τ.

A. Blake (Framingham, Mass.)

STATISTICAL THERMODYNAMICS AND MECHANICS

See also 13107, 13197a–b, 13217.

12782:

De Witt, Cécile; Detoeuf, Jean-François (Editors). ★La théorie des gaz neutres et ionisés: Le problème des *n* corps à température non nulle. Université de Grenoble. École d'été de physique théorique, Les Houches, 1959. Hermann, Paris; John Wiley & Sons, Inc., New York; 1960. 469 pp. $17.50.

A collection of review articles by E. W. Montroll (Topics on statistical mechanics of interacting particles), L. Van Hove (Lectures on statistical mechanics of nonequilibrium phenomena), J.-L. Delcroix (Théorie microscopique des gaz ionisés), M. Kruskal (1. Hydromagnetics and the theory of plasma in a strong magnetic field and the energy principles for equilibrium and for stability;

2. Asymptotic theory of systems of ordinary differential equations with all solutions nearly asymptotic; 3. Laudau damping), A. Kaufman (Plasma transport theory), J.-F. Denisse (Etude des ondes electromagnetiques dans les Plasmas à partir de l'équation de Boltzmann) and E. Schatzman (Les plasmas en astrophysique), all of which will be reviewed individually in these pages.

12783:

Naze, Jacqueline. Propriétés des opérateurs de collision linéaires. Gaz de Lorentz imparfaits. C. R. Acad. Sci. Paris **251** (1960), 2284–2286.

The linear Boltzmann operator obtained for the case of the Lorentzian gas may be studied in two approximations, the "perfect Lorentz operator" K_L and the "imperfect Lorentz operator" K_{LI}; the latter being the higher approximation, as measured in terms of the ratio of electronic to ionic mass. Methods till now known for evaluating K_{LI} restrict it to isotropic distributions as operands. In this paper, the methods of an earlier publication [same C. R. **251** (1960), 651–653; MR **22** #8733] are applied to yield a representation of K_{LI} which eliminates this restriction: The distribution being expanded in spherical harmonics, the author obtains the result of K_{LI} operating on an lth-order term in the form

$$K_{LI}[y(c)c^l Y_{lm}(\mathbf{c})] = c^l Y_{lm}(\mathbf{c})N_l y(c),$$

where $\mathbf{c}$ is the electronic velocity. N_l, whose explicit form is given, is an operator on the isotropic factor $y(c)$. Application to special cases gives agreement with previously known results. *A. Siegel* (Boston, Mass.)

12784:

Fixman, Marshall. Density correlations, critical opalescence, and the free energy of nonuniform fluids. J. Chem. Phys. **33** (1960), 1357–1362.

The author demonstrates the equivalence of three approximate theories of critical opalescence; one due to Ornstein and Zernike [Z. Physik **19** (1918), 134–137; **27** (1926), 761–763], Debye [J. Chem. Phys. **31** (1959), 680–687], and Rocard [J. Phys. Rad. **4** (1933), 165–185]. A modification of the superposition approximation is proposed, but its wider implications are not discussed in the present work. *G. Weiss* (Washington, D.C.)

12785:

Kaufman, Allan N.; Watson, Kenneth M. Linked-diagram expansion for the equation of state of a gas of molecules. Phys. Fluids 4 (1961), 655–662.

Linked-diagram theory is used to obtain a virial expansion for the equation of state for a non-degenerate gas of molecules. The structure of the molecules is taken explicitly into account in that the interaction between the molecules is written as the sum of the Coulomb interactions between the nuclei and electrons of one molecule and those of another molecule. *D. ter Haar* (Oxford)

12786:

Argyres, Petros N. Quantum theory of transport in a magnetic field. Phys. Rev. (2) **117** (1960), 315–328.

Density matrix techniques are used to describe transport phenomena in solids in magnetic and electric fields. It is shown that the fact that Ţiţeica's theory [Ann. Physik. **22** (1935), 129–161] is correct and those of E. M. Lifšic [J. Phys. Chem. Solids **4** (1958), 11–18] and the author [Phys. Rev. **109** (1958), 1115–1128] are wrong because the interference between the electric field and impurities are not taken into account. The theory is developed along the lines of the work of Kohn and Luttinger [ibid. **108** (1957), 590–611; MR **20** #2892], and a transport equation is derived for the density matrix elements. *D. ter Haar* (Oxford)

12787:

Appel, Joachim. Electron-electron scattering and transport phenomena in nonpolar semiconductors. Phys. Rev. (2) **122** (1961), 1760–1772.

From the author's summary: "The effect of electron-electron scattering processes due to Coulomb forces on the transport phenomena in nonpolar isotropic solids is treated in the framework of Kohler's variation principle. By considering the conduction electrons as a Fermi-Dirac gas of noninteracting free quasi-particles, each with charge $-e$ and mass m^*, electron-electron scattering is taken into account as a small perturbation, as is electron-phonon scattering in nonpolar solids. A shielded Coulomb potential which depends on two parameters—the effective dielectric constant and the shielding constant—is used as the interaction potential. These two parameters, for small concentrations of electrons, may be assumed to be independent of the distance between two electrons during a scattering process."

12788:

Stern, Edward A. Effect of interactions on determination of Fermi surfaces. Phys. Rev. (2) **122** (1961), 1773–1780.

From the author's summary: "The effect of both electron-electron and electron-phonon interactions on a degenerate electron gas in a uniform positive background is considered. It is shown that when electron-electron interactions alone are considered the free-electron mass is still measured by cyclotron resonance, the Faraday effect, and optical constants. However, the period of the de Haas-van Alphen oscillations is changed from what one calculates neglecting interactions and is changed in the same way that the specific heat is. When electron-phonon interactions are added everything changes. In particular, it is shown that the cyclotron mass is no longer the free value, and the de Haas-van Alphen period and the specific heat are changed in different ways."

12789:

Baker, George A., Jr. One-dimensional order-disorder model which approaches a second-order phase transition. Phys. Rev. (2) **122** (1961), 1477–1484.

The author considers a one-dimensional array of N equally spaced scalar spins (ν_j), which assume only the value ± 1, where the interaction energy between the jth and kth spin is proportional to $\nu_j\nu_k \exp(-\gamma|j-k|)$, $e^{-\gamma}=r$, with $0 \leq r < 1$. The functional equation for the canonical partition function is solved rigorously for any finite range of interaction. It is shown that in the limit as the range of the force becomes arbitrarily great, but is still

small compared to the total size of the system (i.e., $r\to 1$, $r^N\to 0$), this system approaches a second-order phase transition (the energy is continuous, the specific heat discontinuous). It is also shown that this limit is not identical with the Bragg-Williams approximation ($r\to 1$, $r^N\to 1$) corresponding to the interchange in the order of the limits $r\to 1$, $N\to\infty$.

H. L. Frisch (Murray Hill, N.J.)

12790:

Blount, E. I. Exclusion factors in transport theory. Phys. Rev. (2) **116** (1959), 1365–1368.

Author's summary: "It has recently been shown that the customary exclusion factors should be omitted from the Boltzmann equation [Kohn and Luttinger, Phys. Rev. (2) **108** (1957), 590–611; MR **20** #2892]. This raises some conceptional difficulties in relation to the Pauli principle. The resolution of these difficulties is discussed in this paper."

W. B. Brown (Manchester)

12791:

Domb, C.; Sykes, M. F. Cluster size in random mixtures and percolation processes. Phys. Rev. (2) **122** (1961), 77–78.

Consider an ideal mixed crystal of A and B molecules, where p is the probability that a molecule is an A. The authors calculate the first terms (six to nine) in the expansions in powers of p of the mean size of a cluster of A molecules, i.e., a "site" cluster, for several lattice structures. From the series they make numerical estimates of the critical probability p_c, the number such that the probability of an infinite cluster is 0 or positive according as $p<p_c$ or $p>p_c$. (The authors state that p_c is the radius of convergence of the series. However, this requires proof, since it is conceivable that the mean-cluster size could be infinite on an interval of values of p, for which the probability of an infinite cluster is 0.) Similar series are given for "bond", as opposed to "site", clusters. The work is related to recent work on percolation processes. [See, e.g., Broadbent and Hammersley, Proc. Cambridge Philos. Soc. **53** (1957), 629–641; MR **19**, 989 and Hammersley, ibid., 642–645; MR **19**, 989.]

T. E. Harris (Santa Monica, Calif.)

12792:

Nelkin, M. S. Neutron thermalization. Proc. Sympos. Appl. Math., Vol. XI, pp. 20–42. American Mathematical Society, Providence, R.I., 1961.

This paper constitutes a general survey of neutron thermalization problems and, in particular, of the problems involved in the computation of spatially independent thermal neutron spectra.

In the absence of sources, the neutron transport equation can be written in the form:

$$(1)\qquad \left[\frac{\partial}{\partial t}+\alpha+v\Sigma_s(v)\right]n(v,t) = \int_0^\infty dv'\,v'\Sigma_s(v')n(v',t)P(v'\to v).$$

Here $\Sigma_s(v)$ is the macroscopic scattering cross-section at velocity v, $\alpha=v\Sigma_a(v)$ is the absorption probability per unit time, and $P(v'\to v)dv$ is the probability that a scattering event at velocity v' will take the neutron to the velocity range dv at v. The kernel in equation (1) may be symmetrized through the introduction of a new dependent variable

$$(2)\qquad \nu(v,t)\equiv n(v,t)[M(v)]^{-1/2},$$

where $M(v)$ is the Maxwell distribution. After symmetrization, one finds that

$$(3)\qquad \alpha+\frac{\partial\nu(v,t)}{\partial t} = \int_0^v K(v,v')\nu(v',t)\,dv' - v\Sigma_s(v)\nu(v,t)\equiv H\nu.$$

The author solves equation (3) formally via an expansion in terms of the eigenfunctions ν_i, where $H\nu_i=-\lambda_i\nu_i$. He argues on physical grounds that the ν_i form a complete set and that the eigenvalue expansion is valid. A solution of the time-independent problem is obtained from the time-dependent solution by integrating over all time, and the static solution is discussed in the limit of low absorption.

The author points out that the eigenvalue expansion is too slowly convergent at high energies to be useful in the analysis of practical reactor problems. For this reason part of the energy range $0\le E<\infty$ is commonly treated as a "slowing down" range. The static infinite medium equation then takes the form:

$$[\Sigma_a(E)+\Sigma_s(E)]\varphi(E) = F(E)+\int_0^{E_c}\Sigma(E_0\to E)\varphi(E_0)dE_0,$$

where $F(E)$ is a "slowing down" source, and E_c is the artificial boundary of the thermal-energy range. It is this form of the thermal-neutron equations which is used to determine spectra corresponding to various scattering kernels.

Since experimental information is incomplete, scattering kernels must be derived from simplified theoretical models and the models, to be useful, must incorporate chemical binding effects. Binding acts, roughly, to increase the effective mass of the scatterer. In order to develop insight into binding effects, the author digresses to discuss scattering properties of free particles, and the relation between scattering properties and the mass of the particle. For a more realistic treatment of binding, the author reverts to the formalism of Zemach and Glauber [Phys. Rev. (2) **101** (1956), 118–129]. While the method of Zemach and Glauber is not exposed in detail the author does discuss its basic physical content and describes, within the framework of the method, the important physical processes involved in scattering by graphite, beryllium, zirconium hydride and water. The scattering properties of bound and free particles are compared with each other. It is asserted that the theoretical kernel for water is consistent with presently available cross-section data, but it also appears that binding in water has little effect on infinite medium spectra.

The computation of spatially dependent thermal spectra is considered, briefly, in a concluding section. Here the author proposes and examines a number of computational procedures, including multigroup procedures in addition to some less expensive alternatives.

E. M. Gelbard (Pittsburgh, Pa.)

12793:

Green, H. S. Theories of transport in fluids. J. Mathematical Phys. **2** (1961), 344–348.

A proof is given of the equivalence of the correlation function method and the Chapman-Enskog method for

calculating the transport coefficients of fluids. The macroscopic equations and the correlation function formulas for transport coefficients are rederived by integrating Liouville's equation in the approximation correct to terms linear in the gradients of thermodynamical quantities. It is shown that the computation of the formulas for dilute gases leads to exactly the same results as the Chapman-Enskog method. A brief discussion is given on the extension of these methods to dense systems.

A. Miyake (Tokyo)

12794:

Turner, R. E. The quasi-classical approximation for neutron scattering. Physica **27** (1961), 260–264.

When evaluating Van Hove's space-time correlation function in the quasi-classical approximation for slow neutron scattering by an assembly of atoms, it is required to compute the intermediate scattering function. In this paper it is shown that, to terms of order $\hbar$, at a moderately high temperature the intermediate function expanded as a power series in $\hbar$ can be obtained by calculating the classical function and inserting the Schofield imaginary time shift.

S. Ueno (Santa Monica, Calif.)

12795:

Zubarev, D. N. Double-time Green functions in statistical physics. Uspehi Fiz. Nauk **71** (1960), 71–116 (Russian); translated as Soviet Physics Uspekhi **3**, 320–345.

This important paper gives a comprehensive survey of the application of double-time generalised Green functions (rather than the many-time Green functions involving only creation and annihilation operators used by Martin and Schwinger [Phys. Rev. (2) **115** (1959), 1342–1373; MR **22** #588]) to statistical problems. The author uses the advanced and retarded Green functions because their Fourier transforms can be analytically continued in the complex energy plane.

The applications discussed are those to irreversible processes (a brief discussion leading to the formula of Kubo [J. Phys. Soc. Japan **12** (1957), 570–586; MR **20** #4940a]), to superconductivity, ferromagnetism and electron-phonon interactions.

D. ter Haar (Oxford)

12796:

Alfred, L. C. R. Quantum-corrected statistical method for many-particle systems: the density matrix. Phys. Rev. (2) **121** (1961), 1275–1282.

The single-particle density matrix $\rho(\mathbf{r}_1, \mathbf{r}_2)$ is expanded in a consistent but non-perturbative series in $\hbar$ equivalent to that of Golden [Phys. Rev. (2) **105** (1957), 604–615; MR **19**, 364]. The expansion results from an iterative solution to a differential equation for Φ, where

$$M \equiv \langle \mathbf{r}|\exp \kappa\mathbf{H}|\mathbf{p}\rangle = \exp(\kappa p^2/2m+\Phi).$$

Two different integral representations are discussed that relate M to ρ. The appearance of Φ in the exponent restricts the problems that can be treated exactly. The present method is compared to wave-mechanical, Thomas-Fermi, and Fermi-Almadi-Golden predictions for the density $\rho(\mathbf{r}, \mathbf{r})$ for the case of a linear harmonic oscillator and an hydrogenic atom. The author's results are the closest to the exact ones in these cases.

J. R. Klauder (Murray Hill, N.J.)

12797:

Vlieger, J.; Mazur, P.; de Groot, S. R. On the quantum statistical basis of non-equilibrium thermodynamics. I. Physica **27** (1961), 353–372.

The quantum statistical basis of thermodynamics is developed by using the Wigner function formalism. The time dependence of the latter is studied by means of an appropriate propagator whose properties are developed in detail. The authors introduce a set of extensive variables characterizing the system macroscopically; these variables are defined in terms of commuting operators. A probability distribution function for these variables in equilibrium is introduced; this function is in principle discontinuous in quantum mechanics, and consequently the averages of functions of the extensive variables are expressed as Stieltjes integrals. However, one assumes that a central limit theorem holds for these variables, hence the distribution is approximately equal to a gaussian one and the Stieltjes integrals reduce to ordinary ones. Intensive variables are then associated with the extensive variables by assuming the existence of a Boltzmann entropy.

The present paper deals only with Maxwell-Boltzmann statistics; Fermi-Dirac and Bose-Einstein statistics will be treated in a forthcoming paper. *R. Balescu* (Brussels)

12798:

Huang, Kerson. Bose-Einstein system with attractive forces. Physica **26** (1960), supplement, S 58–S 61.

In this short article, the author treats a Bose system of N particles, interacting via two-body potential which contains a hard core plus a weak long-ranged attractive potential. For simplicity the Fourier transform of the attractive potential is assumed to be constant within the sphere with radius K.

With this rather artificial model, the calculation is divided into two parts: the energy level of the system, and the equation of state of the system. The ground-state energy per particle, in case of an infinite system with finite density, is roughly given by the sum of two terms. One is a negative term proportional to density, and the other is positive with dependence.

For the equation of state, the author shows that, in addition to the usual Bose-Einstein condensation, there exists a first-order phase transition of the gas-liquid type at absolute zero temperature. Making a further simplification for the model, the author considers co-existence of these two types of transition. *M. Tanaka* (Tokyo)

12799:

Tribus, Myron. Information theory as the basis for thermostatics and thermodynamics. J. Appl. Mech. **28** (1961), 1–8.

Information theory is well known to involve a function closely analogous to the entropy function of classical thermodynamics. In this paper the author abandons the idea that one has a mere analogy, and instead derives the whole of statistical thermodynamics on the basis of an information-theoretic definition of entropy. Both closed and open systems are considered, together with Onsager's relations.

H. A. Buchdahl (Hobart)

12800:

Edwards, S. F. The statistical thermodynamics of a gas

with long and short-range forces. Philos. Mag. (8) 4 (1959), 1171–1182.

This paper describes a new and comparatively simple method of obtaining the leading and higher-order terms in the thermodynamic properties of a classical gas with coulombic and hard-sphere interactions. The method avoids the difficulties associated with redundant coordinates in the theory of plasma oscillations and is simpler to extend to higher-order corrections than the method of "summation of diagrams". It is related mathematically to techniques used in quantum field theory, and can be extended to a quantum gas. An interesting variational approach is also discussed which proves that the Debye-Huckel expression for the thermodynamic function PV is an extremum.

W. B. Brown (Manchester)

12801:

Kirkwood, John G.; Baldwin, Robert L.; Dunlop, Peter J.; Gosting, Louis J.; Kegeles, Gerson. Flow equations and frames of reference for isothermal diffusion in liquids. J. Chem. Phys. **33** (1960), 1505–1513.

From the authors' summary: "Equations for testing Onsager's reciprocal relations for isothermal diffusion depend on the frame of reference chosen for the flows. This subject is considered for certain frames of reference, as is the problem of measuring diffusion coefficients when there is a change of volume on mixing (or of any single component). Throughout this article a special effort has been made to present derivations and final equations in a form well adapted for use in experimental work."

J. Ross (Providence, R.I.)

12802:

Пригожин, И. [Prigogine, I.]. ★Введение в термодинамику необратимых процессов [Introduction to thermodynamics of irreversible processes]. Translated from the English by V. V. Mihaĭlov; edited by N. S. Akulov. Izdat. Inostr. Lit., Moscow, 1960. 127 pp. 4 r.

The original English language edition [*Introduction to thermodynamics of irreversible processes*, C. C. Thomas, Springfield, Ill., 1955], was a concise exposition of the equations of the thermodynamics of irreversible processes with special emphasis on chemical applications. This Russian edition is essentially a translation of the original edition with the following additions: an introduction by N. Akulov which gives a resumé of the basic principles and three footnotes to his own work, a section by Prigozin on rate equations for Fourier transforms of thermodynamic variables, and a new chapter (VII) by him on non-linear processes. In this latter, the phenomenological equations for non-linear processes are cast into a form which assures non-negative entropy production, and the equations in this case are shown also to be deducible from a variational principle.

D. Falkoff (Waltham, Mass.)

12803:

Pataki, D'. [Pataki, G.]. On the time dependence of irreversible processes in Knudsen gas. Acta Phys. Acad. Sci. Hungar. **12**, 311–319 (1960). (Russian. English summary)

Author's summary: "In the paper presented here we discuss the time dependence of cross-effects (irreversible processes) in the Knudsen gas by means of two parameters of the conduction matrix. We examined the reversal of sign of thermodynamic forces and found it really—in contrast to Onsager's cross-effect—to be connected with the $g \cdot L$ matrix and not the L matrix being off-diagonal. We found that the reversal of sign of the forces does not occur with a stationary initial condition, whilst in case of zero initial value of one of the forces the other generally reverses its sign."

12804:

Bernard, William; Callen, Herbert B. Irreversible thermodynamics of a nonlinear R-C system. Phys. Rev. (2) **118** (1960), 1466–1470.

The authors consider the fluctuations in the charge, q, on a capacitor in an R-C circuit with non-linear resistance, $R(V)$, maintained at a temperature T. Here $V=\langle q\rangle/C$. It is shown that the correlation function, $\langle q(0)q(t)\rangle$, where the expectation value is taken with respect to a canonical distribution at the temperature T, depends only on $R(0)$ and is thus identical with that for a linear R-C system. Hence the "fluctuation-dissipation theorem" of Callen and Welton [same Rev. (2) **83** (1951), 34–40; MR **13**, 477] is exact also for the non-linear case. Higher correlation moments are not identical, but the aforementioned theorem depends only on the second moment.

D. Falkoff (Waltham, Mass.)

ELASTICITY, PLASTICITY

12805:

Reiner, Markus. ★Deformation, strain and flow: An elementary introduction to rheology. Revised and enlarged edition of "Deformation and flow". Interscience Publishers, Inc., New York, 1960. xvi+347 pp. $9.75.

The aim of this book seems to be to impart some understanding of the behavior of complicated materials using rudimentary analyses based on relatively simple one-dimensional theories, descriptions of observations, and elementary notions concerning microscopic and macroscopic structure of materials. This is embellished with various types of comments; historical, metaphysical, semantic and biblical.

The author has excluded most modern developments in continuum mechanics and in the mechanics of discrete systems. It seems particularly unfortunate that he gives no adequate discussion of the general linear theory of visco-elasticity, as this has been very helpful in organizing and directing experimental and theoretical researches on complicated materials like high polymers.

J. L. Ericksen (Baltimore, Md.)

12806:

Гудьер, Дж. Н. [Goodier, J. N.]; Ходж, Ф. Г. [Hodge, P. G., Jr.]. ★Упругость и пластичность [Elasticity and plasticity]. Translated from the English by N. A. Forsman; edited by G. S. Šapiro. Izdat. Inostr. Lit., Moscow, 1960. 190 pp. 9.35 r.

A translation into Russian of two essays by J. N. Goodier [*The mathematical theory of elasticity*, Elasticity and plasticity, Surveys in Applied Mathematics, Vol. 1,

pp. 1–47, Wiley, New York, 1958; MR **20** #484] and P. G. Hodge, Jr. [*The mathematical theory of plasticity*, ibid., pp. 49–144; MR **20** #485].

12807:

Charrueau, André. Sur diverses questions de géométrie se rattachant à l'étude des milieux continus. Ann. Ponts Chaussées **129** (1959), 375–390. (English summary)

Résumé de l'auteur: "Dans cet article, l'auteur expose d'abord des propriétés de surfaces et de congruences de droites liées au tenseur des contraintes. Il montre ensuite comment ces faits géométriques se rattachent à l'étude de problèmes plus généraux."

12808:

Rajagopal, E. S. The role of initial stresses in lattice dynamics. Ann. Physik (7) (**6**) (1960), 177–181.

The author states that a consistent treatment of classical infinitesimal elasticity theory shows that the elastic behavior of a solid is independent of any initial stresses that may be present in the undeformed lattice. Contrary results of Kun Huang [Proc. Roy. Soc. London Ser. A **203** (1950), 178–197; MR **12**, 375] are attributed to retention of small terms inconsistent with infinitesimal theory. For an earlier discussion of this controversy, see G. Leibfried, *Handbuch der Physik, Bd. VII* (Springer, Berlin, 1955), Part 1, p. 191 (footnote 1).

H. B. Rosenstock (Washington, D.C.)

12809:

Solomon, Liviu. Sur les rapports entre les constantes de Prandtl, la rigidité géométrique à la torsion et la fonction de Green. Bull. Math. Soc. Sci. Math. Phys. R.P. Roumaine (N.S.) **2** (**50**) (1958), 329–341.

In the case of a multiply-connected domain, the limiting values of the Prandtl's function at the inner boundaries have been defined as Prandtl's constants. The relations indicated in the title are then established, but the significance of these relations in solving torsion problems has not been discussed fully.

S. C. Das (Appl. Mech. Rev. **13** #6141)

12810:

Truesdell, C. Invariant and complete stress functions for general continua. Arch. Rational Mech. Anal. **4**, 1–29 (1959).

The paper is concerned with solutions in terms of stress functions of the general dynamical equations for all continuous media in a particular space. The derivations are carried through in general coordinates and the results are proved to be complete within a stated class. The equations solved are

$$(1)\qquad T^{km}{}_{,m} = 0, \quad T^{km} = T^{mk},$$

where the quantities T^{km} are the contravariant components of an absolute tensor field and where the comma denotes covariant differentiation based upon an affine connection in a real space of n dimensions. The problem is completely solved for flat spaces, and the solution yields a valuable coordination of previous work which has mostly relied on haphazard methods. The general problem of solving (1) in an affine space is reduced to one of differential elimination, and a general solution of (1) is given for a space of constant curvature. The paper closes with an application to the classical theory of membranes.

A. E. Green (Newcastle upon Tyne)

12811:

Graiff, Franca. Sul legame tra condizioni di congruenza ed equazioni indefinite nella meccanica dei continui. Atti Accad. Naz. Lincei. Rend. Cl. Sci. Fis. Mat. Nat. (8) **26** (1959), 763–771.

A symmetric tensor S^{ik} on a Riemannian manifold satisfies (1) $S^{ik}{}_{|k}=0$ if and only if (2) $\int S^{ik}\xi_{ik}d\sigma=0$ holds for all tensors ξ_{ik} of the form (3) $\xi_{ik}=s_{i|k}+s_{k|i}$, where the vector s_i vanishes on the boundary of the region of integration [see #12810 and the literature cited there]. Equations of the form (1) and (3) occur in various branches of continuum mechanics, where S^{ik} is interpreted as a stress tensor and ξ_{ik} as an infinitesimal strain tensor. By an ingenious method based on the theorem above the author determines the general solution of (1), in terms of two arbitrary functions, for the case when the manifold is a surface whose total curvature K is such that $K_{|i}$ and $(K_{|k}K^{|k})_{|i}$ are linearly independent. For this case, she also obtains a system of two fourth-order integrability conditions for the differential system (3). She indicates how her method may be generalized to the three-dimensional case but leaves the details to a later paper.

W. Noll (Pittsburgh, Pa.)

12812:

Graiff, Franca. Soluzione generale delle equazioni indefinite della meccanica dei continui. Ist. Lombardo Accad. Sci. Lett. Rend. A **93** (1959), 325–333.

Integration of the system

$$(*)\qquad S_{ik,}{}^{k} = 0, \quad S_{ik} = S_{ki}$$

furnishes a well-known open problem in the theory of differential invariants, having application in several classical and relativistic mechanical theories. [A summary of earlier work is given by the reviewer in #12810.] Guided partially by the formal analogy between this problem and that of finding the explicit form of the conditions of compatibility, where she has previously made notable contributions [Atti Accad. Naz. Lincei. Rend. Cl. Sci. Fis. Mat. Nat. (8) **24** (1958), 415–422; MR **21** #5316], the author sets up a method for solving (*) in a three-dimensional Riemannian space with metric tensor a_{ik}. In the general case, the contracted curvature tensor R_{ik} has three non-constant, functionally independent proper numbers $\omega^{(r)}$. Let $D^{(rs)}=D^{(sr)}$ be any six scalars, and set $D_{ik}\equiv\sum_{rs}D^{(rs)}\omega^{(r)}{}_{,i}\,\omega^{(s)}{}_{,k}$. Then from the author's earlier results it follows that $D_{ik,}{}^{k}=\sum_t d^{(t)}\omega^{(t)}{}_{,i}$. The author shows that a solution to (*) is given by $S_{ik}=D_{ik}+\Phi_{ik}$, where

$$\Phi_{ik} \equiv \nu_{ik,s}{}^{s}-\nu_{is,k}{}^{s}-\nu_{ks,i}{}^{s}+\nu_{pq,}{}^{pq}a_{ik}-2R_{ij}\nu^{j}{}_{,k},$$
$$\nu_{ik} \equiv \sum_t d^{(t)}\lambda_i\lambda_k,$$

where the λ_i are the unit proper vectors of R_{ij}. Thus a solution containing six arbitrary functions is exhibited [but is not shown to be complete]. Since ν_{ik} is defined in terms of the divergence of D_{ik}, the solution effectively involves the third derivatives of that arbitrary tensor, corresponding to the fact that the conditions of compatibility are of third order. Spaces of constant curvature, for which the solution to (*) has long been known, are excluded from the above result. The author gives a special analysis for the case

when $\omega_1=\omega_2\neq\omega_3$, yielding a solution in terms of fourth derivatives. She remarks that the dimension n of the space does not play any essential part in the reasoning, provided $n\neq 2$, and she indicates how her solution includes as special cases the Ricci-Einstein gravitational tensor and a special solution due to Storchi [Ist. Lombardo Sci. Lett. Rend. Cl. Sci. Mat. Nat. **90** (1956), 369–378; MR **19**, 590].

This paper presents a notable advance in the field.

C. Truesdell (Bologna)

12813:

Massonnet, Ch. Quelques méthodes pour la solution de problèmes d'élasticité intervenant en génie civil. Les mathématiques de l'ingénieur, pp. 113–132. Mém. Publ. Soc. Sci. Arts Lett. Hainaut, vol. hors série, 1958.

12814:

Reissner, Eric. On some variational theorems in elasticity. Problems of continuum mechanics (Muskhelishvili anniversary volume), pp. 370–381. SIAM, Philadelphia, Pa., 1961.

The author extends somewhat his (1950) variation principle in infinitesimal elasticity with non-linear stress-strain relation. The original version was the canonical form of the well-known energy principles. By "canonical form" here is meant the direct analogue of Hamilton's Principle expressed in terms of the canonical variables, which yields the full set of Hamiltonian equations when these variables are treated as independent. In the elastic-continuum context the analogous variation principle yields the material law along with the field equations when the displacements and stresses are independently varied.

The present extension covers a certain kind of mixed non-linear boundary condition and also body forces derivable from an arbitrary potential function of displacement. However, most of the paper is concerned with the narrower situation where the field equations and boundary conditions are both linear. In this event it is shown (rather unnecessarily) how the familiar complementary pair of extremum principles of linear elasticity can be deduced, when all the associated quadratic potentials are positive definite. Finally, attention is drawn to the simple form taken by the extended variation principle when the varied quantities are restricted so that the boundary conditions are the only Euler equations. *R. Hill* (Nottingham)

12815:

Lakshminarayana, G. Finite strain in rotating cylinders. Proc. 5th Congr. Theoret. Appl. Mech. (Roorkee, 1959), pp. C.17–C.26. Indian Soc. Theoret. Appl. Mech., Kharagpur.

Displacements and stresses are obtained for a solid, hollow or composite rotating circular cylinder by taking finite components of strain and a linear stress-strain tensor relation. The solution of the non-linear differential equation involved is given to the fourth power of the angular velocity. No numerical results are computed.

B. R. Seth (Madison, Wis.)

12816:

Hopkins, H. G. Dynamic anelastic deformations of metals. Appl. Mech. Rev. **14** (1961), 417–431.

A review paper, with 136 references.

12817:

Shastri, U. A. Some problems of plane stress in an aeolotropic circular disk. Bull. Calcutta Math. Soc. **51** (1959), 154–160.

12818:

Carmichael, A. J.; Holdaway, H. W. Phenomenological elastomechanical behavior of rubbers over wide ranges of strain. J. Appl. Phys. **32** (1961), 159–166.

A theory of rubber-like elasticity for materials in pure homogeneous strain is developed, based upon the work of Mooney [same J. **11** (1940), 582–592]. The theory is employed to examine the experimental results of Treloar, and the properties of rubber are described by means of three parameters. It is not clear how the theory would apply to more general kinds of deformation, since the stress-strain relations are not expressed in terms of the usual invariants. *J. E. Adkins* (Providence, R.I.)

12819:

Hayashi, Takuo. Some plane stress problems of orthotropic material. Bull. JSME **4** (1961), 11–16.

Stresses in elastically orthotropic plates bounded by lines asymmetric with respect to the elastic axes have been investigated by the use of Beltrami-Cayley's conformal mapping method. As examples, stress distributions in a semi-infinite plate, an infinite plate with an elliptic hole and an elliptic plate have been calculated. Results are seen to be significant. *G. Paria* (Kharagpur)

12820:

Hayashi, Takuo. On the plane stress problem in a ring of orthotropic material. Bull. JSME **4** (1961), 6–11.

The title problem is solved by the perturbation method using the orthogonal co-ordinates. Numerical results are obtained for oak, plywood and isotropic plates. The results show that considerably large stress concentration may occur in some aeolotropic plates even in the case of uniform inner pressure. *G. Paria* (Kharagpur)

12821:

Mathurin, Claude. Sur la résolution, en variable complexe, du premier problème de l'élastostatique plane. C. R. Acad. Sci. Paris **251** (1960), 2871–2873.

By use of usual Goursat expression for Airy stress function in terms of analytic functions, the author relates stress-boundary-condition problem of plane elasticity to linear equation in Banach space.

C. E. Pearson (Cambridge, Mass.)

12822:

An, Bunzai. Influence of friction on normal stress distribution occurred by the indentation on the surface of elastic solid. Keio Univ. Centenary Memorial Publ., pp. 47–60. Faculty of Engineering, Keio University, Tokyo, 1958. (Japanese. English summary)

From the author's summary: "The object of this paper is to find the influence of friction on the normal stress distribution due to the indentation of a rigid sphere on the free surface of a semi-infinite elastic medium, with the aid of Hankel transforms."

12823:

Tremmel, E. Ausweitung des kreiszylindrischen Hohlraumes unter örtlichem Innendruck. Ing.-Arch. **29** (1960), 331–350.

The problem of an infinite isotropic elastic medium with a circular cylindrical hole is discussed, the hole being subjected to an axially symmetric local uniform pressure. Using Love's stress function the radial displacement for the load just mentioned as well as for the concentrated distribution of load is found. The improper integrals involved are computed by subdividing the infinite interval of integration and substituting for the integrands reasonably approximating functions.

J. Nowinski (Austin, Tex.)

12824:

Teodorescu, P. P. Über die Berechnung dicker ebener Platten unter örtlicher Belastung. Bull. Math. Soc. Sci. Math. Phys. R. P. Roumaine (N.S.) **2 (50)** (1958), 463–473.

An isotropic elastic infinite layer with a load acting perpendicularly on the external planes is considered. The load, three biharmonic stress-functions, the stresses and the displacements are expressed by double Fourier integrals.

Z. Kączkowski (Warsaw)

12825:

Lang, H. A. Surface displacements in an elastic half-space. Z. Angew. Math. Mech. **41** (1961), 141–153. (German and Russian summaries)

This paper discusses the elastic waves set up in a half-space by the sudden application of a constant normal point load. The integral transform method used is an extension of that used by Sauter [same Z. **30** (1950), 149–153; MR **12**, 215] for the corresponding two-dimensional problem. The presentation is marred by undue contraction, which obscures the argument at crucial points, and by a number of minor misprints.

J. W. Craggs (Newcastle upon Tyne)

12826:

Schaefer, H. Die Spannungsfunktionen des dreidimensionalen Kontinuums; statische Deutung und Randwerte. Ing.-Arch. **28** (1959), 291–306.

In the absence of body forces, the equilibrium conditions of continuum mechanics may be solved by expressing the stress tensor σ_{ik} in terms of a "stress function tensor" $F_{\alpha\beta}$ in the form

$$(1) \qquad \sigma_{ik} = -\varepsilon_{i\alpha\beta}\varepsilon_{k\lambda\mu}\partial_\alpha\partial_\lambda F_{\beta\mu}, \quad F_{\beta\mu} = F_{\mu\beta},$$

where $\varepsilon_{i\alpha\beta}$ are permutation symbols and ∂_α denote partial derivatives. The solution (1) is due to Gwyther and Finzi and may serve as a basis for most theories of stress functions in linear elasto-statics [cf. C. Truesdell's paper reviewed above, #12810, and the literature cited there].

The author shows that the boundary values of the stress function tensor $F_{\alpha\beta}$ may be given the following statical interpretation: Consider a "crust shell" coincident with the boundary of the given body and loaded in the same way as the given body. The boundary values of $F_{\alpha\beta}$ then form a possible system of couple resultants in the shell provided that the stress resultants are taken to be

$$S_{ik} = \varepsilon_{\alpha\beta k}\partial_\alpha F_{\beta i} - \tfrac{1}{2}\partial_i(\varepsilon_{k\beta\gamma}F_{\beta\gamma}).$$

This observation may be used to reduce stress boundary value problems in linear elasto-statics to simpler boundary value problems for the stress functions.

The author uses the method to give elegant solutions of various loading problems.

W. Noll (Pittsburgh, Pa.)

12827:

Chattarji, P. P.; Dutt, S. B. On the stresses due to a nucleus in the form of a centre of dilatation in an elastic infinite solid with rigid spherical inclusion. J. Sci. Engrg. Res. **3** (1959), 175–180.

12828:

Mizuno, Masao. Problem of large deflection of coiled springs. Proc. Fac. Engrg. Keio Univ. **12** (1959), 18–35.

Using expressions for the axial force, flexural and shearing rigidities of springs given by Biezeno and Grammel, *Technische Dynamik, Bd. I* [Springer, Berlin, 1953; p. 616], the large deflections of the center-line of a coiled spring are discussed. Experiments are then carried out on the buckling, under axial compression, of a coiled spring; the results agree generally well with those predicted by the theory. Finally, the characteristics are investigated of springs non-linearized by initial tension, these being used in the common dial gauge. These theoretical characteristics also agree quite well with the results of experiment.

H. D. Conway (Ithaca, N.Y.)

12829:

Brandt, Howard E. On a generalized solution to a torsion problem in linear elasticity. J. Aerospace Sci. **28** (1961), 422–424.

The Galerkin method employed by Stanišić and his coworkers (referred to in [#12830]) is used for the torsion of a regular polygonal cross-section. As pointed out in the paper reviewed below, the numerical results so obtained are unsatisfactory. Some of them are incorrect. Expansion in suitable positive integral powers of the complex variable z gives good results for rectilinear sections.

B. R. Seth (Madison, Wis.)

12830:

Niedenfuhr, F. W.; Leissa, A. W. The torsion of prismatic bars of regular polygonal cross section. J. Aerospace Sci. **28** (1961), 424–426.

The classical torsion problem for a bar of regular polygonal section is discussed by taking a number of terms in the Fourier series solution, which equal the number of sides of the polygon. The constants are determined by satisfying the boundary condition at the angular points of the polygon. Essentially the procedure is equivalent to the known method of taking a limited number of terms in positive integral powers of the complex variable z.

For a regular hexagon it is shown that the method gives better results than those obtained by Stanišić and his coworkers by using the Galerkin method. The authors are apparently unaware of some of the previous work on the subject [the reviewer, Proc. Cambridge Philos. Soc. **30** (1934), 139–149].

B. R. Seth (Madison, Wis.)

12831:

Lakshmikanth, V.; Ramakanth, J. Non-linear torsion

of an orthotropic cylinder. J. Sci. Engrg. Res. **3** (1959), 181–186.

The problem of non-linear torsion of a solid rod of circular cross-section was solved by Seth for an isotropic and, later, a transversely isotropic material [Philos. Trans. Roy. Soc. London Ser. A **234** (1935), 231–264; Bull. Calcutta Math. Soc. **38** (1946), 39–44; MR 7, 501]. Following Seth's analysis, that is, by taking linear stress-strain relations and non-linear strain components and assuming displacements corresponding to a large twist, the authors consider the same problem for a rectilinearly orthotropic material. In the case of topaz, the first few coefficients in the series solution are evaluated numerically.

J. H. Wilkinson (Battersea)

12832:

Lakshmikanta, V.; Ramakanta, J. Non-linear torsion of orthotropic hollow and composite cylinder. Proc. 5th Congr. Theoret. Appl. Mech. (Roorkee, 1959), pp. C.129–C.138. Indian Soc. Theoret. Appl. Mech., Kharagpur.

This paper is the continuation of a previous one by the same authors [#12831] dealing with the non-linear torsion of orthotropic cylinders. In this paper orthotropic hollow and composite cylinders are considered. Numerical results for a hollow cylinder of given orthotropic material are given.

R. M. Morris (Cardiff)

12833:

Chakravorti, A. Torsion of a composite section bounded by intersecting arcs of nonconcentric circles. J. Sci. Engrg. Res. **3** (1959), 108–116.

In this paper the torsion problem for a compound beam of two different isotropic materials is solved when the section is bounded by arcs of the non-concentric circles $\eta=\alpha$, $\eta=\beta$, $\eta=\gamma$ of the transformation

$$x+iy = a\tanh(\xi+i\eta).$$

R. M. Morris (Cardiff)

12834:

Matheson, J. A. L.; Francis, A. J. ★Hyperstatic structures: An introduction to the theory of statically indeterminate structures. Vol. II. With chapters by N. W. Murray and R. K. Livesley. Academic Press Inc., New York; Butterworths Scientific Publications, London; 1960. xi+282 pp. $11.00.

[For review of vol. 1, 1959, see MR **21** #3146.]

Volume 2 consists essentially of worked examples and examples for solution; these problems have been taken mainly from university examinations, and some have been specially constructed. The chapters follow almost exactly those of vol. 1, and each chapter has brief comments on the methods used and references to the earlier volume.

Both volumes are comprehensive, containing almost everything the structural engineer should need. The examples illuminate the theoretical work of vol. 1, are well selected and ordered, and, in cases where alternative methods of solution are possible, enable the reader to evaluate the merits of those methods. There are not many examples on plastic methods; on the other hand, there are problems on stability of frameworks and on the assembly of stiffness matrices not usually found in the literature.

The two volumes together form a most valuable and welcome addition to writings on the theory of structures; very few works achieve the same high standards of comprehensiveness and utility.

J. Heyman (Cambridge, England)

12835:

Seth, B. R. Finite bending of a non-homogeneous aelotropic sheet (elastic bending). Proc. 4th Congress Theoret. Appl. Mech. 1958, pp. 38–44. Indian Soc. Theoret. Appl. Mech., Kharagpur.

An initially plane rectangular plate is bent into the form of a circular cylinder, with the two edges as generators. The elastic body initially isotropic and homogeneous, undergoing large deformations, becomes both anisotropic and non-homogeneous. The author discusses in detail the finite cylindrical bending of a non-homogeneous, transversely isotropic plate. The formula obtained for the bending moment shows, that, if c_{66}/c_{11} is constant, the bending moment remains of the same form as in the isotropic case.

Z. Kączkowski (Warsaw)

12836:

Sherbourne, A. N. The behaviour of a clamped circular plate in compression. Aero. Quart. **12** (1961), 51–64.

Author's summary: "A theoretical solution is presented for the problem of the clamped circular plate loaded in uniform compression. The solution employs a numerical method programmed for a digital computer. Instead of solving the classical von Karman large deflection equations, a step-by-step integration of the elastic differential equations of equilibrium is carried out until suitable boundary conditions are attained. The method is an extension of one developed earlier to explain the behaviour of the simply-supported plate."

J. Heyman (Cambridge, England)

12837:

Lakshminarayana, G. Finite bending of plates. J. Sci. Engrg. Res. **3** (1959), 130–144.

The reviewer's methods on the bending of plates into shells have been extended to aeolotropic and composite plates. The displacements are of the same type as in the isotropic cases. A number of numerical results are obtained and equivalent thicknesses deduced.

B. R. Seth (Madison, Wis.)

12838:

Buckens, F. Stabilité d'une plaque mince annulaire sollicitée sur ses bords intérieur et extérieur. I. Ann. Soc. Sci. Bruxelles. Sér. I **74** (1960), 120–128. (English summary)

The paper deals with buckling of an annular plate under uniform internal and external radial loads. Upper and lower bounds for buckling loads are obtained by variational methods. No numerical results are given.

W. T. Koiter (Delft)

12839:

Kacner, Artur. Bending of semi-infinite plate strips with discontinuous boundary conditions. Arch. Mech. Stos. **12** (1960), 451–481. (Polish and Russian summaries)

12840:

Cristescu, N. On the propagation of elasto-plastic waves for combined stresses. Prikl. Mat. Meh. **23** (1959), 1124–1128 (Russian); translated as J. Appl. Math. Mech. **23**, 1605–1612.

Two plane plates, having their middle surfaces in the same plane, are supposed to impact obliquely with one another along two of their edges so that there is propagation of elastic-plastic waves of so-called combined or complex stress that involve both dilatation and shear. On the basis of deformation theory, Kh. A. Rakhmatulin [same Prikl. **22** (1958), 759–765; MR **22** #1201] first gave attention to this problem. The present paper gives some further discussion of the same problem, elaborating the qualitative picture of stress wave propagation.

H. G. Hopkins (Sevenoaks)

12841:

Slobodianskii, M. G. Estimates of natural frequencies of vibration of clamped plates of constant and variable thickness. Problems of continuum mechanics (Muskhelishvili anniversary volume), pp. 473–482. SIAM, Philadelphia, Pa., 1961.

The problem of obtaining upper and lower bounds to the natural frequencies of vibration of clamped plates of fairly arbitrary shapes and of both constant and variable thickness is discussed here by constructing a "kernel" which in some sense is "similar" to the Green function for the clamped plate. A comparison of the eigenvalues of the equation for the transverse deflection of the plate with the eigenvalues associated with the constructed kernel lead, by means of theorems due to Courant and Weyl, to a set of inequalities giving the desired estimates. The possibility of construction of the kernel required, by means of an approximating sequence, is discussed in principle, but no numerical examples are given to illustrate the efficacy of the method.

G. Temple (Oxford)

12842:

Kantor, B. Ya. Bending of a sectorial plate of variable thickness clamped on the lesser circular edge. Akad. Nauk Ukraïn. RSR. Prikl. Meh. **6** (1960), 420–428. (Ukrainian. Russian and English summaries)

The plate is bounded by arcs of two concentric circles and segments of the radii. It is clamped on the inner edge and free on the other three. The rigidity is variable over the area of the plate. The integro-differential equation for the deflection is set up and approximated by a system of n integral equations in the single variable r, obtained on dividing the plate into n sectorial strips of angle $\Delta\phi$. The integrals are in turn replaced by finite sums, yielding a system of algebraic equations, the solution of which has been programmed for the STRELA. The paper concludes with some comparisons of the calculated results with experimental data.

R. N. Goss (San Diego, Calif.)

12843:

Bassali, W. A.; Hanna, N. O. M. Stresses and deflections in an elastically restrained circular plate under hydrostatic normal pressure over a segment. J. Appl. Mech. **28** (1961), 91–102.

In a recent paper [same J. **26** (1959), 44–54; MR **21** #1737] Bassali and the reviewer obtained the deflection of a thin elastic circular plate, which is elastically restrained along its boundary and uniformly loaded along a chord. The present paper deals with a circular plate, of radius c, under normal loading over a segment of height h and angle 2γ; the load intensity p is given by (1) $p=p_0(x-f)$, $f\leqq x\leqq c$; $p=0$, $x<f$, where p_0 is constant, the origin of coordinates being at the centre of the plate and the x-axis taken along the axis of symmetry. By integrating the complex potential functions $\Phi(z,\theta)$, $\Psi(z,\theta)$, obtained by Bassali and the reviewer in the above-mentioned paper, with respect to θ from $\theta=0$ to $\theta=\gamma$, the authors obtain the complex potential functions $\phi(z,\gamma)$, $\psi(z,\gamma)$ corresponding to the load intensity $p=p_0x\times(f\leqq x\leqq c)$, $p=0(x<f)$. Superposing the result obtained and the results given in the above mentioned paper for uniform loading over a segment, the authors finally obtain expressions giving the deflection and stresses corresponding to the load intensity (1). These expressions are given in series form, which reduce to closed form when the plate is clamped along its boundary. When the segment extends over half the plate, the expressions obtained are much simplified, and the problem is now investigated in detail.

Thus the deflection w along the axis of symmetry is given by

$$\frac{960\pi D}{p_0c^4}w = (1-\xi^2)[120U(\xi)-\frac{44}{3}+\frac{5}{2}\pi(1-\xi^2)\xi+3\xi^3-\frac{19}{3}\xi^{-2}-2\xi^{-4}] -8\varepsilon\xi^3+(2\xi^{-1}-3\xi)(\xi+\xi^{-1})^4\tan^{-1}\xi,$$

where

$$U(\xi) = \frac{2}{15}(2-\lambda)+\frac{\pi\xi}{24(m+1)} -4\sum_{n=1}^{\infty}\frac{(-1)^n\xi^{2n}}{(4n^2-1)(2n+3)(2n+5)(2n+m)};$$

$\xi=x/c$, λ is the constraint parameter; it is equal to 1 when the plate is clamped and $(\eta-1)/(1+\eta)$ when the plate is simply supported; η is the Poisson's ratio, and $\varepsilon=\pm\frac{1}{2}\pi$ according as ξ is negative or positive.

The authors also calculate the numerical values of the deflection along the axis of symmetry and the boundary values of moments, shears and slope both in tabular and graphical forms.

M. Nassif (Assiut)

12844:

Friedrichs, K. O.; Dressler, R. F. A boundary-layer theory for elastic plates. Comm. Pure Appl. Math. **14** (1961), 1–33.

From authors' summary: "The authors consider an elastic plate of uniform thickness in equilibrium loaded by normal and shear stresses which vary arbitrarily along each edge generator and also around the perimeter, and by normal forces distributed arbitrarily over each face. The problem is governed by three-dimensional linear elasticity equations and can be split up into two independent parts, generalized plane-stress case and a pure-bending case. Only the pure-bending problem is considered in detail. Expressions for stresses valid in the interior of the plate and in an edge layer are obtained by two expansion procedures which are interrelated by their boundary conditions. The lowest-order terms for the interior yield the

classical 'thin-plate' theory involving the biharmonic equation. The boundary-layer problem of corresponding order splits into two simpler problems, a plane-strain case and a torsion case, each now in a semi-infinite strip domain. Static equilibrium for the former implies the first Kirchhoff boundary condition on bending moments. The next higher approximation for interior stresses can be derived systematically, showing that its correct boundary conditions involve explicitly the boundary layer stresses of the thick-plate approximation.

"This analysis implies that the Michell 'moderately thick plate' theory is not applicable for the interior problem whenever any edge effect exists in a bent plate. Solutions for these edge stresses involve first other plane-strain and torsion problems for the stress derivatives. Integration then leads to two problems for the integration functions; these may be interpreted as plane-strain and torsion problems in which known boundary-layer stresses act as imposed body forces. Static equilibrium of the semi-infinite strip for this plane-strain case implies the second Kirchhoff boundary condition of the interior thin-plate theory, involving the sum of resultant shear force and derivative of resultant twisting moment."

A. E. Green (Newcastle upon Tyne)

12845:

Weber, C. Einseitig eingespannter Plattenstreifen mit Einzellast. Z. Angew. Math. Mech. **40** (1960), 558–565. (English and Russian summaries)

Author's summary: "A strip of plate infinitely long is clamped at one of its long sides while the other side is free. The strip is acted upon by a force P acting at a point somewhere between the two long sides. A solution is deduced which is expressed by potential functions with singularities."

R. C. T. Smith (Armidale)

12846:

Dundurs, J.; Hetényi, M. The elastic plane with a circular insert, loaded by a radial force. J. Appl. Mech. **28** (1961), 103–111.

Authors' summary: "The problem treated is that of a plate of unlimited extent containing a circular insert and subjected to a concentrated radial force in the plane of the plate. The elastic properties of the insert are different from those of the plate, and a perfect bond is assumed between the two materials. The solution is in a closed form in terms of elementary functions."

R. C. T. Smith (Armidale)

12847:

Van Fo Fi, G. A. Equilibrium of elliptical plates in transverse loads. Akad. Nauk Ukraïn. RSR. Prikl. Meh. **5** (1959), 402–410. (Ukrainian. Russian and English summaries)

Bending by a concentrated load is considered.

R. C. T. Smith (Armidale)

12848:

Ulitko, A. F. General problem of the equilibrium of an elastic cone. Akad. Nauk Ukraïn. RSR. Prikl. Meh. **6** (1960), 302–310. (Ukrainian. Russian and English summaries)

The problem of equilibrium of an infinite elastic cone with stress boundary conditions on the curved surface is considered. Applying the Mellin transform, the author solves the resulting equations by means of infinite series of Legendre and trigonometrical functions. The unknown constants may be found from the boundary conditions. In principle this makes it possible to obtain the solution in the form of Mellin integrals. An example of a hollow cone loaded inside by uniformly distributed pressure is worked out.

H. Zorski (Warsaw)

12849:

Naghdi, P. M. On the general problem of elastokinetics in the theory of shallow shells. Proc. Sympos. Thin Elastic Shells (Delft, 1959), pp. 301–330. North-Holland, Amsterdam, 1960.

Utilizing Marguerre's differential equations of shallow shells, it is shown that the elastokinetic problem may be reduced to a system of three coupled equations involving axial displacement, a stress function, and the middle-surface parallel-shear-stress resultant. By means of a suitable order-of-magnitude-analysis, previously utilized in the linear theory of shallow-shell vibrations [E. Reissner, Quart. Appl. Math. **13** (1955), 169–176; MR **16**, 1070], it is shown that after certain approximations are made, the non-linear equations simplify considerably, through omission of the axial displacement component in the longitudinal inertia terms. Additional approximations, in which the effect of longitudinal inertia is neglected, lead to further simplification of the system of differential equations. A similar analysis is carried out for circular cylindrical shells.

E. Reissner (Cambridge, Mass.)

12850:

Naghdi, P. M. On Saint Venant's principle: elastic shells and plates. J. Appl. Mech. **27** (1960), 417–422.

This investigation is concerned with an examination of the validity of Saint Venant's principle in the theory of thin elastic shells and plates. With the aid of an integral formula derived for the displacements and their relevant partial derivatives of all orders at a fixed point of the shell middle surface, the conclusions reached may be roughly stated as follows. If the loads acting on the shell maintained in equilibrium are purely edge loads, then the orders of magnitude of the displacements and stresses are in accord with the traditional statement of Saint Venant's principle. On the other hand, if the loads on the shell are purely surface loads, then the conclusions concerning the orders of magnitude of the displacements and stresses are the same as those of the Saint Venant principle, modified in the sense of von Mises and Sternberg.

W. T. Koiter (Delft)

12851:

Hu, T. C.; Shield, R. T. Uniqueness in the optimum design of structures. Trans. ASME Ser. E. J. Appl. Mech. **28** (1961), 284–287.

The optimum design of rigid/plastic sandwich shells is considered. It is shown that, even if an optimum design is not unique, there is nevertheless an instantaneous mode of deformation common to all such designs. This result is used to establish uniqueness of particular designs obtained so far.

R. Hill (Nottingham)

12852:

Casacci, S.; Picollier, G. Étude de la flexion des coques coniques d'épaisseur variable soumises à des efforts centrifuges. Houille Blanche **16** (1961), 46–58. (English summary)

The bending-theory equations of a conical elastic shell with linearly varying thickness under a centrifugal load have been solved numerically.

S. Drobot (Notre Dame, Ind.)

12853:

Knowles, J. K.; Reissner, Eric. On stress-strain relations and strain-energy expressions in the theory of thin elastic shells. J. Appl. Mech. **27** (1960), 104–106.

This paper is concerned with stress-strain relations of a more general form than those of Love's first-approximation theory. It is shown that when all terms small of order h^2/R^2 are omitted (h = shell thickness, R = radius of curvature of middle surface), then the stress-strain relations and the corresponding expression for the strain-energy in terms of stress resultants and couples as given by Flügge and Byrne reduce to the improvement of Love's theory considered by Trefftz [Z. Angew. Math. Mech. **15** (1935), 101–108]. In addition, the strain-energy formula of Trefftz for coordinate systems coinciding with the lines of curvature of the middle surface is extended to the case of arbitrary orthogonal coordinates.

H. J. Weinitschke (Los Angeles, Calif.)

12854:

Coupry, G.; Gourbil, L. Vibrations d'un corps de révolution dans un écoulement faiblement supersonique. Rech. Aéro. No. 76 (1960), 41–47.

The effect of vibrations of a thin shell of a slender body of revolution on the external flow is examined using the linearised theory for unsteady flow. Both the body deformation and the flow potential are expressed as Fourier series, the coefficients for the potential solution being found as Bessel functions.

Expressions are obtained for the flutter coefficients. There is fair agreement with experiment at transonic speeds (although the theoretical derivatives obtained by this method are constant). The authors suggest that the method could be applied to the determination of the effects of local vibrations (e.g., panel flutter).

A. W. Babister (Glasgow)

12855:

Aadnesen, Lars. On the behaviour of compression chords for through-bridges. Quart. J. Mech. Appl. Math. **14** (1961), 85–100.

From the author's summary: "A general method is presented for the solution of the differential equations for the stability of a deflected and rotated bar subjected to variable axial loading and constrained elastically along its length against deflexion and rotation."

J. Heyman (Cambridge, England)

12856:

Budiansky, Bernard; DiPrima, R. C. Bending vibrations of uniform twisted beams. J. Math. and Phys. **39** (1960), 237–245.

The vibrations are studied directly by a variational principle, under the usual conditions of beam theory.

J. W. Craggs (Newcastle upon Tyne)

12857:

Caughey, T. K. Random excitation of a system with bilinear hysteresis. J. Appl. Mech. **27** (1960), 649–652.

A nonlinear second-order ordinary differential equation is replaced by a linear second-order ordinary differential equation with constant coefficients. The choice of constants in the linear equation is discussed in detail. The mean-square value of a solution of the linear equation with a Gaussian random inhomogeneous term is determined, and plotted for particular parameter values.

The title, introduction and conclusion misleadingly refer to the original nonlinear problem, for which no results are given. The solution of the linear problem considered is arbitrary since no conditions are imposed to determine it uniquely, i.e., the coefficients of the free vibrations have been chosen arbitrarily. *J. B. Keller* (New York)

12858:

Caughey, T. K. Random excitation of a loaded nonlinear string. J. Appl. Mech. **27** (1960), 575–578.

A system of nonlinear second-order ordinary differential equations for the transverse motion of a set of beads on a massless string is considered. The equations are replaced by linear equations with constant coefficients. The arbitrary constant in each linear equation is chosen to minimize the mean-square difference between the linear and corresponding nonlinear equation when the difference is evaluated in terms of the solution of the linear equations. Then the mean-square deflection of each bead is computed from the linear equation, assuming applied forces which are independent stationary random Gaussian functions of time with zero correlation time.

The title, introduction and conclusion misleadingly refer to a nonlinear problem and to the motion of a string. Actually only a linear problem is considered and that is for a system of particles rather than for a string. The original equations ignore the unequal tensions in the massless string joining the particles. The solution of the linear problem considered is arbitrary since no conditions are imposed to determine it uniquely.

J. B. Keller (New York)

12859:

Hoppmann, W. H., II. Frequencies of vibration of shallow spherical shells. Trans. ASME Ser. E. J. Appl. Mech. **28** (1961), 305–307.

12860:

Claassen, R. W.; Thorne, C. J. Vibrations of thin rectangular isotropic plates. Trans. ASME Ser. E. J. Appl. Mech. **28** (1961), 304–305.

12861:

Sato, Takeshi B. On the flexure-torsional vibrations of a beam with variable section. Keio Univ. Centenary Memorial Publ., pp. 70–85. Faculty of Engineering, Keio University, Tokyo, 1958. (Japanese. English summary)

12862:

Ungar, Eric E. Transmission of plate flexural waves through reinforcing beams; dynamic stress concentrations. J. Acoust. Soc. Amer. **33** (1961), 633–639.

12863:
Lamb, George L., Jr. Input impedance of a beam coupled to a plate. J. Acoust. Soc. Amer. **33** (1961), 628–633.

12864:
Niedenfuhr, F. W. The dynamics of rolling aircraft. J. Aerospace Sci. **28** (1961), 133–140, 157.

The equations of motion are developed for a rolling aircraft with its mass concentrated along a flexible fuselage.

A simple analogous mechanical spring system is first studied (a spring restrained mass in a rotating rigid frame), and it is shown that for a system which is dynamically anisotropic, there exists a finite range of continuously distributed critical speeds. The amplitude response to a forcing function is also considered, and critical-roll rates are found at which a subharmonic response becomes arbitrarily large.

A second equivalent system is that of a ball in a spinning cup. This system is useful for visualising the effect of a variable rate of roll.

For the aircraft, the influence of mode shape on the stability criteria is considered and is shown to be important. Finally, the dynamic response to both fuselage bending and rigid-body motions is considered.

A. W. Babister (Glasgow)

12865:
Bieber, R. E. Missile structural loads by nonstationary statistical methods. J. Aerospace Sci. **28** (1961), 284–294.

Under the assumption of a linear system and of a normal wind-velocity probability density function representing horizontal jet-stream winds, the average, standard deviation and significant extreme design quantiles (95% and 99% probability) of the bending moment distribution over the length of a vertical-rising ballistic missile, considered as a rigid body, are computed at the critical flight altitude at which the dynamic pressure is highest, although the wind velocity is not. The wind-data have been taken from observations at a specific base; all computations have been performed by computers.

A. M. Freudenthal (New York)

12866:
Birkgan, A. Yu.; Vol'mir, A. S. An investigation of the dynamic stability of plates using an electronic digital computer. Dokl. Akad. Nauk SSSR **135** (1960), 1083–1085 (Russian); translated as Soviet Physics. Dokl. **5** (1961), 1364–1366.

A simply-supported square plate with initial curvature is subjected to dynamic compressive load $p(t)$ acting in one direction. Solving the non-linear von Kármán equations by means of the finite differences method and a M-20 digital computer, the authors obtain the deflection of the plate as a function of time for various types of loading $p(t)$. The cases of the load $p(t)$ increasing linearly (with different rates of change in time) up to a certain value, and then remaining constant or decreasing back to zero, have been discussed.

M. Sokolowski (Lawrence, Kans.)

12867:
Samuels, J. Clifton. The buckling of circular cylindrical shells under purely random external pressures. J. Aerospace Sci. **27** (1960), 943–950.

Defining "mean-square stability" by the condition that the mean squares of the displacement-components of the middle surface of a simply supported thin circular-cylindrical elastic shell with velocity-dependent linear damping remain bounded as time goes to infinity, the solution of the equations of motion of such a shell is discussed under the assumption that the external radial pressure is the sum of a constant term and a stationary random function of time (white noise). The equations are solved for the circular ring in plane strain (infinitely long cylinder) and the results discussed and compared to those obtained under the assumption of periodically varying pressure. It is shown that a purely random pressure of any non-zero spectral density does not produce regions of stable modes in an undamped ring, while under periodically varying loads stable and unstable regions exist.

A. M. Freudenthal (New York)

12868:
Nariboli, G. A. A note on the fracture of a hypo-elastic bar. Proc. Sympos. Non-Linear Phys. Probl. (Roorkee, 1959), pp. S.55–S.60. Indian Soc. Theoret. Appl. Mech., Kharagpur.

Interpreting the formation of a singular discontinuity surface in the displacements of an incompressible hypo-elastic bar or fracture, it is shown that the appearance of such a singular surface requires a positive value of the stress-power.

A. M. Freudenthal (New York)

12869:
Mitchell, L. H.; Head, A. K. The buckling of a dislocated plate. J. Mech. Phys. Solids **9** (1961), 131–139.

The buckling conditions are calculated for a thin circular elastic plate which contains an edge dislocation at its center that can be generated by a radial cut along which a constant thickness is removed before the two faces of the cut are rejoined. Using Coker and Filon's equations for the stresses in such a plate and an assumed deflected form of the buckled plate selected for analytical rather than for physical reasons, the critical thickness of the plate is estimated using an energy method.

A. M. Freudenthal (New York)

12870:
Wakasugi, Shōhachi. Buckling of a simply supported triangular plate having inner angles of 30, 60 and 90 degrees. Bull. JSME **4** (1961), 16–20.

12871:
Wakasugi, Shōhachi. Buckling of a simply supported equilateral triangular plate. Bull. JSME **4** (1961), 20–25.

12872:
Kollár, L. Stability of centrally-compressed shell-arches. Acta Tech. Acad. Sci. Hungar. **32** (1961), 11–38. (German, French, and Russian summaries)

12873:
Sretenskii, L. N. Elastic waves arising from normal

stresses applied to the surface of a half-space. Problems of continuum mechanics (Muskhelishvili anniversary volume), pp. 519–536. SIAM, Philadelphia, Pa., 1961.

The author treats the problem of the elastic half-space excited by normal stresses on the boundary that are uniformly distributed over a finite rectangular area and vary harmonically with time. The shear stresses over the entire boundary are taken to be zero for all time. Formal solutions for the surface displacements are written in the form of double Fourier-type integrals. These integrals are then evaluated through contour integrations and residue theory. Finally, asymptotic expansions of the resultant integrals, for the far-field, show the predominance of the Rayleigh wave, as might be expected from Lamb's well-known finding for the related problem involving surface point-load excitation. As in Lamb's problem, the displacement amplitudes associated with this wave decay as $1/\sqrt{r}$. The author presents a detailed study of these far-field Rayleigh wave displacements as a function of surface position. Interesting are the zeros in the displacements that can occur at certain places, related to the characteristic dimensions of the rectangular loading region.

J. Miklowitz (Pasadena, Calif.)

12874:

Stoneley, Robert. The propagation of surface waves in anisotropic media. Partial differential equations and continuum mechanics, pp. 81–93. Univ. of Wisconsin Press, Madison, Wis., 1961.

The author treats the propagation of Rayleigh surface waves and Love waves in two types of crystalline media, namely, transversely isotropic media (i.e., media whose elastic properties are symmetrical to an axis), and crystals of cubic symmetry. He discusses how the method of treatment can be extended to other crystal classes and to the general problem of wave propagation on the surface of a general anisotropic solid. *H. Kolsky* (Providence, R.I.)

12875:

Chao, C. C.; Bleich, H. H.; Sackman, J. Surface waves in an elastic half space. Trans. ASME Ser. E. J. Appl. Mech. **28** (1961), 300–301.

The authors give a solution to the problem of a suddenly applied point load which is valid for points near the surface of the half-space at times near the arrival time of the Rayleigh wave. *G. Eason* (Newcastle upon Tyne)

12876:

Ogurcov, K. I. Stress waves in an elastic plate. Prikl. Mat. Meh. **24** (1960), 438–446 (Russian); translated as J. Appl. Math. Mech. **24**, 640–652.

The author discusses impulsive loading of an elastic plate from the point of view of its relevance to fracture phenomena. Expressions are obtained for the waves generated by a Dirac delta-function type of loading and how this can be generated for arbitrary continuous loading is discussed. *H. Kolsky* (Providence, R.I.)

12877:

Hook, Joseph F. Separation of the vector wave equation of elasticity for certain types of inhomogeneous, isotropic media. J. Acoust. Soc. Amer. **33** (1961), 302–313.

The author extends the method of separation, of both dependent and independent variables of the vector displacement equation of motion for a homogeneous, isotropic, linear elastic solid, to the related elasticity equation governing inhomogeneous media. Separation is restricted to cases where certain mechanical properties of the media are made dependent on one Cartesian coordinate, and Poisson's ratio is taken constant. Axially symmetric waves in three dimensions and waves in two dimensions are considered. Potential representations of the displacement lead to three linearly independent vector solutions of the equation of motion, involving three scalar functions. The author is able to identify each of these functions with a generalized SH (horizontally polarized shear), SV (vertically polarized shear), or P (dilatational) wave, and to show each satisfies an independent second-order wave equation. In the problems treated, he finds that the SH waves are solenoidal, but that the P and SV waves generally are not purely irrotational and solenoidal, respectively. *J. Miklowitz* (Pasadena, Calif.)

12878:

Pao, Yih-Hsing; Mindlin, R. D. Dispersion of flexural waves in an elastic, circular cylinder. J. Appl. Mech. **27** (1960), 513–520.

The exact solution of the Pochhammer equations for the flexural vibration of a circular cylinder leads to very heavy numerical computation. The present paper shows how an approximate procedure using a grid of simpler curves and the asymptotic equations for long and short wavelengths leads to a qualitative description of the various branches of the curves which relate phase velocity to frequency for such flexural vibrations. *H. Kolsky* (London)

12879:

Miklowitz, Julius. Recent developments in elastic wave propagation. Appl. Mech. Rev. **13** (1960), 865–878.

This paper is a review of the work that has been published in recent years on the propagation of elastic waves in bounded semi-infinite and unbounded solids. It contains a discussion of exact and approximate theories for the propagation of longitudinal, flexural and torsional waves in rods and plates. Waves in cylindrical shells and surface waves are also discussed and progress in diffraction problems is described. The paper is a very complete summary of the present position and gives 135 references.

H. Kolsky (London)

12880:

Redwood, Martin. ★Mechanical waveguides: The propagation of acoustic and ultrasonic waves in fluids and solids with boundaries. Pergamon Press, New York-Oxford-London-Paris, 1960. ix + 300 pp. $9.00.

This book is an introduction to the subject of wave propagation in bounded elastic fluids and solids. It is basically a survey of recent research, but one in which the author has sacrificed broadness of coverage to devote space to detailed physical and mathematical analysis in the subject matter. The first two chapters are brief treatments of the usual introductory material in the subject, i.e., wave propagation in unbounded elastic media, and the reflection and refraction of plane harmonic waves at an interface. The third chapter logically begins this study of dispersive waves with a treatment of the simpler perfect

fluid waveguide. The treatment is based on infinite harmonic wave train solutions (continuous waves) with the plate and cylindrical rod as the vehicles. Analysis by reflected plane waves and direct solutions of the boundary value problems are used to generate the dispersion relations and bring out the physical features of the modes of propagation. The fourth chapter is on pulse propagation in the fluid waveguide, a review being given of the use of Fourier series and integral, and simple Laplace transform techniques in this problem.

In the next several chapters this approach of dealing with the continuous wave first, and then the pulse propagation problem, is extended to the more complicated solid wave guide. In this material continuous wave propagation in rods and plates, based on approximate theories, is also reviewed briefly, as well as the propagation of such waves in solid guides of other cross-sections (rectangular rods and cylindrical shells). The material on pulse propagation in solid wave guides is broken down into chapters on narrow- and wide-bandwidth pulses. In the first, the problem of trailing pulses in the propagation of a dilatational wave at high frequencies, which has been of particular interest to the author, is treated in detail. In the second, a review is given of recent integral transform work that has led to the exact theory of far-field solutions for the semi-infinite circular cylindrical rod subjected to end excitation (of mixed condition type).

The final chapters of the book are devoted to brief treatments on multi-layered waveguides, solid resonators, and waves in anisotropic media. The reference lists on the various subjects in the book are extensive, but only a moderate amount of these are discussed in the text. The reviewer believes the author has accomplished his task well. He can be criticized, however, for not bringing out certain important continuous wave features of the rod and plate associated with complex wave numbers (so-called "edge waves"). Also, the important role approximate theories of the elastic rod and plate have played, in pulse propagation, is not brought out very well. The reader can find a discussion of these topics in the survey by the reviewer [#12879 above]. *J. Miklowitz* (Pasadena, Calif.)

12881:

McNiven, H. D. Extensional waves in a semi-infinite elastic rod. J. Acoust. Soc. Amer. **33** (1961), 23–27.

This paper discusses a mode of symmetrical vibration which can take place at the end of an elastic rod as a result of coupling between pure extensional waves and waves with complex wave numbers which attenuate rapidly in amplitude with distance from the free end of the rod. The author shows that such coupling can result in waves of the type observed experimentally by J. Oliver in his work on wave propagation.

H. Kolsky (Providence, R.I.)

12882:

Flinn, Edward A. Exact transient solution of some elementary problems of elastic wave propagation. J. Acoust. Soc. Amer. **33** (1961), 623–627.

The author obtains exact solution for two problems in the propagation of elastic waves, namely, the wave patterns generated by (a) a point source of SH waves in a free solid plate and (b) an arbitrary distribution of torsional stress across a normal section of a solid free cylinder of infinite length. *H. Kolsky* (Providence, R.I.)

12883:

Buldyrev, V. S.; Molotkov, I. A. On non-stationary propagation of waves in homogeneous and isotropic media separated by a cylindrical or a spherical boundary. Leningrad. Gos. Univ. Uč. Zap. Ser. Mat. Nauk **32** (1958), 261–321. (Russian)

This paper is concerned with solutions of the wave equation in spherical (r, φ, θ) or cylindrical (r, φ, z) coordinates in two different media separated by a boundary $r=R$. The waves issue from sources with intensity $a(t)$ uniformly distributed along $r=R_1$, $\theta=0$, or $r=R_1$, $\varphi=0$, in the respective cases. Four problems are distinguished, determined by the cases $R_1 \gtrless R$ and $\gamma \gtrless 1$, where γ is the ratio of the velocities in the two media. By means of the Laplace transform $F(s)=\int_0^\infty a(t)e^{st}dt$, the solutions are expressed in the usual way as infinite sums of contour integrals involving spherical and cylindrical functions.

The body of the paper is devoted to the systematic computation of the integrals by the method of stationary phase. The asymptotic representations of the cylinder functions are used to determine the singularities of the integral. The location of saddle points is reduced to the solving of a system of four algebraic equations, and this process is effectively illustrated by an example. The various types of phase functions occurring in the problems under consideration are completely catalogued. Finally, the selection of contours is made, and the calculation of the wave fields is completed. Physical implications of the analysis are everywhere recognized. Like previous papers in this field from this school, the treatment is exhaustively detailed and does not take note of similar work being done elsewhere. *R. N. Goss* (San Diego, Calif.)

12884:

Perzyna, Piotr. General analysis of the problem of propagation of plane elastic-plastic waves in a non-homogeneous medium. Arch. Mech. Stos. **12** (1960), 371–378. (Polish and Russian summaries)

The propagation of plane elastic-plastic waves when the strain-displacement relation is of the form $\varepsilon(x, t)= u_x(x, t)+\frac{1}{2}[u_x(x, t)]^2$ is considered, transverse motion being disregarded. The body is considered non-homogeneous, that is, $\sigma=f(\varepsilon, x)$. The characteristics and the differential relations satisfied on them are obtained for the loading and unloading processes. Then, for a non-homogeneous linear work-hardening material, using also the dynamical continuity conditions and considering the fronts of elastic and plastic waves as fronts of shock waves, the variation of the derivatives of the displacement on these front-waves is obtained. In the loading elastic and plastic domains the usual solution of the differential equations is indicated, as well as the inverse method for the unloading domain.

N. Cristescu (Bucharest)

12885:

Leonov, M. Ya.; Panasyuk, V. V. Development of slight cracks in a solid body. Akad. Nauk. Ukraïn. RSR. Prikl. Meh. **5** (1959), 391–401. (Ukrainian. Russian and English summaries)

The methods of the theory of elasticity applicable to the growth of microcracks in solid bodies are used by the introduction of the interaction forces of the edges of the microcracks. It is demonstrated that the presence of a

single linear dislocation in an extended body reduces the strength of the body by about one-half.

R. M. Evan-Iwanowski (Syracuse, N.Y.)

12886:

Goren, S. L.; Gavis, J. Transverse wave motion on a thin capillary jet of a viscoelastic liquid. Phys. Fluids **4** (1961), 575–579.

The tension and velocity are assumed to vary sufficiently slowly along the jet for the transverse vibrations to be governed by the wave equation with respect to axes moving with a particle. The equation, transformed to axes fixed in space, is solved neglecting variations in velocity and assuming exponential decay for the tension. {Reviewer's remarks: (i) The velocity and tension are determined by the longitudinal equation of motion and the constitutive equation of the liquid. (ii) Exponential-type decay of stress is a property of visco-elastic materials only when held at constant strain.}

D. R. Bland (Manchester)

12887:

Èstrin, M. I. The equations of the dynamics of a compressible plastic medium. Dokl. Akad. Nauk SSSR **135** (1960), 36–39 (Russian); translated as Soviet Physics. Dokl. **5** (1961), 1349–1352.

A small-strain theory of the kinetics of an elastic-perfectly plastic material in plane strain is considered. The Tresca yield condition is used in the form valid when the stress normal to the plane of deformation, σ_z, is the intermediate principal stress. A compressibility equation is assumed which is appropriate to a rigid-plastic material, but in order to be true for the above, the author's K should be replaced by $K+\frac{1}{3}G$, where K and G are the usual elastic bulk and shear moduli (not defined in the paper).

Two propagation velocities for surfaces of weak plastic discontinuity are derived from characteristics theory, and are found to depend on the angle between the wave-front normal and a principal-stress direction, and the possibility of shock formation is suggested. As a special case, the author derives a compression wave velocity, $(K/\rho)^{1/2}$, where ρ is the mass density, i.e., $[(K+\frac{1}{3}G)]^{1/2}$ in usual notation, which is not the familiar velocity of uniaxial strain theory. This difference arises because there σ_z ($\equiv\sigma_y$) is not strictly the intermediate principal stress, and the flow rule allows the plastic strain-rate vector to have a z-component. Under these circumstances the form assumed for the compressibility equation is incorrect.

In conclusion, the equations for similarity motion in an incompressible medium, for which there is only one wave velocity, are derived.

L. W. Morland (Sevenoaks)

12888:

Ziegler, Hans. On the theory of the plastic potential. Quart. Appl. Math. **19** (1961), 39–44.

The author shows that basic properties of Prager's generalized stresses and strains may be proved from von Mises' plastic potential hypothesis, assumed valid for each element of body. The advantage of the concept of generalized stress and strain is emphasized for rigid-plastic solids, and also for limit analysis of elastic-plastic solids.

W. T. Koiter (Delft)

12889:

Ziegler, Hans. Über den Zusammenhang zwischen der Fliessbedingung eines starrplastischen Körpers und seinem Fliessgesetz. Z. Angew. Math. Phys. **11** (1960), 413–426. (English summary)

The author supplies a proof, free from any additional physical assumption, that von Mises' theory of the plastic potential, if valid for the elements of a rigid-plastic body, also holds for the representation in generalized strains and stresses.

W. T. Koiter (Delft)

12890:

Ziegler, Hans. Bemerkung zu einem Hauptachsen-problem in der Plastizitätstheorie. Z. Angew. Math. Phys. **11** (1960), 157–163. (English summary)

In an ideally plastic material any principal axis of the stress tensor is also a principal axis of the strain-rate tensor. It is shown here that in plastic flow also the reverse is true. Besides, the validity of either statement is discussed for materials obeying Prager's hardening rule or its modification proposed by the author.

W. T. Koiter (Delft)

12891:

Gurtin, M. E. Extension of Nadai's sand hill analogy to multiply connected cross sections. Trans. ASME Ser. E. J. Appl. Mech. **28** (1961), 301–302.

In relation to a doubly-connected section the proposed method is to (i) determine by the original analogy the stress surface for the simply-connected section having the same external contour C; (ii) level this sand-hill to the height of the contour line C' just enclosing the inner boundary C''; (iii) determine by a second experiment the sand-hill for a section bounded by C' and C''. Finally, put the second hill on top of the levelled first hill.

No mention is made of a non-uniqueness difficulty when the problem is not statically determined: namely, how to be sure in an experiment that one has found the sand-hill of greatest volume when there are an unknown number of different hills having the limiting slope at all points.

R. Hill (Nottingham)

12892:

Il'yušin, A. A. Problems of the general theory of plasticity. Prikl. Mat. Meh. **24** (1960), 399–411 (Russian); translated as J. Appl. Math. Mech. **24**, 587–603.

This paper suffers from several ambiguities (either of translation or of thought), but contains some relevant criticism of the assumptions of accepted theories of plasticity.

J. W. Craggs (Newcastle upon Tyne)

12893:

Horne, M. R. The stability of elastic-plastic structures. Progress in Solid Mechanics, Vol. II, pp. 277–322. North-Holland, Amsterdam, 1961.

This paper is a review of stability of elastic and elastic-plastic beam- and frame-type structures, beginning from first principles. It is divided into three main sections concerned with general principles, methods, and solutions, respectively.

In the first two sections careful attention is given to distinguishing those principles which depend on the

stress-strain relations to those which are independent of them. Conventional elastic methods are reviewed and the concept of stability of elastic and inelastic structures stated in various forms.

The final section reviews the concepts of first-yield load λ_y, rigid-plastic collapse load λ_p, and elastic critical load λ_c as criteria for the failure load λ_F, this last being defined as the first relative maximum point on a load-displacement diagram. Various methods of estimating λ_F based on a knowledge of one or more of the idealized yield loads are given. *P. G. Hodge, Jr.* (Chicago, Ill.)

12894:

Ma, B. M. Creep analysis of rotating solid disks with variable thickness and temperature. J. Franklin Inst. **271** (1961), 40–55.

A creep analysis is presented for stress distributions in rotating solid disks of variable thickness and various temperature. An exponential-function creep law is used, and numerical examples are worked, based on the Tresca criterion and associated flow rule.

J. Heyman (Cambridge, England)

12895:

Odqvist, F. K. G. Engineering theories of creep. Proc. Sympos. Non-Linear Phys. Probl. (Roorkee, 1959), pp. S.1–S.18. Indian Soc. Theoret. Appl. Mech., Kharagpur.

The author surveys certain simple structural problems (bending of beams and stretching of circular membranes) under the assumption that the material response can be represented by that of an incompressible non-linear viscous fluid, with non-linearity according to a power-function with integral exponents.

A. M. Freudenthal (New York)

12896:

McNamee, John; Gibson, R. E. Displacement functions and linear transforms applied to diffusion through porous elastic media. Quart. J. Mech. Appl. Math. **13** (1960), 98–111.

In 1941 M. A. Biot [J. Appl. Phys. **12** (1941), 155–164] proposed a theory of the deformation of an elastic medium containing a distribution of connected pores completely filled with fluid. The equilibrium equation of the porous solid, assumed to be isotropic and homogeneous, is (1) $\nabla^2\mathbf{u}-\operatorname{grad}[(2\eta-1)e+\sigma/G]=0$, where $\mathbf{u}$ is the displacement vector and $e=-\operatorname{div}\mathbf{u}$ the dilatation of the solid skeleton, and σ is the pressure of the fluid in excess of the stress level in the surrounding elastic material (compressions and compressive stresses being counted positive in accordance with standard usage in soil mechanics). G, ν are, respectively, the shear modulus and Poisson's ratio of the solid skeleton and $\eta=(1-\nu)/(1-2\nu)$. Since there are four unknowns, the components of $\mathbf{u}$ and σ, a further equation is required. This governs the flow of fluid through the pores and, when the fluid is incompressible or when the overall compressibility of the medium is zero, can be written in the form (2) $c\nabla^2 e=\partial e/\partial t$, where $c=2G\eta k$, k being a permeability coefficient.

The present authors take up Biot's theory in this form and show that a solution of equations (1), (2) is given by $\mathbf{u}=-\operatorname{grad}(E+zS)+2z\operatorname{grad}S$, $\sigma=2G(\partial S/\partial z-\eta\nabla^2 E)$, where the displacement functions E, S satisfy the equations (3) $c\nabla^4 E=\nabla^2(\partial E/\partial t)$, $\nabla^2 S=0$. This form of solution is appropriate to problems involving the consolidation (i.e., settlement under suddenly applied surface loading) of a semi-infinite medium $z\geqq 0$ or an infinite layer $0\leqq z\leqq h$, and particular attention is devoted to the cases in which conditions of plane strain and axial symmetry apply. The solution of equations (3) by the repeated application of integral transforms of appropriate type is discussed as a preliminary to the separate presentation of specific solutions [see #12897].

{In view of the considerable weight of analysis which the proposed techniques entail, the reviewer would have welcomed a more detailed discussion of the basic equations (1), (2). The authors do not define precisely the various material constants which enter into the theory, and make no mention of the incompressibility assumption implicit in equation (2). Furthermore, no attempt is made to justify the neglect of inertial effects during the initial stages of settlement.} *P. Chadwick* (Sheffield)

12897:

McNamee, John; Gibson, R. E. Plane strain and axially symmetric problems of the consolidation of a semi-infinite clay stratum. Quart. J. Mech. Appl. Math. **13** (1960), 210–227.

The method of solving the equations governing the deformation of a porous solid, developed by the authors in an earlier paper [see #12896], is here applied to two particular boundary-value problems. They concern respectively the consolidation of the semi-infinite medium $z\geqq 0$ under loading applied uniformly at the surface $z=0$ over the infinite strip $-b\leqq x\leqq b$ and over the circular area $\rho\leqq b$. In addition to the specification of the surface tractions, a boundary condition on the excess pore pressure σ must be provided. Both problems are solved under the extreme assumptions that the pore fluid can escape at the surface without hindrance ($\sigma=0$ at $z=0$), and that no escape is possible ($\partial\sigma/\partial z=0$ at $z=0$). From each of the four solutions, expressions are then derived, in the form of infinite integrals, for quantities of particular engineering significance, namely, the surface displacement and dilatation of the solid skeleton and the excess pore pressure. *P. Chadwick* (Sheffield)

12898:

Sandor, I. Die Differentialgleichung der Bodenkonsolidation und ihre Lösung mittels des Matrizenkalküls. Les mathématiques de l'ingénieur, pp. 382–385. Mém. Publ. Soc. Sci. Arts Lett. Hainaut, vol. hors série, 1958.

12899:

Odqvist, F. K. G.; Mellgren, A. Thermal stresses in long circular cylinders at periodic surface temperature variations. Proc. 5th Congr. Theoret. Appl. Mech. (Roorkee, 1959), pp. C.1–C.8. Indian Soc. Theoret. Appl. Mech., Kharagpur.

The title problem is solved with the neglect of both thermo-mechanical coupling and inertia effects. Numerical values are presented. The effect of the non-uniform distribution of thermal stresses upon uni-axial creep tests is discussed. *J. H. Weiner* (New York)

12900:

Nariboli, G. A. Spherically symmetric thermal shock in a medium with thermal and elastic deformations coupled. Quart. J. Mech. Appl. Math. **14** (1961), 75–84.

The thermoelastic problem in an infinite medium containing a spherical cavity with a sudden change in temperature imposed on the surface of the cavity has been treated by Sternberg and Chakravorty [Quart. Appl. Math. **17** (1959), 205–218; MR **21** #6149] by use of the uncoupled linear thermoelastic theory, which includes the inertia terms in the equations of motion, but neglects the mechanical coupling term in the energy equation. The present author treats the same problem by use of the coupled theory which includes both effects. The Laplace transform is used. The inversion is not carried out exactly, but two different types of approximations are considered. One involves the first few terms of an expansion of the transform in inverse powers of the transform parameter and yields information regarding the short-time behavior of the solution while in the second approximation the first two terms of the expansion of the transform in powers of the small coupling parameter in the energy equation are used. The author concludes that the effect of this coupling is to reduce the stress discontinuity, propagated with a velocity corresponding to that of a dilatational wave, to an almost negligible amount except at points close to the cavity.

J. H. Weiner (New York)

STRUCTURE OF MATTER

See also 12733, 13054.

12901:

Boys, S. F. Construction of some molecular orbitals to be approximately invariant for changes from one molecule to another. Rev. Mod. Phys. **32** (1960), 296–299.

12902:

Chester, G. V.; Thellung, A. The law of Wiedemann and Franz. Proc. Phys. Soc. **77** (1961), 1005–1013.

A proof is given of the Wiedemann-Franz law for non-interacting electrons, forming a degenerate Fermi-Dirac assembly and undergoing elastic scattering. No Boltzmann equation is employed in the derivation, which is true for strong- as well as weak-coupling.

S. Simons (London)

12903:

Wahl, F. Klassische nichtlineare Gitterstatik der Stufenversetzung. I. Theorie. Z. Naturforschg. **14a** (1959), 901–912.

In continuation of three previous papers [E. Fues and H. Stumpf, same Z. **10a** (1955), 136–145; E. Fues, H. Stumpf and F. Wahl, ibid. **13a** (1958), 962–978; H. Gross and F. Wahl, ibid. **14a** (1959), 285–294], the model of a slip dislocation in a crystal is discussed with the aid of classical non-linear lattice statics. The equations of equilibrium of a monoatomic cubic lattice with dislocation describes the deviation of the crystal configuration from the ideal configuration. This is done by the introduction of a stress field in the crystalline medium with the help of a translation theorem. Consideration of convergence require symmetrisation of the compliance coefficients. As a result certain dipoles of a mathematical nature are introduced. Numerical methods for calculating the stable positions of the lattice and a survey of the necessary tools are reserved for a later paper.

B. R. Seth (Madison, Wis.)

12904:

Kroupa, F. Circular edge dislocation loop. Czechoslovak J. Phys. **10** (1960), 284–293. (Russian summary)

A solution of the stress, deformation and deformation energy is given for an edge dislocation with its dislocation line having the shape of a circle in an unlimited isotropic medium. The possibility of using this solution in studying the dislocation loop in a crystal is discussed.

Werner Nowacki (Bern)

12905:

Lothe, Jens. Lorentz force on screw dislocations and related problems. Phys. Rev. (2) **122** (1961), 78–82.

The concept of a Lorentz force on screw dislocations, first introduced by Nabarro, is analyzed. It is concluded that the Nabarro-Lorentz force on screws and the Lorentz force of electromagnetism are not analogous, and that the term "Lorentz force on screws" should be dropped in order to avoid confusion. Only when the screw is constrained to move on one slip plane is the analogy with electromagnetism complete. Total quasi-momentum is not generally conserved when screw dislocations interact with elastic waves.

Werner Nowacki (Bern)

12906:

Vonsovskiĭ, S. V.; Giterman, M. Š. The many-electron theory of ionic crystals. Fiz. Tverd. Tela **2** (1960), 1793–1805 (Russian); translated as Soviet Physics. Solid State **2** (1961), 1622–1632.

A many-electron treatment of ionic crystals is carried out from the phenomenological and model point of view by means of the method of elementary excitations. The energy spectra of the boson and fermion branches of the elementary excitations is obtained by taking into account the interactions between electrons and the electron-phonon interaction. The general theory is applied in particular to the study of "polaron" excitations, of the anomaly of the kinetic coefficients near the Curie point in ferromagnetic and antiferromagnetic semiconductors, and of the Stark effect for exciton levels.

Werner Nowacki (Bern)

12907:

Striževskiĭ, V. L. The theory of the dispersion and absorption of light in crystals. Fiz. Tverd. Tela **2** (1960), 1806–1815 (Russian); translated as Soviet Physics. Solid State **2** (1961), 1633–1641.

The interaction of a crystal with a monochromatic light wave in the case in which the states to which phototransitions are allowed form a continuous spectrum is considered. The specific dipole moment of the dielectric polarization is calculated, and formulas are derived that determine the index of refraction and the absorption coefficient of the light. It is shown that the calculation of the absorption coefficient as a quantity proportional to the probability of phototransition does not always lead

to the correct result. The theory is applied to the case of a molecular crystal in which excitons are produced, with the exciton-phonon coupling not treated as small. A preliminary determination is made of the wave function and energy levels of a molecular crystal for arbitrary exciton-phonon coupling on the assumption that the crystal is in thermal equilibrium.

Werner Nowacki (Bern)

12908:

Simons, S. The interaction of longitudinally polarized vibrations in an isotropic dielectric. Proc. Cambridge Philos. Soc. **57** (1961), 86–95.

Author's summary: "A theoretical treatment is given of the relaxation time for the absorption of longitudinally polarized vibrations in an isotropic medium. Employing standard phonon interaction theory, the relaxation time is obtained in terms of certain integrals, depending parametrically on the ratio of transverse to longitudinal sound velocity. These integrals are evaluated numerically over the range which this ratio takes in practice. The results are applied to a consideration of solid argon, sodium chloride and germanium."

H. L. Frisch (Murray Hill, N.J.)

12909:

Klimontovič, Yu. L.; Silin, V. P. The spectra of systems of interacting particles and collective energy losses during passage of charged particles through matter. Uspehi Fiz. Nauk **70** (1960), 247–283 (Russian); translated as Soviet Physics. Uspekhi **3**, 84–114.

All theories of matter, considered as a system of interacting particles, require the development of procedures for describing ordered states of motion of the particles. In recent years this question has become acute in several fields of physics, where it is known as the "many-body problem". In the present paper, which is a review article, the authors discuss the theory for such phenomena as electron plasma oscillations in solids and phonon-roton excitation in He II. The scope of the paper is somewhat broader than is suggested by the title, but the authors do not attempt a systematic review of the theory, which is still very imprecise and qualitative. Rather, they describe in general terms the efforts, their own among others, which have been made to formulate a mathematical description of collective and plasma motions.

An attempt is made (Sects. 1, 2) to base the theory on the properties of a statistical distribution function of quantum mechanical character. This approach, which is a generalization of the Boltzmann distribution function of classical statistical mechanics, is so abstract and mathematically complicated that it can scarcely be said to have been effective in practice. Progress becomes possible only when the physical model is made more concrete. More specific considerations are given on the application of the Hartree-Fock method of the self-consistent field (Sect. 3), and on the theory of correlation of the motions of particles (Sects. 4, 5). Lastly, the interaction of a charged particle with a plasma is discussed (Sects. 6, 7). Various formulas which have been derived for the description of effects predicted by the theory are quoted but, for the most part, it is not known how accurate they may be in a quantitative sense. A brief discussion of some comparisons with experimental data are given at the end of the paper.

{For other recent papers in this field, see P. Nozières, Ann. Physique **4** (1959), 865–903 [MR **22** #417] and I. M. Lifšic and M. I. Kaganov [#12910].}

E. L. Hill (Minneapolis, Minn.)

12910a:

Lifšic, I. M.; Kaganov, M. I. Some problems of the electron theory of metals. I. Classical and quantum mechanics of electrons in metals. Uspehi Fiz. Nauk **69** (1959), 419–458 (Russian); translated as Soviet Physics. Uspekhi **2** (1960), 831–855.

12910b:

Lifşiţ, I. M.; Kaganov, M. I. Some problems of the electron theory of metals. I. Classical and quantum mechanics of electrons in metals. Acad. R. P. Romîne. An. Romîno-Soviet. Ser. Mat.-Fiz. (3) **14** (1960), no. 2 (33), 85–130. (Romanian)

[Various translations of same article.] This is the first part of a projected review article, to be completed in three parts, on the properties of electrons in metals. It is written along somewhat unusual lines. Instead of trying to give a unified review of the general literature on the subject, the authors develop a line of thought, and to some extent a philosophy, which they indicate to be that of the group in solid state theory at Kharkov. This can be characterized roughly as an attempt to derive the maximum amount of information possible from a consideration of the general topological features of the energy-momentum relation, $\varepsilon(\mathbf{p})$, for the electrons in the solid. This is called the dispersion law for the electrons. {Such an approach has many attractive features, despite the fact that at present it must depend in large measure on arguments which are really to be justified only by reference to earlier work based on more specific models.}

In this first paper the ground covered is largely that of the usual Bloch-Peierls theory, expressed in more abstract terms than is customary. Many of the considerations are given only in qualitative form, so that the reader will need to be familiar with the standard discussions in order to appreciate them. The distinction between the real particles (electrons) and the excitation states (quasi-particles) in the lattice is kept in the foreground of the discussion. The remaining parts of the review are to cover the thermodynamic and kinetic properties of electrons in metals.

E. L. Hill (Minneapolis, Minn.)

12911:

Pirenne, Jean. Thermodynamic properties of an isotopic mixed crystal. Physica **27** (1961), 385–402.

In an earlier paper [Physica **24** (1958), 73–92; MR **20** #528], the author obtained an expansion in terms of the moments of the deviations of the inverse masses for the frequency distribution of an isotopically disordered crystal. Here this expansion is used to derive expressions for various thermodynamic functions. The main difficulty encountered is that, whereas the perfectly ordered ("demixed") phase may be the stable one at low temperatures, the totally disordered one may nonetheless be "frozen in" in actual cases. Tractable expansions of the Thirring form are presented for high temperatures and the Debye approximation is used at low temperatures.

H. B. Rosenstock (Washington, D.C.)

12912:

Maradudin, A. A.; Weiss, G. H. Dispersion relations and vibrational frequency spectra. Nuovo Cimento (10) **15** (1960), 408–415. (Italian summary)

A common problem is the finding of the distribution-in-λ, $g(\lambda)$, from an expression $\lambda = f(\mathbf{x})$ which gives the value of λ at any point $\mathbf{x}$ in a suitably defined space. (The authors, for example, are mainly interested in deriving the frequency distribution $g(\omega)$ of atomic vibrations from the "dispersion relation" $\omega = f(k_1, k_2, k_3)$; another example might be a geographer who knows the altitude above sea level at any point in some country, and wants to know what fraction of the area of the country is located at any given altitude range.) A formal solution is $g(\lambda) = V^{-1}\int_V \delta(\lambda - f)d\mathbf{x}$; in this paper the authors show that many other previously used expressions for $g(\lambda)$ can be derived from this one by using different representations for the Dirac delta function δ.

H. B. Rosenstock (Washington, D.C.)

12913:

Klemens, P. G. Anharmonic attenuation of localized lattice vibrations. Phys. Rev. (2) **122** (1961), 443–445.

It was shown by Montroll and Potts [Phys. Rev. (2) **100** (1955), 525–543; MR **17**, 568] that point defects can give rise in certain circumstances to localized modes of vibration in crystal lattices, with extraband frequencies. The present author, considering the cubic terms of the anharmonic part of the potential, estimates the lifetime of such modes. It is found that the relaxation time at absolute zero for a typical case is $\tau_0 \sim 100/\omega$, where ω is the angular frequency of the mode. This means that the anharmonic interaction is not so strong as to destroy the character of the localized mode. The broadening of an optical line at this frequency would be of 10^{-3} ev or less. A certain observed infrared adsorption line has been in fact interpreted as being due to a localized mode [see R. F. Wallis and A. A. Maradudin, Progr. Theoret. Phys. **24** (1960), 1055–1077; MR **22** #10406].

J. Peretti (Montpellier)

12914:

Karle, J.; Hauptman, H. Seminvariants for non-centrosymmetric space groups with conventional centered cells. Acta Cryst. **14** (1961), 217–223.

The relationship of phase to the choice of origin, enantiomorph or frame of reference is clarified for those non-centrosymmetric space groups for which the conventional unit cell is not primitive. The theory employs special linear combinations of the phases, the structure seminvariants. Simple procedures are developed for selecting the origin by first fixing the functional form of the structure factor, then specifying the sign of a seminvariant when required, and, finally, specifying arbitrarily the values of a suitable set of phases. The study of the seminvariants for all the space groups is completed.

Werner Nowacki (Bern)

12915:

Viglin, A. S. The texture function—a quantitative measure of the texture of a polycrystalline material. Fiz. Tverd. Tela **2** (1960), 2463–2476 (Russian); translated as Soviet Physics. Solid State **2** (1961), 2195–2207.

Various polycrystalline materials with the same chemical composition can be distinguished from one another by the size of the grain (single crystals) of which it is composed and the orientations of (the crystallographic axes of) the various grains relative to one another. The texture function $p(g)$, the probability distribution of the orientations of the various grains with respect to a reference direction, is introduced as a quantitative measure of the texture of the material. The symmetry of $p(g)$ then determines the type of texture, i.e., the symmetries of the directions of the predominating orientations of the grains. For a given type, $p(g)$ also describes the degree of perfection of the texture, i.e., the dispersion of orientations about the predominating ones.

For a ferromagnetic material it is shown that $p(g)$ is determined by the values of the torques which result when the material is placed in an external magnetic field in different positions relative to the direction of the field.

H. A. Hauptman (Washington, D.C.)

12916:

Luzzati, V.; Benoit, H. Diffusion centrale des rayons X par des particules filiformes. Acta Cryst. **14** (1961), 297–300.

The authors derive an asymptotic expression for the intensity of x-rays (or of light) scattered by an assembly of filament-like particles having uniform linear density. Their formula reduces (for large s) to

$$i(s) \approx \frac{K}{s} + \frac{K'}{s^2},$$

where s depends on the scattering angle. The first term, important for large s, is related to previous work. The second term, which is negligible for large s, is important for moderate values of s. Hence expressions for K', which the authors obtain and which depend on the configuration of the collection of filaments, e.g., the relative numbers of filament which are broken, or bent, or crossed, are important in order to interpret the scattered density in terms of the nature of the filament structure.

H. A. Hauptman (Washington, D.C.)

12917:

Otte, Henry M. Estimation of a crystallographic orientation relationship. Acta Cryst. **14** (1961), 360–361.

Two methods are described for obtaining the orientation between two specified axes of two crystal structures, such as occur, for example, in the case of twinning. The methods employ standard Laue photographs and manipulations with a stereographic net. In comparing these methods with one due to Mackenzie, the claim is made that the speed is greater and the precision comparable.

H. A. Hauptman (Washington, D.C.)

12918:

Hosemann, Rolf. Möglichkeiten für die Gewinnung röntgenographisch gesicherter Informationen über die Struktur von Atomen bei anomaler Dispersion. Acta Cryst. **13** (1960), 794–802. (English summary)

Author's summary: "Expressions are given of the observable diffracted intensity for a sample producing anomalous dispersion. From the intensities two structure-describing functions of $\mathbf{x}$, the coordinate in crystal-space, can in many cases be obtained, viz., $a\rho(\mathbf{x})$ and $\delta(\mathbf{x})$, where ρ is the actual electron density, $a(\mathbf{x})\rho(\mathbf{x})$ the effective scattering density, and $\delta(\mathbf{x})$ the scattering phase ($a = 1$,

$\delta = 0$ when dispersion effects may be neglected). The complex scattering coefficients necessitate the introduction of two $Q(\mathbf{x})$ functions, Q_α even and Q_β odd in x, which are both obtainable by Fourier transformation of the intensities and expressible in terms of a_ρ and δ. For the case of a non-absorbing and unbounded crystal the Q_α function goes over into the Patterson function of the crystal structure, whilst Q_β degenerates to the $P_s(u)$-function of Okaya, Saito and Pepinsky [Phys. Rev. (2) **98** (1955), 1857–1858]."

FLUID MECHANICS, ACOUSTICS

See also 12782, 12805, 12880, 13030, 13102, 13260, 13261, 13265.

12919:

Birkhoff, Garrett. ★Hydrodynamics: A study in logic, fact and similitude. Revised ed. Princeton Univ. Press, Princeton, N. J., 1960. xi + 184 pp. (1 plate) $6.50.

This book is mainly concerned with two aspects of fluid mechanics: the inter-relation between theory and experiment, and the application of group theory to the subject. It is a revision in detail of the first edition, and includes much new material uncovered during the last decade. The chapter headings are as follows: (I) Paradoxes of non-viscous flow; (II) Paradoxes of viscous flow; (III) Jets, wakes and cavities; (IV) Modelling and dimensional analysis; (V) Groups and fluid mechanics; (VI) Added mass. A detailed discussion of the contents of the first edition, 1950, has been given in an earlier review [MR **12**, 365].

In the reviewer's opinion the whole of this book is of major importance to all writers on fluid mechanics who are not inclined to make sacred cows either of purely mathematical arguments, however elegant, or of fudged mathematics whose chief merit is apparent agreement with experiment.

Particularly important are the first three chapters in which the necessity of a close collaboration between theory and experiment is driven home by means of paradoxes and discussions of difficulties in the subject. Judging from many modern textbooks these are still not sufficiently well-known. To take one example, hardly any book mentions that there is a fundamental difficulty in compressible viscous flow; one does not know how to relate the thermodynamic pressure to the stress tensor in a general fluid.

In his final conclusions the author says that he has tried to throw two bridges across the widening gap between pure mathematics and physics. Whether he has been successful remains to be seen, of course, but there is every hope and expectation that he will be, and that many fluid dynamicists, finding this brilliant book stimulating, will adopt his creed as their own.

K. Stewartson (Durham)

12920:

Sommet, Jean. Sur le calcul de la masse virtuelle d'un navire animé d'un mouvement oscillatoire en profondeur finie. C. R. Acad. Sci. Paris **250** (1960), 4280–4282.

Author's summary: "Les équations du mouvement du centre de gravité d'un navire peuvent être formées commodément lorsqu'on a déterminé ce qu'on est convenu d'appeler la masse virtuelle de la carène. On se propose d'évaluer celle-ci dans le cas d'un navire animé d'un mouvement oscillatoire horizontal dans son plan longitudinal. Ce résultat s'applique en particulier aux mouvements des navires amarrés dans un port soumis à des seiches."

12921:

Bouligand, Georges. Groupements de problèmes et théories unitaires. C. R. Acad. Sci. Paris **250** (1960), 1409–1412.

Partant de considérations de philosophie des sciences, l'auteur examine le problème particulier des transformations conservant les volumes. En utilisant des transformations d'origine arithmétique, il montre qu'il existe pour les fluides incompressibles des mouvements avec diffusion totale, dont l'emploi n'est pas habituel en hydrodynamique et qui pourraient servir à la compréhension de phénomènes aussi complexes que la turbulence. *J. Bass* (Paris)

12922:

Bouligand, Georges. Théorie des ensembles et mouvement turbulent. C. R. Acad. Sci. Paris **250** (1960), 1948–1950.

L'auteur reprend les idées d'une note antérieure [#12921] concernant les transformations qui conservent les mesures de volume. Il en signale les difficultés et les généralisations possibles, ainsi que l'influence, dans le cas hydrodynamique, de la structure moléculaire de la matière. *J. Bass* (Paris)

12923:

Moreau, Jean-Jacques. Une méthode de "cinématique fonctionnelle" en hydrodynamique. C. R. Acad. Sci. Paris **249** (1959), 2156–2158.

Résumé de l'auteur: "Au mouvement d'un fluide incompressible sur un domaine fixe de l'espace est associé celui d'un hypersolide à nombre infini de dimensions. . . ." *J. P. Guiraud* (Paris)

12924:

Graffi, Dario. Sul teorema di unicità nella dinamica dei fluidi. Ann. Mat. Pura Appl. (4) **50** (1960), 379–387.

The author strengthens his earlier uniqueness theorem for incompressible fluid flow in an unbounded domain [Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Nat. (6) **12** (1930), 129–135] by eliminating convergence requirements on the velocity at infinity. He states that this will permit its utilization in the theory of turbulence. Specifically, he proves the uniqueness of the solution to the flow problem in the domain D external to one or several closed surfaces σ for the time interval $T: 0 \leq t \leq T$, subject to the following conditions: (1) velocity $\mathbf{v}$ given in D for $t=0$ and on $\sigma \times T$; (2) body forces given in $D \times T$; (3) $\mathbf{v}$ and all first derivatives bounded and continuous, second space derivatives piecewise continuous in $D \times T$; (4) pressure p continuous with piecewise continuous first derivative in $D \times T$; (5) at infinity, $p = p_0$, a given constant, and $p - p_0 = O(r^{-1})$.

H. C. Kranzer (Garden City, N.Y.)

12925:

Benjamin, T. B. Dynamics of a system of articulated pipes conveying fluid. I. Theory. Proc. Roy. Soc. Ser. A **261** (1961), 457–486.

The free motion of a system of articulated pipes, through which there is a constant flow of incompressible fluid, is considered. The neglect of frictional forces and pressure variations over the resilient joints is justified, and the fluid is replaced by a system of particles each travelling with uniform velocity relative to the pipe containing it. The equations of motion are formulated and reduced to Lagrangian form, from which the appropriate form of Hamilton's principle is derived. When the oscillations are infinitesimal, two particular examples are discussed in some detail: (i) the equation governing the sinuations of a flexible tube; (ii) the solution for two pipes. In the second of these it is pointed out that the presence of the fluid can cause instability either through buckling (small disturbance independent of time) or through over-stability (amplified oscillations).

K. Stewartson (Durham)

12926:

Benjamin, T. B. Dynamics of a system of articulated pipes conveying fluid. II. Experiments. Proc. Roy. Soc. Ser. A **261** (1961), 487–499. (1 plate)

Here the theoretical predictions of #12925 are tested by experiments with two articulated pipes each free to oscillate in a vertical or horizontal plane. The conclusions are in good agreement with experiment.

K. Stewartson (Durham)

12927:

Ludford, G. S. S. Longitudinal vortices in shear flow. Z. Angew. Math. Mech. **41** (1961), 153–158. (German and Russian summaries)

From the author's summary: "The effect of a trailing or longitudinal vortex on shear flow near a wall is investigated. It is found that the vortex tunnels under and across the shear flow, lifting the flow away from the wall by an amount depending on the distance of the vortex from the wall but not on its strength."

S. I. Pai (College Park, Md.)

12928:

Pârvu, A. Sur un problème de hydrodynamique. An. Univ. "C. I. Parhon" Bucureşti. Ser. Şti. Nat. No. 22 (1959), 49–54. (Romanian. Russian and French summaries)

Author's summary: "L'auteur établit des formules de calcul du mouvement des liquides compressibles autour d'obstacles cylindriques."

12929:

Stark, Valter. Aerodynamic forces on rectangular wings oscillating in subsonic flow. Svenska Aeroplan A.B. Tech. Note **TN 44** (1960), 29 pp.

The digital-computation method developed by the author in a previous paper [same Notes **TN 41** (1958); MR **21** #4670] is extended to wings with swept leading edges and unswept trailing edges with the aid of the reverse-flow theorem. Extensive numerical results are presented for rectangular wings undergoing polynomial-type deformations at $M=0$ and $M=0.9$. Comparisons with previous theoretical and experimental results are given. Agreement with experiment is, for the most part, within the rather wide scatter of the available data. Future results for cropped delta wings are promised.

J. W. Miles (Los Angeles, Calif.)

12930:

Meister, E. Flow of an incompressible fluid through an oscillating staggered cascade. Arch. Rational Mech. Anal. **6**, 198–230 (1960).

To each member of a group of N successive blades of an infinite cascade there is prescribed a harmonic motion either in phase or 180 degrees out of phase with any other member of the same group. A similar pattern is considered to persist throughout every such group, an indefinite number of which are imagined to develop the infinite cascade. The solution to the lifting surface problem is sought by a method which depends upon rearrangement of the integral equation relating downwash to free and bound vorticity into a form suggestive of a boundary relation appropriate to a steady Dirichlet flow with circulation. Hydrodynamical qualifications necessary to render this re-interpretation admissible are not discussed. The whole procedure is tailored to meet the formulation of a "modified Dirichlet problem", but it would seem that tractability has been sacrificed early in the attempt. Indeed, the author submits that the lift and moment calculations are too complex to be of value. The theoretical interest which otherwise attaches would have been enhanced by cognizance, at least, of the improbability of the hydromechanical existence of a quasi-steady state fluid motion.

A. Billington (Melbourne)

12931:

Kulakowski, L. J.; Haskell, R. N. Solution of subsonic nonplanar lifting surface problems by means of high-speed digital computers. J. Aerospace Sci. **28** (1961), 103–112, 176.

A method for calculating the flow pattern in the vicinity of an airfoil at subsonic speeds by the use of a digital calculator is discussed. Comparisons are made with results obtained by other methods and with experimental data.

H. Polachek (Washington, D.C.)

12932:

Clarke, Joseph H. The forces on wing-fuselage combinations in supersonic flow. J. Fluid Mech. **8** (1960), 210–226.

The use of reverse-flow relations is shown to simplify the calculation of aerodynamic forces acting on a wing-fuselage combination when the wing is nearly-plane and the fuselage is a cylinder of arbitrary cross-section upstream and downstream of the wing. The aerodynamic forces can be obtained from the solution for the pressure on that part of the fuselage surface within the dependence domain of the wing. The reverse-flow theorem does not apply to all the cases considered; the difficulty is overcome by introducing a derived configuration which is equipollent to the given configuration, the two configurations differing by a two-dimensional cylinder flow.

The analysis is general, and no applications are made to special problems.

G. N. Ward (Cranfield)

12933:

Laitone, E. V. The second approximation to cnoidal and solitary waves. J. Fluid Mech. **9** (1960), 430–444.

The expansion method introduced by Friedrichs [Comm. Pure Appl. Math. **1** (1948), 81–85] for the systematic development of shallow-water theory is extended so as to obtain second approximations to both cnoidal and solitary waves. It is shown that in this approximation the vertical motions are no longer negligible, nor is the pressure hydrostatic [see also the reviewer's paper, Proc. Cambridge. Philos. Soc. **49** (1953), 685–694; MR **15**, 260]. The author finds that his necessarily laborious calculations lead to results which differ from those of Korteweg and de Vries (1895), but agree with more recent calculations relating to the solitary wave.

An attempt is made to extrapolate these small-amplitude results to the waves of limiting height.

F. Ursell (Cambridge, England)

12934:

Wiegel, R. L. A presentation of cnoidal wave theory for practical application. J. Fluid Mech. **7** (1960), 273–286.

A set of convenient formulae and numerical data is derived from the leading results of two approximate theories describing long water waves of finite amplitude and permanent form, i.e., the original cnoidal wave theory of Korteweg and de Vries [Philos. Mag. (5) **39** (1895), 422–443] and the alternative theory of Keulegan and Patterson [J. Res. Nat. Bur. Standards **24** (1940), 47–101; MR **1**, 284]. Comprehensive numerical results for the wave profile and velocity are presented graphically, and some results for the water particle velocity and acceleration are also included. A few experimental measurements are recalled for comparison.

The paper largely fulfils its declared aim to detail the properties of cnoidal waves in a way that is directly useful to hydraulic engineers, but the absence of any critical comparison between the alternative theories detracts somewhat from its value. The respective predictions of the theories differ appreciably, and though it appears likely that the differences would be unimportant with respect to practical applications, no assurance is given on this point.

T. B. Benjamin (Cambridge, England)

12935:

Curtet, Roger. ★Sur l'écoulement d'un jet entre parois. Publ. Sci. Tech. Ministère de l'Air, No. 359, Paris, 1960. xiii+114 pp. 29.50 NF.

This is the most comprehensive and original treatment of confined jets yet published. If a turbulent jet emerges into a channel or pipe and the surrounding fluid has too small a velocity in the same direction to supply the normal entrainment, recirculation occurs when the widening jet reaches the wall and a standing eddy is created in which material of the jet is re-entrained. The profiles of the jets confined between plane walls and in circular pipes are found to differ very little from those emitted into extensive surroundings except near the regions of recirculation.

Plane jets emitted between parallel plane walls are not stable at high speeds but tend to cling to one side if the flux of surrounding fluid is sufficiently small to cause recirculation. The various velocity ratios at which the regime changes are given with considerable accuracy.

The velocity and pressure distribution is measured extensively, including close to the nozzle, and comparison made with theory. The theory, like much theory in which the effects of turbulence are represented by transfer coefficients, is open to much objection because of the uncertain variation of these coefficients with position. The boundary layer at the wall outside the jet is also studied. The comparison made with the ideas of Townsend, Thring and Newby, Coles, von Kármán is of special interest because of the extensiveness of the experiments.

R. S. Scorer (London)

12936:

Chester, W. The effect of a magnetic field on the flow of a conducting fluid past a body of revolution. J. Fluid Mech. **10** (1961), 459–465.

The fluid is incompressible, viscous and conducting, the Reynolds number and the magnetic Reynolds number of the flow being finite while the magnitude H of the imposed uniform magnetic field is allowed to become indefinitely large. It is shown that when $H\to\infty$ the limiting flow pattern consists of an undisturbed uniform stream outside a cylinder C circumscribing the body with generators parallel to the stream (and to the magnetic field). It is also shown that a 'boundary layer' develops on the body and from an examination of its structure the drag is computed.

K. Stewartson (Durham)

12937:

Chester, W.; Moore, D. W. The effect of a magnetic field on the flow of a conducting fluid past a circular disk. J. Fluid Mech. **10** (1961), 466–472.

The solution obtained in #12936 contained a velocity discontinuity across the cylinder C. The flow in the region of this discontinuity is examined in more detail when the body is a circular disc broad-side on to the flow. If H is very large but not infinite it is replaced by a transition region of thickness $H^{-1/2}$ which increases with distance from the body until at a distance of the order of H it merges into the over-all flow field. Using known solutions of the diffusion equations a correction factor to the formula for the drag obtained in the previous paper is calculated.

K. Stewartson (Durham)

12938:

Ballabh, Ram. Superposable axially symmetric flows. Proc. Sympos. Non-Linear Phys. Probl. (Roorkee, 1959), pp. S.61–S.68. Indian Soc. Theoret. Appl. Mech., Kharagpur.

Two incompressible viscous flows are said to be superposable if the vector sum of the two velocity vectors is again a velocity vector of a flow. The author first restates the condition obtained earlier [Gaṇita **6** (1955), 15–21; MR **18**, 691]. Then, he considers the case of a self-superposable flow, for which he derives a linear equation for Stokes' stream function. Finally he solves the known linear case of unsteady flow in a tube of uniform circular cross-section with given axial pressure gradient by superposition.

S. S. Shu (Lafayette, Ind.)

12939:

Khamrui, S. R. On the flow of a viscous liquid between two confocal elliptic cylinders under a periodic pressure gradient. J. Sci. Engrg. Res. **4** (1960), 395–400.

This paper deals with the oscillating viscous flow between two confocal elliptic cylinders under a periodic pressure gradient. The velocity distribution is expressed in terms of Mathieu functions, and its explicit form is given for two limiting cases of very small and very large frequencies of the pressure oscillation. For very small frequencies the velocity oscillation is in phase with the pressure oscillation and the instantaneous velocity profile is exactly the same as that for the constant pressure gradient. For very large frequencies, on the other hand, the velocity profile consists of a core of uniform amplitude and thin shear layers along the solid boundaries. These results are essentially the same as those already obtained for flows of the same type but with simpler geometrical configuration.

T. Tatsumi (Kyoto)

12940:

Rüdiger, D. Ein Variationsprinzip in der Hydrodynamik. Z. Angew. Math. Mech. **41** (1961), 66–72. (English and Russian summaries)

The author considers the linearized steady flow equations of a viscous incompressible fluid. By requiring that the variation of the divergence of the velocity vanish, the author obtains an expression whose first variation vanishes. This expression is determined in terms of the surface intervals of the stress forces and velocities on the boundaries and of the volume integrals of the body forces over the space region. By decomposing the stress tensor, force due to stress, and body force into partial sums with unknown coefficients and using the Galerkin type of equations, the above first variational equations may be analyzed. In a similar manner, the Trefftz equations can be treated. *N. Coburn* (Ann Arbor, Mich.)

12941:

Volkov, R. A. Jet flow of a viscous liquid from a round tube. Akad. Nauk. Ukraïn. RSR. Prikl. Meh. **5** (1959), 428–433. (Ukrainian. Russian and English summaries)

The theory of the jet flow of a viscous liquid under the influence of gravity tube is developed using the Navier-Stokes equations. The flow conditions for low Reynolds numbers are assumed to be stationary and laminar. This permits the linearization of the Navier-Stokes equations. The problem is a three-dimensional one, and is solved by means of a two-dimensional Laplace transformation. In this paper the following particular cases are considered: (1) flow from a vertical tube, (2) flow from a horizontal tube. The distributions of velocities and pressures at the end of the tube, at the point of origin of the jet, are taken as boundary conditions.

D. H. Hyers (Los Angeles, Calif.)

12942:

Chamberlain, Joseph W. Interplanetary gas. III. A hydrodynamic model of the corona. Astrophys. J. **133** (1961), 675–687.

The ionized interplanetary gas is regarded as an extension of the solar corona. Its motion is discussed in terms of hydrodynamics together with the first law of thermodynamics with the heat conduction term present. No magnetic fields are postulated. A solution exists in which both the expansion velocity and the density vanish at infinity. A steady "solar wind" would imply that an accelerating mechanism existed in the solar corona. Coronal observations do not support the view that there is a solar wind with a speed greater than several kilometers per second at the earth's orbit. In this region the temperature is governed by adiabatic equilibrium, its value is calculated to be 15,000–20,000° K. and the density of the gas is about 30 electrons/cm³. Closer to the sun, conduction appears to become important.

G. C. McVittie (Urbana, Ill.)

12943:

Nanda, R. S.; Jain, M. K. Viscous flow near a stagnation point with suction. J. Sci. Engrg. Res. **4** (1960), 387–394.

Navier-Stokes' equations for axisymmetric flow in the vicinity of a stagnation point with steady suction are integrated numerically. Introducing a stream function $\varphi(s)$ in the usual manner, $s=(a/\nu)^{1/2}z$ (a is a constant for the potential flow prevailing at great distance and ν is the kinematic viscosity), $u=ra\varphi'(s)$ and $w=-2(a\nu)^{1/2}\varphi(s)$, leads to the ordinary differential equation for $\varphi(s)$

$$(1)\qquad \varphi''' + 2\varphi\varphi'' - \varphi'^2 + 1 = 0$$

{a misprint occurs in this equation in the article} with the boundary conditions: $\varphi(0)=w_0/2(a\nu)^{1/2}=k$ (w_0 = suction velocity), $\varphi'(0)=0$ and $\varphi'(\infty)=1$. For large values of k, a power series solution in powers of $1/k$ is presented, without proof of convergence, however. For smaller values of k, the differential equation (1) is integrated numerically. Tables for φ, φ', φ'', the boundary layer thickness and shearing stress are given for selected values of k.

H. Oser (Washington, D.C.)

12944:

Ivanilov, Yu. P. On the stability of plane-parallel flow of a viscous fluid over an inclined bottom. Prikl. Mat. Meh. **24** (1960), 380–381 (Russian); translated as J. Appl. Math. Mech. **24**, 549–552.

The principal result, obtained by using a long wavelength approximation, is that the flow is unstable if

$$\frac{gh^3}{\nu^2} > \frac{5}{2}\frac{\cos\alpha}{\sin^2\alpha},$$

where h is the depth of the stream and α its inclination to the horizontal. A more general discussion of this problem, including the present result, has been given previously by T. B. Benjamin [J. Fluid Mech. **2** (1957), 554–574].

W. H. Reid (Providence, R.I.)

12945:

Bhatnagar, P. L. Vorticity and circulation in a compressible viscous flow in the presence of magnetic field. Golden Jubilee Research Volume, pp. 194–199. Indian Institute of Science, Bangalore, 1959.

The author discusses the vorticity and circulation in conducting fluids. He takes the curl of the momentum equation and analyses the physical situations under which some of the terms in that equation can be made to vanish. He further attempts to integrate the vorticity equation along a moving circuit.

The reviewer would like to draw attention to the paper on the same subject by H. C. Brinkman in Physica **25** (1959), 1063–1066 [MR **22** #1285].

R. P. Kanwal (University Park, Pa.)

12946:

Bearman, Richard J.; Jones, Peter F. Statistical mechanical theory of the viscosity coefficients of binary liquid solutions. J. Chem. Phys. **33** (1960), 1432–1438.

Kirkwood's theory of the transport coefficients of liquids [J. G. Kirkwood, F. P. Buff, and M. S. Green, same J. **17** (1949), 988–994], based on the Fokker-Planck equation, is used to obtain a formula for the viscosity of a binary mixture of spherically symmetric atoms. For "regular solutions" in which the radial distribution function is independent of composition, the formula implies that the product of the viscosity with the self-diffusion coefficient of either component is independent of concentration. This and other consequences of the formula are found to be consistent with a variety of published experimental results. *O. Penrose* (London)

12947:

Mamaladze, Yu. G.; Matinyan, S. G. The hydrodynamics of an oscillating disk in a rotating fluid. Prikl. Mat. Meh. **24** (1960), 473–477 (Russian); translated as J. Appl. Math. Mech. **24**, 691–697.

The Navier-Stokes equations for a viscous incompressible fluid are linearized for the case of small amplitudes of oscillation of the disk. Approximate solutions are obtained which are appropriate to various parts of the field near the disk, allowance being made for edge effects. The moment due to the viscous forces is calculated; it depends critically on the ratio of the frequency of the disk to that of the fluid. Good agreement is obtained between theory and experiment. *A. W. Babister* (Glasgow)

12948:

Schlichting, Hermann. ★Boundary layer theory. Translated by J. Kestin. 4th ed. McGraw-Hill Series in Mechanical Engineering. McGraw-Hill Book Co., Inc., New York-Toronto-London; Verlag G. Braun, Karlsruhe; 1960. xx + 647 pp. $16.50.

This is a revised edition of the book first published in 1951 [Verlag G. Braun, Karlsruhe; MR **13**, 177]. The chapters on compressible and thermal boundary layers and on transition have meantime been considerably increased in size. In addition, Gersten has collaborated with the author in several of the sections on heat transfer. As a result, a number of the criticisms of the book, set out in the review of the first edition, have been met, particularly those related to heat transfer through a turbulent boundary layer.

There is no doubt that the present book is by far the best general work of reference on boundary layers and indeed on viscous flow which is available at the present time and that all workers in the field should have it at hand. The great majority of topics are covered and the presentation is good, thanks partly to Kestin's careful translation. However, the book is not a success from all points of view. Partly, the reason is the almost exponential growth of the subject, referred to by Dryden in a foreword, which means that at the present time such topics as creeping motion and hydrodynamic instability which occupy relatively small parts of this book can command fair-sized monographs on their own. A further reason, however, is that the book is in no sense critical, and important developments in the last decade have not been placed in context as they warrant. Thus Coles' view of turbulent boundary layers is relegated to three lines on p. 543, although it is possibly more important now than the theory of the mixing length. Again the revolution in the view of the relation between Stokes' and Oseen's flow is not mentioned.

Although other examples can be adduced it is perhaps unfair to harp on them for surely the subject is now too big for any one man fully to comprehend. Grateful thanks are in fact due to the author for the tremendous effort required in making his view of the subject available to us. *K. Stewartson* (Durham)

12949:

Wadhwa, Y. D. Unsteady boundary layers. Proc. Sympos. Mech. Real Fluids (Kharagpur, 1958). J. Sci. Engrg. Res. **4** (1960), 7–22.

This is a review of certain aspects of unsteady (but not turbulent) boundary layers. The principal conclusion is that the method of expansion in series has two serious defects: (i) it is only valid at small time and (ii) it fails for a flat plate since the effect of the leading edge does not appear in it. *K. Stewartson* (Durham)

12950:

Pozzi, Amilcare. Raffreddamento con trasporto di massa alle alte velocità. Aerotecnica **40** (1960), 223–230.

Author's summary: "Si esaminano alcuni problemi riguardanti l'applicazione dei metodi integrali al calcolo dello strato limite con iniezione allo scopo di renderli semplici e precisi. Le equazioni dello strato limite vengono trasformate in due equazioni algebriche di terzo e quarto ordine disaccoppiate. La precisione del metodo viene definita per confronto con alcune soluzioni esatte."

12951:

Nickel, Karl. Parabolic equations with applications to boundary layer theory. Partial differential equations and continuum mechanics, pp. 319–330. Univ. of Wisconsin Press, Madison, Wis., 1961.

With the aid of parabolic differential inequalities, useful qualitative statements can be made about boundary layers in a viscous fluid. The author outlines results already obtained and then makes some extensions to two-component fluids. He shows that separation cannot occur with a favorable pressure gradient if the initial profiles are not of the separation type, and also shows that the velocities inside the boundary layer will not exceed the outer velocities if this is true initially. (The analysis is for incompressible two-dimensional steady flow; "initial" conditions refer to an upstream reference section across the boundary layer.) The author proves a uniqueness theorem and estimates the difference between the exact solution and any approximate solution of the equations and boundary conditions. *L. A. Segel* (Troy, N.Y.)

12952:

Ovčinnikov, O. N. Longitudinal flow past a cylinder immersed in an inhomogeneous flow in the presence of steady boundary-layer suction. Prikl. Mat. Meh. **24** (1960), 376–378 (Russian); translated as J. Appl. Math. Mech. **24**, 541–545.

An exact solution is presented for the velocity and temperature distributions in the laminar flow near a cylinder placed lengthwise in a stream of viscous incompressible fluid, with a free-stream velocity of the form $U=U_0+\omega_0 r^2$ and a constant suction velocity v_0 at the cylinder. For large enough values of the suction Reynolds number v_0a/ν, where a is the cylinder radius, even a weak inhomogeneity (i.e., a small value of the parameter ω_0a^2/U_0) appreciably affects the results. Since suction is usually used to delay transition to turbulent flow, the author suggests that such large Reynolds numbers will be of most interest in practice. {This statement is of doubtful validity, in the reviewer's opinion.}

D. W. Dunn (Ottawa, Ont.)

12953:

Roy, Ajit Kumar. Boundary layers and up-stream influence. Proc. Sympos. Mech. Real Fluids (Kharagpur, 1958). J. Sci. Engrg. Res. **4** (1960), 23–38.

This paper gives a brief survey of the interaction problem between the shock wave and the boundary layer. The main results of existing experimental and theoretical works on this subject are described and discussed.

T. Tatsumi (Kyoto)

12954:

Reid, W. H. The effects of surface tension and viscosity on the stability of two superposed fluids. Proc. Cambridge Philos. Soc. **57** (1961), 415–425.

The effects of surface tension on the stability of two superimposed fluids with constant properties is discussed. In the case that the viscosity of the upper and lower fluids is the same, the characteristic value problem reduces to that of solving a quartic equation which is identical to that derived by Chandrasekhar [same Proc. **51** (1955), 162–178; MR **16**, 639] except for a term due to the surface-tension effect. The roots of this equation are carefully analyzed for the cases when the density of the lower fluid is zero (an unstable case), and when the density of the upper fluid is zero (a stable case).

The author also briefly discusses a variational principle derived by Chandrasekhar to treat this problem in the case when the fluid properties are not constant; and he points out that in some applications of this principle by Hide [ibid. **51** (1955), 179–201; MR **16**, 639] one term has been omitted due to an incorrect limiting process. As a consequence of this additional term, the variational principle is of less usefulness than originally thought.

R. C. DiPrima (Troy, N.Y.)

12955:

Graebel, William P. Stability of a Stokesian fluid in Couette flow. Phys. Fluids **4** (1961), 362–368.

The author shows that stability of a special type of Reiner-Rivlin fluid in Couette flow depends on the Taylor number as well as a dimensionless parameter proportional to the "coefficient of cross-viscosity". The effect of cross viscosity on stability seems to be considerable. The results disagree with those of Jain [#12956].

A. E. Green (Newcastle upon Tyne)

12956:

Jain, M. K. The stability of certain non-Newtonian liquids contained between two rotating cylinders. J. Sci. Engrg. Res. **1** (1957), 195–202.

The stability of a non-Newtonian fluid in Couette flow is studied. The fluid is assumed to be characterized by the two (constant) coefficients of viscosity and "cross-viscosity". The basic velocity distribution is then the same as for a Newtonian fluid but the basic pressure distribution is modified by the presence of a cross-viscosity. The disturbance equations are derived after the manner of the usual normal-mode analysis. The problem is then further simplified by making the small-gap approximation, by assuming that the principle of exchange of stabilities holds, by assuming that the cylinders rotate in the same direction so that the basic velocity distribution can be replaced by its average value, and by retaining only linear terms in the cross-viscosity. These simplifications then lead to an equation for the radial component of the disturbance velocity which is linear, of sixth-order, and with constant coefficient and in which there now appears, in addition to the usual Taylor number, an additional parameter depending on the coefficient of cross-viscosity. This equation is then solved by a variational method, and one numerical example is given from which it is concluded that the cross-viscosity inhibits the onset of instability. This conclusion has recently been questioned by W. P. Graebel [#12955], who finds that the cross-viscosity has a destabilizing effect.

W. H. Reid (Providence, R.I.)

12957:

Morton, B. R. On a momentum-mass flux diagram for turbulent jets, plumes and wakes. J. Fluid Mech. **10** (1961), 101–112.

From the author's summary: "It is shown that with suitable simplifying assumptions the relationship between the momentum flux and the mass flux in the column of moving fluid can be found from the solution of a single ordinary differential equation. The character of many related flows can then be represented immediately (except for actual distribution in space) on a single momentum-mass flux diagram. Some approximate solutions are outlined for buoyant and non-buoyant wakes and jets directed parallel to a uniform main stream, and momentum-mass flux curves are presented."

D. W. Dunn (Ottawa, Ont.)

12958:

Ber, L. É. On a method for solving problems of non-isothermal turbulent convection in a channel formed by parallel planes. Soviet Physics. Tech. Phys. **29** (4) (1959), 52–60 (61–69 Ž. Tehn. Fiz.).

The assumptions are those normally used for the solution of the title problem. The Reynolds equations and the Boussinesq assumption of turbulent transfer coefficients are postulated. The turbulent Prandtl number is assumed constant and equal to 0.7 without much discussion. The "law of the wall" is modified to provide a smooth transition from the laminar sublayer to the universal logarithmic law (disregarding all previous attempts to achieve the same object).

Any new set of heuristic calculations, such as the ones advanced in this paper, will meet with much skepticism unless it can be demonstrated that they are decidedly superior to the existing schemes. This must be done by careful comparison with experiment at all stages, but the

author dismisses the matter with a few half-hearted remarks.

The translation is of a barely acceptable standard.

J. Kestin (Providence, R.I.)

12959:

Constantinescu, V. N. On turbulent lubrication. Proc. Inst. Mech. Engrs. **173** (1959), 881–900d.

The general equations for turbulent lubrication are derived from the Reynolds equations, using the standard "thin film" approximation of excluding the inertia terms. The Prandtl mixing-length hypothesis is then used to give an exact solution for the velocity profile in terms of pressure gradient and Reynolds number. Some approximations are made to simplify the results, and a comparison with some measured friction coefficients in journal bearings is said to show satisfactory qualitative agreement.

D. A. Spence (Farnborough)

12960:

Howells, I. D. The effect of turbulence and magnetic field on electron density fluctuations in the ionosphere. J. Fluid Mech. **8** (1960), 545–564.

This is an elaboration of the work of Dungey [J. Atmos. Terrest. Phys. **8** (1956), 39–42] on electron density fluctuations in the ionosphere. The main formal result is the derivation of a transport equation for the electron density $n(x, t)$ (or its Fourier transform $N(k, t)$), which includes the effect of the Earth's magnetic field and the electric space charge effects. This equation has the structure

$$\frac{\partial N(k,t)}{\partial t}+ik\cdot\int dk'V(k')N(k-k',t)$$
$$+D(k^2+\sum_{i,j}^{3}k_ik_jL_{ij})N(k,t)=i\sum_{i,j}^{3}k_iL_{ij}U_j(k)n_0.$$

n_0 is the average electron density, U_i the components of the wind velocity U, D a diffusion constant; V is an effective transport velocity, and differs from U by terms arising from magnetic field and space charge effects; L_{ij} describes the effects of the magnetic field on diffusion. The right-hand term is a source term, and is n_0 times the divergence of the velocity field V. (U is assumed to be solenoidal.) This equation is used to construct the spectrum $\Gamma(k)=|N(k)|^2$ of electron density fluctuation and to relate it to the turbulence spectrum $E(k)$ of U. Its form is discussed for a variety of situations, corresponding to different heights in the E-region of the ionosphere.

Special reference is made to the fact that experimental data indicate an anisotropic spectrum, with irregularities in electron density elongated along the magnetic field lines. The mechanism assumed here does not give to rise such an effect: The spectrum is nearly isotropic, or allows only elongated irregularities perpendicular to the magnetic field. If $E(k)\sim k^{-5/3}$, then $\Gamma(k)$ is a mixture of a $k^{-5/3}$ and a k^{-1} dependence.

F. Villars (Cambridge, Mass.)

12961:

Lacaze, J. Tourbillons atmosphériques d'axes verticaux (tourbillons de sable). II. Théorie: aspect hydrodynamique, aspect convectif. J. Méc. Phys. Atmos. (2) **1** (1959), 25–41. (English and Spanish summaries)

[For part I, see J. Sci. Météorol. **10** (1958), 133–147; MR **21** #7794.]

12962:

Dorodnicyn, A. A. Numerical methods in gas-dynamics. Arch. Mech. Stos. **12** (1960), 13–27. (Russian. Polish and English summaries)

An expository article which discusses the following techniques for computing flows with two independent variables: (i) von Neumann's and Richtmyer's artificial viscosity method [J. Appl. Phys. **21** (1950), 232–237; MR **12**, 289]; (ii) S. K. Godunov's integrated conservation law method [Mat. Sb. (N.S.) **47** (**89**) (1959), 271–306; MR **22** #10194]; (iii) approximate reduction of the steady flow equations to systems of ordinary differential equations by integration with respect to one variable along certain curves, followed by appropriate interpolation, as employed by P. I. Čuškin [Prikl. Mat. Meh. **21** (1957), 353–360; MR **19**, 1006] and O. M. Belocerkovskiĭ [Dokl. Akad. Nauk SSSR **113** (1957), 509–512; MR **19**, 1121].

J. H. Giese (Aberdeen, Md.)

12963:

Komarovskiĭ, L. V. On spatial gas flows with degenerate hodographs. Prikl. Mat. Meh. **24** (1960), 491–495 (Russian); translated as J. Appl. Math. Mech. **24**, 717–723.

The author discusses the reduction of the general equations for isentropic flow of a polytropic gas for the case when (1) one velocity component and the density are functionally dependent on the other two velocity components u_1, u_2, and (2) all the loci, in four-dimensional space-time, on which u_1 and u_2 are constant, are planes. The essential problem is to find equations specifying the said functional dependence, and for the various cases that may arise it is shown how this may be done. Two of the simpler cases are those of simple waves and conical flow.

T. M. Cherry (Melbourne)

12964:

Мизес, Р. [von Mises, Richard]. ★Математическая теория течений сжимаемой жидкости [Mathematical theory of compressible fluid flow]. Translated from the English by P. P. Koryavov, A. M. Ter-Krikorov and P. I. Čuškin; edited by N. N. Moiseev. Izdat. Inostr. Lit., Moscow, 1961. 588 pp. 2.66 r.

The original [Academic Press, New York, 1958] was reviewed in MR **20** #1504.

12965:

Serruys, Max; Magot-Cuvrû, Pierre. Influence de la viscosité chimique et d'un transfert de chaleur sur la célérité du son et la vitesse au col d'un écoulement permanent. C. R. Acad. Sci. Paris **250** (1960), 3134–3136.

A very perfunctory analysis which starts with the old misconception that the velocity of flow in the throat of a convergent-divergent nozzle, as well as that in a channel of constant cross-sectional area, must always necessarily be that of sound, written here $W=(dp/d\rho)^{1/2}$. The reviewer is not clear about what is meant by "chemical viscosity", but is certain that a reader interested in calculating the velocity at the throat in a nozzle traversed by a chemically reacting perfect gas will be misled.

J. Kestin (Providence, R.I.)

12966:

Hosokawa, Iwao. Theoretical prediction of the pressure distribution on a non-lifting, thin symmetrical aerofoil at

various transonic speeds. J. Phys. Soc. Japan **16** (1961), 546–558.

In this paper the author uses the improved linearized transonic theory which he has developed in an earlier publication [same J. **15** (1960), 149–157; MR **22** #1263] to calculate transonic pressure distributions on symmetrical airfoils whose thickness distribution is expressible by polynomials. The results for the circulararc airfoil are compared with various experimental and other theoretical results available in the literature. In particular, the position of the sonic point and the pressure drag coefficient are discussed.

This paper contributes substantially to demonstrate the usefulness of the so-called linearized transonic flow theory and its improvements by the author of the paper.

P. F. Maeder (Providence, R.I.)

12967:

Tricomi, Francesco G. Trans-sonic gas flow and the equations of mixed type. Partial differential equations and continuum mechanics, pp. 207–216. Univ. of Wisconsin Press, Madison, Wis., 1961.

Dans cette communication, l'auteur rappelle le lien entre l'étude des écoulements transsoniques et celle des équations aux dérivées partielles de type mixte. La remarque la plus intéressante concerne l'approximation de Tomotika et Tamada. Pour un changement de variable l'équation dont dépend l'étude est ramenée à une équations à coefficients linéaires.

P. Germain (Paris)

12968:

Kvashnina, S. S.; Chernyi, G. G. Steady state flow of detonating gas around a cone. J. Appl. Math. Mech. **23** (1959), 252–259 (182–186 Prikl. Mat. Meh.).

Steady state flow of an inert gas with attached conical shock about a non-yawing cone of revolution has been discussed by A. Busemann [Z. Angew. Math. Mech. **9** (1929), 496–498] and G. I. Taylor and J. W. MacColl [Proc. Roy. Soc. London Ser. A **139** (1933), 278–311]. The authors discuss the corresponding problem for the case of an attached detonation front. For prescribed Mach number of the incident flow, specific heat ratio, and ratio of stagnation temperatures at the front, there is a maximum permissible value $\theta_{\max}$ of the semi-vertex angle θ of the conical obstacle. For each $\theta \leqq \theta_{\max}$ there are two possible types of flow. Furthermore, the nature of the solution depends on the location of θ in the interval $0 \leqq \theta \leqq \theta_{\max}$. In an interval adjacent to $\theta_{\max}$ there are continuous (behind the detonation front) compressive solutions analogous to those of the inert case. In the complementary interval adjacent to $\theta = 0$ there appears a new type of solution which starts with a Chapman-Jouguet detonation front, followed by a conical rarefaction wave, a conical shock, and a compressive wave down to the obstacle.

J. H. Giese (Aberdeen, Md.)

12969:

Ryhming, I. L. The supersonic boom of a projectile related to drag and volume. J. Aerospace Sci. **28** (1961), 113–118, 157.

The strength of a supersonic boom depends on the body shape through a factor $T^{1/2}$, where

$$T = \int_0^{y_0} F(y)\,dy,$$

$$F(y) = \frac{1}{2\pi}\int_0^y S''(x)(y-x)^{-1/2}\,dx, \quad F(y_0) = 0,$$

and the body is assumed pointed and axisymmetrical with cross-sectional area $S(x)$ at distance x from the nose; only the forward part of the body, $0 \leqq x \leqq y_0$, is involved. The drag D on this forward part is proportional to $\int_0^{y_0} F^2(y)dy$ and its maximum thickness $S(y_0)$ can also be obtained in terms of F. The author determines the function $F(y)$, and hence the body shape, which makes T a minimum for given D and $S(y_0)$. This optimum T is itself a minimum when D is a minimum for given $S(y_0)$. Thus minimum boom and minimum drag require the same body shape. The greatest variation with D of the optimum $T^{1/2}$ is approximately ten percent, and this is also found to be the typical order of variation in $T^{1/2}$ for various body shapes with the same length and thickness ratio.

G. B. Whitham (Cambridge, Mass.)

12970:

Dorrance, William H. On the approach to chemical and vibrational equilibrium behind a strong normal shock wave. J. Aerospace Sci. **28** (1961), 43–50.

Author's summary: "The concurrent approach to chemical and vibrational equilibrium of a pure diatomic gas passing through a strong normal shock wave is investigated. It is demonstrated that the equilibrium degree of dissociation behind the shock front, and hence the density, for the case where the vibrational degrees of freedom are frozen out, can exceed the degree of dissociation, and hence the density, for the case where all degrees of freedom are in equilibrium. Thus the necessary condition for a maximum of the density between the shock front and the position of full equilibrium flow downstream of the shock front is established. The sufficient condition that such a maximum be observable is shown to be that the approach to equilibrium of the vibrational degrees of freedom (or any other internal degrees of freedom) must lag the approach to dissociation equilibrium by a significant amount; that is, there must be at least an order of magnitude difference in the respective relaxation times before such a maximum might be observed. An example calculation for a Mach 13 strong shock wave in oxygen illustrates the appearance of such a maximum of the density and its dependency upon the relative values of the vibration and dissociation relaxation times."

R. M. Evan-Iwanowski (Syracuse, N.Y.)

12971:

Cabannes, Henri; Stael, Claude. Calcul de la singularité des ondes de choc attachées dans les écoulements de révolution. C. R. Acad. Sci. Paris **251** (1960), 1960–1962.

This is a preliminary summary, by the same authors, of results reported in #12978.

H. Polachek (Washington, D.C.)

12972:

Oppenheim, A. K.; Urtiew, P. A.; Stern, R. A. Peculiarity of shock impingement on area convergence. Phys. Fluids **2** (1959), 427–431.

Authors' summary: "General study of wave systems which can result from the interaction of a plane shock with an area convergence has led to the discovery of an interesting regime, specified in terms of functional relationships

between shock Mach numbers and area ratios, where two or three solutions exist at the same time, all satisfying the dynamic boundary conditions imposed by the conservation laws. The ambiguity is resolved by invoking the extremum principle of irreversible thermodynamics."
H. Polachek (Washington, D.C.)

12973:

Lun'kin, Yu. P. Variation of the parameters of a gas in connection with the nonequilibrium dissociation behind a shock wave. Ž. Tehn. Fiz. **30** (1960), 622–626 (Russian); translated as Soviet Physics. Tech. Phys. **5**, 585–589.

Author's summary: "An approximate method is proposed for the solution of the system of equations which describes the nonequilibrium dissociation of a gas behind a shock wave." *H. Polachek* (Washington, D.C.)

12974:

Manson, N. ★Détermination par la "méthode inverse" des caractéristiques des ondes explosives. Publ. Sci. Tech. Ministère de l'Air, No. 366, Paris, 1960. viii+46 pp. 12.00 NF.

Author's summary: "Using the classical hydrodynamic theory of detonation, mathematical relations are derived which permit the determination of the thermodynamic characteristics of the detonated gases from the variation of the velocity of the detonation wave as a function of certain thermo-elastic properties of the explosive material (gaseous, liquid or solid).

"Illustrative applications with two gaseous explosive mixtures and a liquid explosive are treated and the practical possibilities for the use of this method are examined." *H. Polachek* (Washington, D.C.)

12975:

Van Tuyl, A. H. Rational approximations formed from Chester's series for the body behind a given axially symmetric bow shock wave. J. Aerospace Sci. **28** (1961), 162–163.

The author develops a rational approximation (by the use of Padé fractions) for finding the body shape producing a given bow shock-wave for large M (free-stream Mach number) and values of γ (adiabatic exponent) near unity. The rational approximation gives accurate results for the shape of the body over a wider range of M and γ than can readily be obtained by series expansion. This advantage has, however, not been achieved in calculating the pressure on the body. *H. Polachek* (Washington, D.C.)

12976:

Adamson, T. C., Jr. On the structure of plane detonation waves. Phys. Fluids **3** (1960), 706–714.

Author's summary: "A steady planar detonation wave, considered to be a shock wave followed by a reaction zone, is studied with both irreversible and reversible first-order reaction kinetics. A perturbation solution with first-order transport effects, valid in the reaction zone for those cases where the ratio of the characteristic collision time to the characteristic chemical time is small compared to one, is presented with sample calculations of temperature and concentration distributions for typical irreversible and reversible reaction cases. Analysis of the solution shows that simple series solutions and hence the given perturbation solutions do not hold near the hot boundary for all possible final Mach numbers. In the irreversible reaction case, the perturbation solution is a valid approximation for final Mach numbers less than $(1-B)^{1/2}$, where B is the ratio of characteristic times, the approximation becoming less accurate as the Mach numbers tend toward this limiting value. In the reversible reaction case, the perturbation solution is a valid approximation for final Mach numbers up to the Chapman-Jouguet value of unity, if the Mach number is based on the equilibrium speed of sound." *H. L. Frisch* (Murray Hill, N.J.)

12977:

Wood, W. W. Existence of detonations for small values of the rate parameter. Phys. Fluids **4** (1961), 46–60.

The author proves the existence of, and finds an asymptotic approximation to, the solution of a simplified one-dimensional steady-state Navier-Stokes detonation problem formulated by Hirschfelder and Curtiss [J. Chem. Phys. **28** (1958), 1130–1147; MR **21** #1108] even in the limit of arbitrarily small values of the (unimolecular) reaction rate. These limiting solutions which approach the von Neumann model of a shock preceding a flame are obtained through an expedient use of an ignition temperature to make the steady state problem mathematically well-defined. *H. L. Frisch* (Murray Hill, N.J.)

12978:

Cabannes, Henri; Stael, Claude. Singularities of attached shock waves in steady axially symmetric flow. J. Fluid Mech. **10** (1961), 289–296.

The differential equations of motion are integrated numerically by the use of a high-speed calculator in order to determine the curvature of the attached shock wave. The case studied is one in which the body consists of a nose cone attached to a cylindrically symmetric section, such as a cylinder, in which the material velocity produced on the cone is subsonic. Results are tabulated for twenty-two cases for cones of 10°, 20° and 30°.
H. Polachek (Washington, D.C.)

12979:

Pai, S. I. Cylindrical shock waves produced by instantaneous energy release in magneto-gas dynamics. Proc. 4th Congress Theoret. Appl. Mech. 1958, pp. 89–100. Indian Soc. Theoret. Appl. Mech., Kharagpur.

Author's summary: "The behaviour of a cylindrical shock wave produced by instantaneous energy release along a straight line of infinite extent in a conducting gas subjected to a magnetic field with circular lines of force has been analyzed. Initially the gas is at rest and has constant temperature. Both the initial density and the initial magnetic field $H_{\theta 0}$ are assumed to be inversely proportional to some power of the radial distance r. It was found that a similar solution exists only if $H_{\theta 0}$ is proportional to $1/r$. Similar solutions for various initial density distributions have been obtained."
H. Polachek (Washington, D.C.)

12980:

Germain, Paul; Guiraud, Jean-Pierre. Conditions de choc dans un fluide doué de coefficients de viscosité et

de conductibilité thermique faibles mais non nuls. C. R. Acad. Sci. Paris **250** (1960), 1965–1967.

12981:

Isakovič, M. A. Nonlinear effects involved in certain acoustical problems. Akust. Ž. **6** (1960), 321–325 (Russian); translated as Soviet Physics. Acoust. **6** (1961), 321–325.

The square correction term to the linearized equation for free axial oscillations of a gas in a capped tube leads to beats between the fundamental and first overtone when overtone frequencies are not integrally related to the fundamental, or to indefinite growth of the first overtone when they are integrally related. These effects do not occur in an open tube. However, similar phenomena are shown to exist for wave propagation in an acoustic waveguide and may be extended to natural waveguides in the sea or atmosphere caused by laminar inhomogeneity of the medium. The beats are due to the dispersive properties of the waves. In such cases a finite-amplitude wave can be propagated over greater distances and with smaller decay than in the case of nondispersive waves.

W. W. Soroka (Berkeley, Calif.)

12982:

Masterov, E. P.; Muromceva, V. N. An example of antiwaveguide propagation of sound in laminar-inhomogeneous media. Akust. Ž. **6** (1960), 335–339 (Russian); translated as Soviet Physics. Acoust. **6** (1961), 335–339.

A harmonic point source is located at $(0, 0, z_0)$ in a medium whose index of refraction varies as $1+(pz)^2$. The field of the source is developed in terms of sums of normal modes for the two cases: (1) a perfectly yielding boundary at $z=0$; (2) a perfectly rigid boundary at $z=0$. At small distances a large number of modes is needed to describe the field, whereas at large distances the field is primarily determined by the zero-order normal mode, whose law of decay with distance is independent of frequency.

W. W. Soroka (Berkeley, Calif.)

12983:

Morrow, Charles T.; Troesch, B. Andreas; Spence, Harry R. Random response of two coupled resonators without loading. J. Acoust. Soc. Amer. **33** (1961), 46–55.

The root mean-square response of two coupled mechanical resonators with viscous damping excited at the base by a power spectrum of constant density is investigated under the assumptions that one resonator does not affect the motion of the mass of the other ("no loading effect"), that the individual resonant frequencies of the two resonators are not too close together and that the damping is relatively small. Approximate equations are obtained and solved, and compared with the respective solutions of the exact equations obtained on a digital computer. Curves of the responses of the indirectly excited resonator as a function of the ratio of resonant frequencies of both resonators are obtained for various values of their damping. The use of these curves and of their derivatives in the design of equipment is discussed.

A. M. Freudenthal (New York)

12984:

Ivlev, D. D. The structure of the hydrodynamics of viscous fluids. Dokl. Akad. Nauk SSSR **135** (1960), 280–282 (Russian); translated as Soviet Physics. Dokl. **5** (1961), 1169–1171.

The author proposes to establish the most general constitutive equation that expresses the velocity strain as an isotropic function of the stress and reduces to Newton's law of viscous friction for simple shear. The generality of the discussion is unnecessarily restricted by the assumption that there exists a stress invariant from which a typical component of the velocity strain can be obtained by partial differentiation with respect to the corresponding stress component.

W. Prager (Providence, R.I.)

12985:

Walters, K. The motion of an elastico-viscous liquid contained between coaxial cylinders. II. Quart. J. Mech. Appl. Math. **13** (1960), 444–461.

[For part I, see same J. **13** (1960), 325–333; MR **22** #6216.] For the present treatment of elastico-viscous liquids a new definition of relaxation spectrum is used. So far the distribution function has been defined by considering the rigidity of the Maxwell elements as distributed over a continuous range of relaxation times. Here a distribution function of the viscosity (rather than the rigidity) is used. It is shown that a relatively simple continuous spectrum characterized by three constants can represent satisfactorily the experimental results.

B. Gross (Rio de Janeiro)

12986:

Wooler, P. T. Instability of flow between parallel planes with a coplanar magnetic field. Phys. Fluids **4** (1961), 24–27.

The equations determining the growth of three-dimensional disturbances when a coplanar magnetic field is not parallel to the flow are shown to be similar to those determining the growth of two-dimensional disturbances when the field is parallel to the flow. Previous results for the latter case can then be used to determine, for a given magnetic field, the dependence of the minimum critical Reynolds number on the direction of propagation of the disturbance. When the field is not parallel to the flow, the critical Reynolds number is found to be finite for disturbances propagated in a certain direction, whatever the strength of the magnetic field. From this result, it follows that an analog of Squire's theorem is not generally valid when the magnetic field is not parallel to the flow.

D. W. Dunn (Ottawa, Ont.)

12987:

Imai, Isao. Asymptotic behavior of the flow past a body of a compressible, viscous or electrically conducting fluid. Partial differential equations and continuum mechanics, pp. 177–205. Univ. of Wisconsin Press, Madison, Wis., 1961.

This paper discusses the forces and moments on a body in a uniform flow by considering the asymptotic behavior of the flow at infinity. Three cases have been investigated in detail: (1) Subsonic flow of an inviscid flow of a compressible fluid in which both the two-dimensional and the three-dimensional irrotational flows are discussed. In the two-dimensional cases, both the continuous flow past an arbitrary cylinder and the discontinuous flow of Kirchkoff type are treated. (2) Two-dimensional flow of a viscous incompressible fluid in which Oseen's approximation on

flow past an arbitrary cylinder and boundary layer flow over a semi-infinite plate parallel to a uniform flow are considered. (3) Magnetohydrodynamic flow of an electrically conducting fluid in which the aligned magnetic field has been discussed. It has been shown that there are two wakes with different orientations in such a flow.
S. I. Pai (College Park, Md.)

12988:
Pacholczyk, Andrzej G. Sulla instabilità magneto gravitazionale di un mezzo compressibile non uniforme con rotazione anche non uniforme. Atti Accad. Naz. Lincei. Rend. Cl. Sci. Fis. Mat. Nat. (8) **28** (1960), 357–363.

The stabilizing effect of a magnetic field on the formation of galactic spiral arms is studied by considering the stability of a non-uniformly rotating compressible perfectly conducting fluid in a non-uniform purely azimuthal magnetic field. Both unperturbed and perturbed variables are assumed to have cylindrical symmetry, and the perturbed velocity and magnetic fields are taken to be purely azimuthal. Stability criteria are derived for the cases where the unperturbed density is (a) uniform and (b) proportional to the square of the magnetic field and therefore to a certain negative power of distance from the axis of symmetry.
O. Penrose (London)

12989:
Sirotina, E. P.; Syrovatskiĭ, S. I. Structure of low intensity shock waves in magnetohydrodynamics. Ž. Èksper. Teoret. Fiz. **39** (1960), 746–753 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 521–526.

Ce travail est une nouvelle étude de la structure d'onde de choc en magnéto-dynamique des fluides. Dans les discontinuités considérées, la valeur absolue des grandeurs physiques change peu, mais les changements de direction de la vitesse et du champ magnétique sont arbitraires. La difficulté réside dans le fait que le problème n'est pas plan. Les auteurs déterminent le coefficient d'amortissement pour les ondes de faible amplitude et indiquent la relation qui existe entre ce coefficient et l'intensité de la discontinuité.
H. Cabannes (Paris)

12990:
Polovin, R. V. Shock waves in magnetohydrodynamics. Uspehi Fiz. Nauk **72** (1960), 33–52 (Russian); translated as Soviet Physics. Uspekhi **3** (1961), 677–688.

Cet article est une revue des principaux résultats relatifs aux ondes de choc en magnéto-dynamique des fluides. L'auteur rappelle d'abord le cas des ondes simples: ondes d'Alfven, magnéto-acoustiques et ondes d'entropie, et il énonce le théorème de Zemplen: conditions pour que la pression et la masse spécifique augmentent à travers une onde de choc.

La connaissance des conditions de discontinuité est insuffisante, en général, en magnéto-dynamique des fluides, pour déterminer une solution unique avec onde de choc. Une perturbation infiniment petite se propage suivant une loi du type exp $i(kx-\omega t)$, x et t désignants l'abscisse et le temps. Si ω est réel pour toute valeur de k, la discontinuité est stable; si la partie imaginaire de ω est positive pour certaines valeurs de k, la discontinuité est instable; dans les autres cas la discontinuité est dite "évolutive". Cette notion correspond à la stabilité au sens fort de Liapounoff. Seules les ondes "évolutives" auraient une existence physique. Les conditions d'évolution sont discutées en fonction des vitesses avant et après le choc. Comme conséquence, l'auteur indique quelques propriétés des ondes évolutives, liées à leur vitesses de propagation relatives.

La courbe de choc, et particulièrement la portion qui correspond à une onde évolutive, le problème du piston et les dégénérescence des chocs sont étudiés ensuite.
H. Cabannes (Paris)

12991:
Polovin, R. V. The motion of shock waves along a magnetic field. Ž. Èksper. Teoret. Fiz. **39** (1960), 1005–1007 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 699–700.

L'auteur considère une onde de choc, engendrée dans un milieu magnéto-aerodynamique, par un piston conducteur animé d'un mouvement parallèle au champ magnétique. Les conditions pour que le choc soit "non-évolutif" [#12990] sont établies; le choc dégénère alors en deux ondes magnéto-acoustiques.
H. Cabannes (Paris)

12992:
Kakutani, Tsunehiko. Axially symmetric stagnation-point flow of an electrically conducting fluid under transverse magnetic field. J. Phys. Soc. Japan **15** (1960), 688–695.

The two-dimensional treatment of the stagnation point flow of an incompressible viscous conducting fluid of Neuringer and McIlroy [J. Aero. Sci. **25** (1958), 332–334] is extended to the three-dimensional axially symmetric case.
A. Herzenberg (Manchester)

12993:
Napolitano, Luigi G. Strato limite turbolento in magnetofluidodinamica. Aerotecnica **40** (1960), 311–322.

The paper begins with a general discussion following that of Chandrasekhar [Proc. Roy. Soc. Ser. A **233** (1955), 322–330, 330–350; MR **17**, 920, 921] of the effect of a magnetic field on the structure of the turbulent energy spectrum, using a linearisation valid for small magnetic Reynolds numbers. It is shown that the kinetic energy at all wave numbers is an order of magnitude greater than the magnetic energy, but that it is nevertheless reduced by the presence of a magnetic field. The author concludes that the general features of the mean flow in a turbulent boundary layer will be sufficiently preserved to permit the standard type of dimensional analysis, and shows that the universal 'defect' law for the outer part of the layer can still be derived for small values of $\sigma B_0^2 x/\rho U$ (σ=conductivity, B_0=applied induction). The 'law of the wall', on the other hand, depends explicitly on R_m. It is predicted that the skin friction will be reduced as a result of work done by pressure forces against the magnetic field, reducing the turbulent energy supplied to the layer.
D. A. Spence (Farnborough)

12994:
Hargitai, Cs.; Szabó, J. Die Eindeutigkeit der Lösung des magnetohydrodynamischen Randwertproblems. Z. Naturforschg. **16a** (1961), 92–94.

Direct proof of the uniqueness of the solution of the following problem in magnetohydrodynamics: Given an incompressible fluid with constant finite viscosity and conductivity, a finite domain with smooth boundary is considered. The initial state in the domain and the velocity and magnetic field on the boundary are given. It is assumed that the solution has a sufficient number of smooth derivatives in order to allow the required applications of the theorems of Gauss and Green.

L. J. F. Broer (Delft)

12995:

Čekmarev, I. B. One-dimensional flow of a compressible gas in a pipe in the presence of a transverse magnetic field. Prikl. Mat. Meh. **24** (1960), 382–383 (Russian); translated as J. Appl. Math. Mech. **24**, 553–555.

The author discusses the one-dimensional motion of a compressible inviscid fluid with finite electrical conductivity in the presence of transverse magnetic field. If the local Mach number $M_0=u_0^2/(kp_0/\rho_0)$ at $x=0$ is greater than unity, the stream decelerates from a certain velocity u_0u_1 at $x=-\infty$ to $u_0(u_1u_2)^{1/2}$; if, on the other hand, $M_0<1$, then the stream accelerates continuously from the velocity $u=u_0u_2$ at $x=-\infty$ to $u_0(u_1u_2)^{1/2}$, where u_1, u_2 are the roots of the equation

$$u^2-\frac{2kh}{k+1}u+\frac{2kh}{k+1}-\frac{kS}{k+1}-1 = 0,$$

$k=C_p/C_v$, $S=\mu H_0^2/u_0^2$, $R_m=\sigma\mu u_0L$ (L, some characteristic length), $h=1+1/kM_0^2+S/2$, and the suffix zero denotes the value of a physical quantity at $x=0$. The density varies inversely as velocity, and the magnetic field is given by

$$M = \left\{\frac{k+1}{kS}\frac{(u_1-u)(u-u_2)}{u}\right\}^{1/2}.$$

P. L. Bhatnagar (Bangalore)

12996:

Chopra, K. P. On simulation studies of motion of bodies in ionized atmosphere. Z. Physik **161** (1960/61), 445–453.

Author's summary: "The principle of dimensional similitude is applied to the laboratory model studies of the motion of bodies in an ionized atmosphere pervaded by a magnetic field. The correspondence relationships of various characteristic parameters in the actual and laboratory model scale cases are obtained. It is shown that the scaling relationships satisfy the conditions for aerodynamic, magnetohydrodynamic and electrodynamic similitude. These relationships indicate that it is not necessary to obtain actual atmospheric densities in the laboratory to perform such studies. By adopting a suitable scaling factor for the linear dimension of the body, it should be possible to simulate flight conditions corresponding to the upper atmosphere of the earth. Outline details of a suitable experiment in a hypersonic wind tunnel are described."

K. C. Westfold (Clayton)

12997:

Gupta, A. S. On the laminar flow of an electrically conducting fluid under pressure gradient in channels with porous walls in the presence of a transverse magnetic field. J. Sci. Engrg. Res. **4** (1960), 401–407.

The Hartman flow through a two-dimensional channel is extended to the case in which uniform suction or injection of fluid is distributed along the channel walls. This paper deals only with the case in which one wall has a uniform suction and the other a uniform injection of the same strength. This case is mathematically much simpler than the more realistic cases in which both walls have either suction or injection. The velocity and the temperature distributions are calculated and the effect of the magnetic field on the viscous stress at the wall is discussed.

T. Tatsumi (Kyoto)

12998:

Epstein, Melvin. Nonlinear behavior of the electrical conductivity of a slightly ionized gas. Phys. Fluids **3** (1960), 1016–1018.

A relationship between the current density in an ionized gas and the strength of an applied electromagnetic field of angular frequency ω is derived on the assumption that

$$v^2 \ll \omega^2\lambda^2,$$

where v is the average electron velocity and λ its mean free path. Except for this change the procedure follows Margenau [Phys. Rev. (2) **69** (1946), 508]. The result is a simpler non-linear relationship which reduces to Margenau's linear approximation in the limit of high frequencies and vanishing field strength.

R. D. Kodis (Providence, R.I.)

12999:

Polubarinova-Kočina, P. Ya. Ground water movements at water level fluctuations in a reservoir with a vertical boundary. Prikl. Mat. Meh. **23** (1959), 540–545 (Russian); translated as J. Appl. Math. Mech. **23**, 762–769.

The author solves rigorously the problem of groundwater motion in soil of infinite depth, initiated by the sinusoidal fluctuations of water level in a reservoir. The Laplace transform technique is employed; and the solution is obtained quite elegantly in terms of the sine- and cosine-integrals, and elementary functions.

The author's solution is slightly more general than those obtained by Carrier and Munk [cf. Proc. Sympos. Appl. Math., Vol. 5, pp. 89–96, McGraw-Hill, New York, 1954; MR **16**, 763] and Meyer [cf. Houille Blanche, Vol. 10, Nos. 1 and 5 (1955), Vol. 11, No. 1 (1956)].

K. Bhagwandin (Stockholm)

13000:

Chakraborty, B. B. On the stability of a gravitating liquid cylinder carrying a non-uniform volume current and surface charge. Z. Astrophys. **51**, 107–118 (1961).

The author has discussed the stability of an infinite gravitating liquid cylinder carrying a volume current, along the axial direction, which is proportional to r^n, where r is the distance from the axis. The liquid is assumed to be ideally conducting and surrounded by a non-conducting liquid whose density is very small compared to the density of the conducting liquid. It is shown that for all values of n, there exist axi-symmetric disturbances for which the equilibrium configuration is unstable. The proof is based on Sturm's oscillation theorems.

F. C. Auluck (Delhi)

13001:

Tkalič, V. S. Finite-amplitude waves in a multi-

component conducting medium. Ž. Èksper. Teoret. Fiz. **39** (1960), 73–77 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 52–55.

It is shown that there is a particular set of solutions (helical motion) for the magnetohydrodynamic equations that satisfy linear partial differential equations, with arbitrarily large amplitudes. The following model is accepted: electrically charged, incompressible, multicomponent plasma. Then the dielectric constant, the conditions for the validity of the hydrodynamic approximations, and the proper boundary conditions are studied.

G. Kalman (Haifa)

13002:

Gotoh, Kanefusa. Magnetohydrodynamic flow past a sphere. J. Phys. Soc. Japan **15** (1960), 189–196.

There are now at least three papers on the magnetohydrodynamic flow past a sphere, of which the one being reviewed here appears to have been received and published first. [The other two are due to R. Van Blerkom, Thesis, Harvard University, 1960, cf. #13003, and G. S. S. Ludford, #13004.] Each of these papers deals with the same subject matter, reaches the same conclusions and presents similar figures. The investigations concern the flow of an incompressible viscous electrically conducting fluid past a sphere in the presence of a uniform applied magnetic field directed parallel to the free stream velocity. The Oseen approximation is used and the governing equations are solved in the manner of S. Goldstein [Proc. Roy. Soc. London Ser. A **123** (1929), 225–235]. The drag is computed for all values of $\beta=\mu H^2/\rho V^2$ and it is found to be nonzero at $\beta=1$. (The drag coefficient is zero at this value for two-dimensional flows.) The form and structure of the precursory disturbance which occurs at super-Alfvén velocities, $\beta>1$ [see for example, H. P. Greenspan and G. F. Carrier, J. Fluid Mech. **6** (1959), 77–96; MR **21** #6907; or H. Hasimoto, Phys. Fluids **2** (1959), 337–338] as well as the viscous wake are described and discussed in detail.

H. Greenspan (Cambridge, Mass.)

13003:

Van Blerkom, Richard. Magnetohydrodynamic flow of a viscous fluid past a sphere. J. Fluid Mech. **8** (1960), 432–441.

The time-independent velocity and magnetic fields are parallel at $\infty(U_1, H_0\mathbf{i})$; the fluid is incompressible with finite conductivity σ. The magnetohydrodynamic equations valid for $r>a$ (a is the spherical radius) reduce to two relations between the velocity field and the magnetic field, involving non-linear terms $(\mathbf{V}\cdot\nabla)\mathbf{V}$, $(\mathbf{V}\cdot\nabla)\mathbf{H}$, $(\mathbf{H}\cdot\nabla)\mathbf{V}$, and $(\mathbf{H}\cdot\nabla)\mathbf{H}$. In an extension of the Oseen approximation, the first factor in these expressions is replaced by its value at ∞ (the reasoning behind this technique is, however, not quite Oseen's). If this is done, the nondimensional form of the two relations is: $R\partial\mathbf{V}/\partial x=-\nabla p_0+\nabla^2\mathbf{V}+(M^2/R_m)\partial\mathbf{H}/\partial x$, $R_m^{-1}\nabla^2\mathbf{H}=\partial(\mathbf{H}-\mathbf{V})/\partial x$. Here R and R_m are viscous and magnetic Reynolds numbers, M is the Hartmann number $\mu H_0 a(\sigma/\rho\nu)^{1/2}$, and $p_0=p+M^2H^2/2R_m$. By introducing $\mathbf{v}=\mathbf{V}-\mathbf{i}$, $\mathbf{h}=\mathbf{H}-\mathbf{i}$, and $\mathbf{v}_j=\mathbf{v}-(\alpha_j/R_m)\mathbf{h}$, $j=1, 2$, where α_j solves $\alpha^2+(R-R_m)\alpha-M^2=0$, one obtains the pair of Oseen equations $R_j\partial\mathbf{v}_j/\partial x=-\nabla p_0+\nabla^2\mathbf{v}_j$, where R_j satisfies $R_j^2-(R+R_m)R_j+(RR_m-M^2)=0$. In addition, div $\mathbf{v}_j=0$. For energy-density ratio $\mu H_0^2/\rho U^2\equiv\beta<1$, both $R_j>0$; otherwise one $R_j\leqq 0$.

Goldstein's expansion method (1929) can now be applied and leads here to 4 infinite sets of equations, of which two express the conditions for v_r and v_θ, and two the continuity of μh_r and h_θ, for $r=a$. The paper contains first-order formulas for the drag if R_j is small and $\beta\gtrless 1$, and numerical results for $R=0.5$ including the terms $\sigma(R_j^2)$. A note on the point-force problem (no obstacle, but a body force in the form of a δ-function) and another on the case $\sigma=\infty$ concludes the paper.

G. Kuerti (Cleveland, Ohio)

13004:

Ludford, G. S. S. The effect of an aligned magnetic field on Oseen flow of a conducting fluid. Arch. Rational Mech. Anal. **4**, 405–411 (1960).

This paper is one of three concerning the same problem that appeared in the literature almost simultaneously. The other two are by K. Gotoh [#13002] and R. Van Blerkom [#13003]. The Oseen approximation is used and it is found that for super-Alfvén flow there is no essential change in the value of the drag. For sub-Alfvén flow, however, the drag increases with increasing magnitude field. In the latter case, the existence of an upstream "wake" or precursor is confirmed.

H. P. Greenspan (Cambridge, Mass.)

13005:

Yosinobu, Hirowo. A linearized theory of magnetohydrodynamic flow past a fixed body in a parallel magnetic field. J. Phys. Soc. Japan **15** (1960), 175–188.

Steady flow of a conducting fluid past a fixed body with undisturbed magnetic field parallel to the stream is treated for finite Reynolds and magnetic Reynolds numbers and finite value of the ratio S of magnetic to dynamic pressures. An approximate method analogous to Oseen's well-known linearization is used. Study of the asymptotic behavior of the equations reveals two wake-like disturbances of paraboloidal shape, both of which extend downstream for $S<1$, but one upstream and one downstream for $S>1$. For $S=1$, a wake extends downstream and the other disturbance extends over the whole field. After this general study, the equations are specialized for two-dimensional flow, and solutions are found that are mathematically analogous to those for Oseen flow. Detailed calculations are carried out for flow past a circular cylinder, and an approximate formula for the drag per unit span is obtained. This is related to various special cases previously published. It is clear that the present method fails for $S=1$.

Calculations for flow past a sphere have also been made and are presented by Gotoh [#13002].

W. R. Sears (Ithaca, N.Y.)

13006:

Żelazny, Roman. Derivation of hydrodynamic equations for the quantum systems of diatomic molecules. Phys. Rev. (2) **117** (1960), 1–11.

Author's summary: "Bogolyubov's method of derivation of the hydrodynamic equations from a quantum-statistical formalism, based on the array of distribution operators for clusters of s molecules, is adapted to the derivation of the hydrodynamic equations for a fluid composed of diatomic molecules. The general form of the

hydrodynamic equations with an additional equation of angular momentum, which is coupled with the momentum equation through the antisymmetric part of the stress tensor, is obtained and all the interesting hydrodynamic quantities are calculated. A general procedure of derivation of the hydrodynamic equations by successive approximations is proposed and the equations of zeroth approximation are discussed."

13007:

Crupi, Giovani. Una soluzione delle equazioni di Euler-Minkowski e sua interpretazione fisica. Atti Sem. Mat. Fis. Univ. Modena **9** (1959/60), 1–12.

A solution is obtained of the Euler-Minkowski equations for wave motion in an incompressible, inviscid, electrically conducting fluid acted on by a constant external magnetic field. This solution is interpreted as a plane wave propagated in an arbitrary prescribed direction (not perpendicular to the field direction). It is found that under certain conditions the waves may be attenuated, stationary or amplified, and their group velocities are correspondingly greater than, equal to or less than the phase velocity. *F. A. E. Pirani* (London)

13008:

Fogel'son, R. L. The equation of diffusion. Fiz. Tverd. Tela **2** (1960), 903–907 (Russian); translated as Soviet Physics. Solid State **2**, 824–827.

Author's summary: "In this paper, it is shown that, in the solution of problems on heterogeneous diffusion, it is necessary to use a diffusion equation in which the dependent variable is not the relative concentration of the diffusing substance, but its absolute concentration. It is shown that the application of this equation leads to the best agreement between theory and experiment. As a consequence of this, for binary systems there exist, instead of a single coefficient, two diffusion coefficients, corresponding to the two components."

13009:

Bear, Jacob. On the tensor form of dispersion in porous media. J. Geophys. Res. **66** (1961), 1185–1197.

The dispersion of tracer particles injected at a point into the liquid flowing in a porous medium, has been sought to be explained by the statistical method from two different viewpoints. Assuming that the spreading of the tracer takes approximately the form of a bivariate normal distribution depending upon a longitudinal and a transverse constant of dispersion, it is shown that the variance of the distribution is a second-rank tensor, whereas the constant of dispersion is a fourth-rank one. The same results are also obtained by using the variance of distribution as a measure of the dispersion. It is also shown that the displacement in the porous medium can be taken as a second-rank tensor instead of a vector.

When a point injection is subjected to a sequence of uniform movements in different directions, the final concentration distribution can be obtained by a summation of the tensors corresponding to various movements. The concentration distribution across a transition zone, which develops when an abrupt interface between two miscible fluids is subjected to a sequence of uniform movements, can be determined by integrating the result for a single point injection over the entire tracer region. *G. Paria* (Kharagpur)

13010:

Gheorghiţă, Şt. I. Sur les mouvements non-linéaires avec gradient initial. An. Univ. "C. I. Parhon" Bucureşti. Ser. Şti. Nat. No. 22 (1959), 39–48. (Romanian. Russian and French summaries)

The flow of an incompressible fluid through a porous medium is studied, assuming that the filtration law is non-linear and that there exists a limit value of the pressure gradient below which the filtering velocity is equal to zero. The form $\mathbf{J}(1-K^*J^{-1})=f(V)\mathbf{v}$, where $\mathbf{J}=-\operatorname{grad}(p/\rho g+z)$ and K^* is the limit value of $|\mathbf{J}|$ (for $J<K^*$, $\mathbf{v}\equiv 0$), is admitted for the filtration law; $f(V)=K_0+K_1V$, in which $V=|\mathbf{v}|$, is considered, the coefficients K_0 and K_1 being point functions. By means of the equation of continuity the differential equation of the function $H=p/\rho g+z$ is deduced. Besides the boundary conditions known for H, the conditions which must be satisfied on the boundaries between the domain D_0 in which $\mathbf{v}\not\equiv 0$ and the domains D_j ($j=1, 2, \cdots, n$) in which $\mathbf{v}\equiv 0$ ($J<K^*$) also appear. As for the function H, it is assumed that in D_j it satisfies the same equation as in D_0, or that it is harmonic. As examples, the linear, radial and spherical flows are studied, by particularizing the results when K_0 and K_1 are constant. Finally, it is shown that for the flow below an impermeable dam, resting on a homogeneous porous half-plane (K_0 and K_1 being constant), the rate of flow is finite, as distinguished from the case in which a value of the pressure gradient below which $\mathbf{v}\equiv 0$ does not exist. *T. Oroveanu* (Bucharest)

OPTICS, ELECTROMAGNETIC THEORY, CIRCUITS

See also 12782, 13127, 13128, 13235.

13011:

Baudoux, Pierre. ★Électricité. Tome I. Lois fondamentales–milieux–systèmes–circuits. Presses Académiques Européennes, Brussels, 1959. 242 pp.

A general textbook and handbook.

13012:

Bruder, K. Die Bildfehler dritter Ordnung in anamorphotischen Systemen. Optik **17** (1960), 663–670. (English and French summaries)

Author's summary: "This paper represents a continuation of earlier work by Horst Köhler [Optik **13** (1956), 145–157; MR **18**, 169] and gives expressions for the third order aberrations of double-symmetric systems used for anamorphic imagery. The expressions for the aberrations are given both for systems composed of toric surfaces and for those using cylindrical surfaces with parallel axes." *H. A. Buchdahl* (Hobart)

13013:

Leathem, J. G. ★The elementary theory of the symmetrical optical instrument. Reprinting of Cambridge Tracts in Mathematics and Mathematical Physics, No. 8.

Hafner Publishing Co., New York, 1960. vii+74 pp. $3.00.

This was an acceptable textbook when it was originally published in 1908 [Cambridge Univ. Press, London].

G. L. Walker (Providence, R.I.)

13014:

Buchdahl, H. A. Optical aberration coefficients. IX. Theory of reversible optical systems. J. Opt. Soc. Amer. **51** (1961), 608–616.

[For part VIII, see same J. **50** (1960), 678–683; MR **22** #7595.] This paper develops a theory of symmetrical reversible optical systems (sometimes called "symmetrical" or "holosymmetrical") which are axially symmetrical systems possessing a normal plane of symmetry if the position of the diaphragm is ignored. The principal result can be stated as: if f is the focal length, m the magnification associated with the conjugate object and image planes, s the magnification associated with the pupil planes of a symmetrical reversible optical system K and $\mathscr{R}$ is the real field, then the primary (Seidel) aberration coefficients of K and f^{-2} are linearly dependent over $\mathscr{R}[m, s]$. The proof follows from a consideration of the Hamiltonian of K. It follows that K can be fully corrected for Seidel aberrations only when $m^2=1$. A further result is: the number of linearly independent aberration coefficients of order $2n-1$ is $\frac{1}{4}(n+2)^2$ if n is even and $\frac{1}{4}(n+1)(n+3)$ if n is odd. *G. L. Walker* (Providence, R.I.)

13015:

Biot, A. Sur l'aberration chromatique latérale des systèmes optiques. II. Le dioptre. III. Discussion de la formule générale. Ann. Soc. Sci. Bruxelles. Sér. I **74** (1960), 23–30.

In a previous article [same Ann. **72** (1958), 107–117; MR **22** #10523], the author has given an expression for the chromatic (lateral) aberration of an optical system. Starting with this general expression for the lateral chromatic aberration, he proceeds to analyze the chromatic properties of a spherical and planar dioptric system. Explicit expressions are derived for the lateral chromatic aberration for arbitrary and particular positions of the entrance pupil and for different type of lenses (microscopic objectives, oculars and a thick doublet).

N. Chako (New York)

13016:

Biot, A. L'aberration chromatique latérale d'un système optique baigné par des milieux différents. Ann. Soc. Sci. Bruxelles. Sér. I **74** (1960), 100–105.

In a previous publication [same Ann. **72** (1958), 107–117; MR **22** #10523], the author has obtained an expression for the lateral chromatic aberration in the image space for an optical system immersed in air. In this paper, it is extended to the case where the object and the image space lie in media with different refractive indices. As special cases of this optical arrangement, the author gives formulas for the lateral chromatic aberration when both object and image are located at infinity (afocal arrangement), the entrance pupil and the image are at infinity, (ocular system) and for the image at infinity and the entrance pupil fixed at some finite distance (ordinary immersion microscope). *N. Chako* (New York)

13017:

Azpiroz Yoldi, M. Remarks about one of the formulas of optical geometry. An. Real Soc. Españ. Fis. Quim. Ser. A **56** (1960), 125–130. (Spanish. English summary)

Author's summary: "Some results concerning light propagation in isotropic heterogeneous media, obtained in a previous paper, are transformed in such a way that the local properties of the μ=Const. surfaces will appear in the formulae giving the change in the principal curvatures of the wavefront and the rotation of the trihedron formed by the directions of these curvatures and that of light propagation. As an application to the particular case where a wave traverses the discontinuity surface limiting two isotropic homogeneous media with different refraction indexes, the classical results are obtained. Some considerations with regard to the outcomes attained are briefly discussed."

13018:

Mollet, Pol (Editor). ★Optics in metrology. Colloquia of the International Commission for Optics, 6–9 May 1958. Pergamon Press, New York-Oxford-London-Paris, 1960. xix+436 pp. (78 plates) $17.50.

The 59 papers in this collection are principally on instrumentation and techniques for high resolution optical measurements, but the following two articles are devoted to theoretical considerations. "Mesures de longueurs par méthodes interférométriques, et principes d'incertitudes en optique à trois dimensions", by B. Vittoz, pp. 73–81, is a study of the influence of a non-zero spectral line width upon interferometric measurement of length. It is shown that there is an absolute limitation of accuracy, which is analogous to the general principle of uncertainty in quantum mechanics, and which is a complement of the optical uncertainty principle of E. Ingelstam [Ark. Fys. **7** (1953), 309–322]. Fourier transforms are used to obtain the optical analogue of the photon in quantum mechanics. "Die Abbildung von Phasenobjekten in der optischen Übertragungstheorie", by E. Menzel, pp. 283–293, is devoted to an extension of the transfer theory of illumination [P.-M. Duffieux, *L'intégrale de Fourier et ses applications à l'optique*, chez l'auteur, Faculté des Sciences, Besançon, 1946; MR **10**, 273; and H. H. Hopkins, Proc. Roy. Soc. London Ser. A **217** (1953), 408–432; MR **14**, 1042] to phase-contrast objects. Examples are given of the application to phase-contrast processes, Schlieren processes, and interference processes.

G. L. Walker (Providence, R.I.)

13019:

Slansky, Serge. Images d'un disque clair ou sombre en éclairage partiellement cohérent. Rev. Opt. **39** (1960), 555–577. (English summary)

Author's summary: "The light distributions in the images of a bright disk on a darker background and of a dark disk on a brighter background are calculated in the case of partially coherent illumination, considering three values of disk radius, two values of object contrast, and two values of the ratio of the aperture of the illuminating system to that of the imaging system, the latter being supposed stigmatic. These images are compared with those of similar coherent or incoherent objects, in order to get some information about the evolution of the image according to the degree of coherence. It appears that the

image of a small dark disk tends to broaden as the coherence is increased (by decreasing the aperture of the illuminating system). The results obtained from application of general formulae of the partial coherence theory are compared with those deduced from an approximate treatment, valid for low-contrast objects: the image of a partially coherent object is calculated like that of a coherent one, with a modified expression of the function representing the image of a point. In some cases this low-contrast approximation gives satisfactory accurate results up to contrast values of the order of 0.5."

G. B. Parrent, Jr. (Medford, Mass.)

13020:

Naze, Jacqueline. Calcul de la conductivité électrique et de la température d'un gaz faiblement ionisé. C. R. Acad. Sci. Paris **251** (1960), 2464–2466.

The Boltzmann equation for ions and electrons in a slightly ionized gas can be solved in a closed form when the force between charged and neutral particles is proportional to $1/r^5$, the inverse fifth power of the distance. The author calculates in this case the temperature and the tensor of electric conductivity of ions and electrons in a gas in a magnetic field; the electric field need not be weak. *T. Kihara* (Tokyo)

13021:

Bernstein, Ira B.; Kulsrud, Russell M. Ion wave instabilities. Phys. Fluids **3** (1960), 937–945.

The authors discuss ion waves in a uniform collision-free plasma, with general distribution functions and in the presence of a uniform magnetic field. The dispersion relation for electrostatic oscillations in a magnetic field is derived. The assumptions used are: (1) the thermal velocities of the particles and the phase velocity of the wave are small compared to those of light; (2) the component of the wave vector **k** normal to the magnetic field is small compared to the reciprocal of the gyration radius of an ion at the larger of the mean ion, and electron thermal energies; (3) the magnetic field **B** is uniform. The dispersion relation has the same structure as that for electrostatic oscillations in the absence of a magnetic field. The authors then approximate the solutions of this dispersion relation and in those cases where the mean thermal energy of the ions is small compared to that of the electrons the unstable situations are classified. It is found that a very small current along the lines of force leads to an instability of the ion waves. Also the phase velocities and growth rate of the waves are determined. Furthermore some results on the "two stream" instability are given. *R. S. B. Ong* (Leiden)

13022:

Simons, Lennart. Some properties of a thin plasma of variable density. Soc. Sci. Fenn. Comment. Phys.-Math. **24** (1959/60), no. 3, 16 pp.

A cylindrical low-density plasma confined by an axial magnetic field is studied from the particle viewpoint. The particle density is assumed time-independent, but is allowed to vary as a smooth function $f(r)$ of the radial coordinate, decreasing monotonically to zero at the plasma boundary. The plasma, initially cold, is subjected to ion-cyclotron heating. The resulting electron and ion currents are calculated. The sheet current occurring in "constant density" plasmas is obtained as a limiting case by taking density profiles which fall off to zero more and more sharply at the plasma boundary. In the general case, the magnetic field induced by the ion current is found to initiate phase differences among ions at different radii. These phase differences, which increase with time and lead eventually to collisions, are related in the same way to $|f'(r)|$ as the electron currents. A specific consequence of this is that a strong sheet current is followed by strong turbulence at the boundary.

H. C. Kranzer (Garden City, N.Y.)

13023:

Simons, Lennart. The influence of induction on ion and electron currents in a thin plasma. Soc. Sci. Fenn. Comment. Phys.-Math. **24** (1959/60), no. 6, 9 pp.

The influence of the induced magnetic field discussed in #13022 is here calculated in detail for energies up to the fusion threshold for both constant and non-constant $f(r)$. Numerical results are given for a particular geometry.

H. C. Kranzer (Garden City, N.Y.)

13024:

Katz, Sylvan. Configuration of a plasma in an axially symmetric magnetic field. Phys. Fluids **4** (1961), 204–209.

Using moment equations of the collisionless Boltzmann equation, solutions are obtained for the static configuration of a fully ionized, low β, adiabatic plasma immersed in an applied axial symmetric magnetic field. The moment equations are expanded to zeroth order in β and closed by specifying that the diagonal elements of the divergence of the heat flux tensor vanish. Introducing a curvilinear coordinate system, one of whose axes lies along the applied field, these equations are integrated to yield results which depend on the imposed boundary conditions. These results are applied to trapping in a mirror machine.

E. A. Jackson (Princeton, N.J.)

13025:

Yadavalli, S. A note on the relativistic Boltzmann equation and some applications. J. Franklin Inst. **271** (1961), 368–375.

13026:

Velihov, E. P. Stability of a plasma-vacuum boundary. Ž. Tehn. Fiz. **31** (1961), 180–187 (Russian); translated as Soviet Physics. Tech. Phys. **6** (1961), 130–133.

13027:

Bekefi, G.; Hirshfield, Jay L.; Brown, Sanborn C. Kirchhoff's radiation law for plasmas with non-Maxwellian distributions. Phys. Fluids **4** (1961), 173–176.

Authors' summary: "Calculations are given for the radiation temperature in terms of the average electron energy, to be used for interpreting microwave radiation from plasmas with non-Maxwellian distributions of electron velocities." *E. A. Jackson* (Princeton, N.J.)

13028:

Anderson, N. Oscillations of a plasma in a static magnetic field. Proc. Phys. Soc. **77** (1961), 971–979.

The propagation of electromagnetic waves through an infinite homogeneous plasma permeated by a static magnetic field is considered. The Boltzmann equation is linearized and the distribution function obtained as a trajectory integral. From this the current density is calculated and hence the components of the dielectric tensor. These are evaluated to first order in the temperature. Insertion into Maxwell's equations gives a dispersion relation. Explicit expressions are given for the case when the wave velocity is considerably greater than the mean thermal velocity of the particles. Thus the plasma wave is lost and the two electromagnetic waves are discussed only for propagation along or across the static magnetic field. The results agree with those previously obtained by Bernstein, Ahiezer, and others.

W. P. Allis (Cambridge, Mass.)

13029:

Hagfors, Tor. Density fluctuations in a plasma in a magnetic field, with applications to the ionosphere. J. Geophys. Res. **66** (1961), 1699–1712.

Author's summary: "General expressions are developed for the fluctuation in density of electrons, ions, and charge in a plasma in thermal equilibrium in an external magnetic field taking only Coulomb interaction into account. The spectral distribution of the spatial Fourier components of these fluctuations is derived from basic principles.

"The fluctuations in electron density are discussed in some detail, and spectra are computed under conditions that are thought to prevail in the outer ionosphere. Frequency spectra of general validity are computed for electron-density fluctuations along the magnetic field. It is shown by means of examples that the frequency spectra under ionospheric conditions are little influenced by the magnetic field except for density fluctuations fairly close to perpendicularity to the magnetic field. Applications to incoherent backscattering are discussed, and it is shown that, under suitable conditions, backscatter techniques can give valuable information about electron density, temperature, and constituents of the ionosphere."

K. C. Westfold (Clayton)

13030:

Goto, Keniti. Characteristic functional for plasma turbulence. Progr. Theoret. Phys. **25** (1961), 603–612.

The differential equation of the characteristic functional of Hopf has been derived for the magnetohydrodynamic turbulence of an incompressible plasma flow. An exact stationary solution for an ideal plasma, inviscid and of infinitely electrical conductivity, has been found. It is the characteristic functional of an isotropic Gaussian distribution for which the equi-partition of the kinetic and magnetic energies holds. A formal general solution by the functional Fourier transformation is also given.

S. I. Pai (College Park, Md.)

13031:

Gerstenkorn, Horst. Über die Entstehung von Plasmawellen in stark ionisierten Gasen. Z. Physik **162** (1961), 363–381. (English summary)

The author discusses the properties of plane space-charge waves of small amplitude in a thermal-electron plasma. He supplements the usual Vlasov equation by collision terms of various forms, and shows that such alterations have little effect on the solution. As usual, the dispersion relation is such that, in the complex half-plane for which it is defined, there exists no phase velocity to correspond to a given wave number. An exception occurs, in the case of one form of the collision term, at very low wave numbers. In general the response of the plasma to given initial conditions must still be found by contour integration and by analytical continuation of the dispersion relation into the other half-plane.

F. D. Kahn (Green Bank, W. Va.)

13032:

Žigulev, V. N. The phenomenon of magnetic pinch in a free-molecular plasma stream (the theory governing the flow of solar-corpuscular streams past the earth's magnetic dipole). Dokl. Akad. Nauk SSSR **135** (1960), 1364–1366 (Russian); translated as Soviet Physics. Dokl. **5** (1961), 1306–1308.

The author argues that a low density plasma coming from a field free region is specularly reflected by a strong enough magnetic field. There is a transition region between the field-free plasma and the plasma-free field, and this region is usually so thin that it can be treated like a surface. The author deduces that the interaction is analogous to that between a perfectly conducting fluid and a magnetic field, and can be discussed by conventional hydromagnetic theory. An application of this result can be made to the problem of the flow of a solar corpuscular stream past the Earth.

No discussion is given of the possible plasma instabilities in such a configuration.

F. D. Kahn (Green Bank, W. Va.)

13033:

Bourret, R. An hypothesis concerning turbulent diffusion. Canad. J. Phys. **38** (1960), 665–676.

An integral equation for isotropic turbulent diffusion is proposed as a generalization of a thermodynamic argument. One particular choice of the velocity auto-correlation function permits reduction to the Goldstein-Michelson equation. The author believes the formalism is of value if spatial velocity correlations are negligible.

W. V. R. Malkus (Los Angeles, Calif.)

13034:

Bourret, R. C. Velocity autocorrelations of charged particles in a magnetoionic medium with applications to turbulent diffusion. Canad. J. Phys. **38** (1960), 1213–1223.

The total force on a particle is taken as the sum of a stochastic force $\mathbf{F}(t)$, a magnetic interaction $e\mathbf{v}\times\mathbf{B}$, and a dynamical friction $-mg\mathbf{v}$ which is split off from the stochastic force. The truncated Fourier transform of $\mathbf{F}$ is $F_T(\omega)$, and the velocity $\mathbf{v}_T(\omega)$ of the particle is assumed related to $\mathbf{F}_T(\omega)$ by the standard a.c. mobility. The power spectrum $R_{\alpha\beta}(\omega)$ and the velocity autocorrelation function

$$R_{\alpha\beta}(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{+T/2} v_\alpha^*(t)v_\beta(t+\tau)\,dt$$

are then derived and explicit expressions found when $F(t)$ is isotropic and also white, Markovian, or due to collision impulses.

The white velocity autocorrelation function is then introduced in a diffusion equation previously advanced [#13033] to obtain an equation for "turbulent diffusion":

$$\frac{\partial n}{\partial t} = D\left(\nabla^2 n - \frac{2m}{kT}\frac{\partial^2 n}{\partial t^2}\right)$$

in which D has the conventional form.

W. P. Allis (Cambridge, Mass.)

13035:

Boerboom, A. J. H. Numerical calculation of the potential distribution in ion slit lens systems. II. Z. Naturforschg. **15a** (1960), 244–252.

Author's summary: "The potential distribution is computed in certain ion slit lens systems, consisting of three parallel slits in three parallel electrodes. In a previous paper [same Z. **14a** (1959), 809–816] the case was treated where the slit widths were smaller than the distances to the neighbouring electrodes. In the present paper this requirement has been dropped; for the sake of simplicity, however, the computations are confined to the case, where the central electrode represents a plane of symmetry.

"Various approximation and iteration methods are given to find the necessary parameters to perform the Schwarz-Christoffel transformation. Several typical examples are given."

13036:

Boerboom, A. J. H. Numerical calculation of the potential distribution in ion slit lens systems. III. Z. Naturforschg. **15a** (1960), 253–259.

Author's summary: "In previous papers [see #13035] the potential distribution was calculated in ion slit lens systems, consisting of three slits in three parallel electrodes and satisfying certain conditions concerning their shape.

"In the present paper the computing methods are generalized to slit systems of an arbitrary number of electrodes, with as the only restriction, that slits broader than the distances to neighbouring slits are separated by slits, narrower than the respective distance, and that a pair of electrodes with a mutual distance smaller than their slit widths are separated from the neighbouring slits by distances greater than the respective slit widths.

"For slit systems, satisfying this condition the parameters are computed, necessary to perform the Schwarz-Christoffel transformation. Formulae are given to compute the potential distribution and field strength. In a typical example the potential distribution and field strength are computed in the region around two parallel electrodes with broad slits compared with the distance between the electrodes."

13037:

Cairó, Lorenzo; Kahan, Théo. Principe variationnel relatif aux systèmes d'équations à valeurs propres communes. Application au calcul de la constante de propagation des ondes électromagnétiques dans des guides à milieux anisotropes. C. R. Acad. Sci. Paris **251** (1960), 1865–1867.

Les auteurs partent d'un principe variationnel établi dans: Morse et Feshbach, *Methods of theoretical physics, Vol. II* [McGraw-Hill, New York, 1953; MR **15**, 583; p. 1109]; ils en font l'application aux systèmes d'équations qui fournissent la constante de propagation des ondes électromagnétiques dans les guides d'ondes gyromagnétiques, obtenus par W. Hauser [Quart. Appl. Math. **16** (1958), 259–272; MR **21** #3201]. Cet exposé fait suite à la publication: T. Kahan, G. Rideau et P. Roussopoulos, *Les méthodes d'approximation variationnelles dans la théorie des collisions atomiques et dans la physique des piles nucléaires* [Memor. Sci. Math. no. 134, Gauthier-Villars, Paris, 1956; MR **18**, 975; p. 48].

P. M. Poincelot (Issy-les-Moulineaux)

13038:

Oster, Ludwig. Cyclotron radiation from relativistic particles with an arbitrary velocity distribution. Phys. Rev. (2) **121** (1961), 961–967.

The author derives the intensity of the radiation emitted by a single particle with arbitrary momentum in a magnetic field. He then obtains the total radiation for a Maxwell-Boltzmann distribution by direct integration. His reference to the Drummond and Rosenbluth paper uses the wrong initial for that Drummond. This reference should read, W. E. Drummond and M. N. Rosenbluth, Phys. Fluids **3** (1960), 45–51 [MR **22** #2296]. The interested reader should also see their later comment, ibid. **4** (1961), 277–278.

J. E. Drummond (Seattle, Wash.)

13039:

Dyhne, A. M.; Pokrovskiĭ, V. L. Change of the adiabatic invariant of a particle in a magnetic field. I. Ž. Èksper. Teoret. Fiz. **39** (1960), 373–377 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 264–267.

Authors' summary: "The change of the adiabatic invariant is found for a particle moving in an axially symmetrical inhomogeneous magnetic field. The problem is solved for the usual model Hamiltonian."

13040:

Gotō, Tetsuo. On the nature of the electromagnetic field. Nuclear Phys. **24** (1961), 388–399.

An attempt is made to introduce the electromagnetic field in connection with the affine connection for spinors. The foundation of the charge independence is discussed on this ground.

O. Hara (Minneapolis, Minn.)

13041:

Tihonov, A. N.; Šahsuvarov, D. N. The electromagnetic field of a dipole in a distant zone. Izv. Akad. Nauk SSSR. Ser. Geofiz. **1959**, 946–955. (Russian)

The paper gives analytic expressions and graphs for the far electromagnetic field of dipoles in a variety of material media. The results are expressed in terms of elliptic functions.

D. E. Spencer (Storrs, Conn.)

13042:

Day, T. B. Radiation from fast particles moving through magnetic materials. Phys. Rev. (2) **122** (1961), 1028–1036.

Author's summary: "The problem of the generation of a changing magnetic field due to the interaction of a fast particle with a magnetic medium is studied. This combined Čerenkov-spin wave effect is shown to give rise to a 'ringing' of the spin system under certain conditions of frequency and angle of observation, at least within an

approximate evaluation of the general Green's function for the problem. Some striking differences from the usual Čerenkov effect are discussed and possibilities of using this effect as a neutral magnetic moment detector or as a probe of magnetic materials are mentioned briefly."

13043:

Önal, Hasan. Integralrechnung der magnetischen Induktion einer stromdurchflossenen Spule, die unendlich lang ist und einen beliebigen Querschnitt hat. Bull. Tech. Univ. Istanbul **12** (1960), 59–67. (Turkish summary)

Author's summary: "In der Literatur ist die magnetische Induktion einer unendlich langen Spule mit Hilfe des Durchflutungsgesetzes unter Berücksichtigungen der Symmetrieeigenschaften des magnetischen Feldes berechnet worden. In diesem Aufsatz wird zu diesem Zweck eine Integralrechnung durchgeführt, die keine Symmetrieeigenschaft und auch kein Durchflutungsgesetz erfordert. Das Zwischenergebnis dieser Integralrechnung kann auch für die Ermittlung der Lösungen einiger anderen Probleme nützlich sein. Als Beispiel hierzu wird die von einem unendlich grossen Flächenstrom hervorgerufene magnetische Induktion und der auf die Fläche einer Spule ausgeübte Druck berechnet."

13044:

Kielich, S.; Piekara, A. A statistical molecular theory of electric, magnetic and optical saturation phenomena in isotropic dielectric and diamagnetic media. Acta Phys. Polon. **18** (1959), 439–471.

The authors apply the statistical mechanical procedure for calculating dielectric constants developed by Kirkwood [J. Chem. Phys. **4** (1936), 592–601] to a variety of saturation phenomena. Expressions in terms of molecular dipole moments, polarizabilities, etc., are derived for the effect of dielectric, magnetic, and optical (oscillating electric) fields upon the dielectric constant, magnetic susceptibility and refractive index of the system. In addition to the usual saturation effects, cross-saturation phenomena, such as the alteration of the dielectric constant by a magnetic field, are also treated. To make the calculations tractable, local fields are replaced by their averages over the entire system; this avoids the difficulties inherent in a detailed treatment of the interactions between different molecules. Numerical estimates are given for several cross-saturation effects which have not yet been observed experimentally.

S. Prager (Minneapolis, Minn.)

13045:

Pringle, G. E. The potential of a circular current. Proc. Cambridge Philos. Soc. **57** (1961), 385–392.

The author gives a review of methods based on the use of toroidal coordinates for deriving the potential of a charged circular ring as an elliptic integral and extends these methods to derive similar expressions for the potential of a circular current.

W. D. Collins (Newcastle upon Tyne)

13046:

Desirant, M.; Michiels, J. L. (Editors). ★Electromagnetic wave propagation. International conference sponsored by the Postal and Telecommunications Group of the Brussels Universal Exhibition. Academic Press, London-New York, 1960. xiii + 730 pp. $22.00.

A set of 55 papers of which those of mathematical interest will be reviewed separately.

13047:

Wait, James R.; Conda, Alyce M. Radiation from a slot on a large corrugated cylinder. Electromagnetic wave propagation, pp. 103–109. Academic Press, London, 1960.

The radiation patterns of an axial-magnetic line source on a large circular cylinder with an inductive surface reactance are computed in this paper using a representation in terms of a complex integral valid for cylinders with a circumference much greater than a wavelength. The integral is evaluated by a combination of the saddle-point method, residue series and numerical means depending upon the magnitude and sign of the azimuthal angle. The results show a broader pattern than that obtained for a perfectly conducting cylinder and this effect is ascribed to the trapping of waves by the reactive surface. *L. O. Goldstone* (Brooklyn, N.Y.)

13048:

Hirai, Masaichi; Fukushima, Madoka; Kurihara, Yoshitaka. Correlation between amplitudes of radio waves of different frequencies in UHF beyond-the-horizon propagation. J. Radio Res. Lab. **7** (1960), 509–529.

Authors' summary: "Selective fading in UHF beyond-the-horizon propagation is caused by the interference between multipath waves. Based on this phenomenon, a correlation coefficient between amplitudes of radio waves of different frequencies is derived theoretically. Relations of this coefficient with amplitude ratio distribution and multipath wave distribution are also derived. These relations are examined by use of the experimental data in 600 and 2,120 Mc bands on circuits of 226 km from Kokubunji to Furukawa, and thus the applicability of the theoretical formulas is illustrated."

S. A. Bowhill (University Park, Pa.)

13049:

Beckmann, Petr. A generalized Rayleigh distribution and its application to tropospheric propagation. Electromagnetic wave propagation, pp. 445–449. Academic Press, London, 1960.

This paper derives the amplitude distribution of the sum of a number of random vectors of equal amplitudes, the phases of which have zero mean and arbitrary, unsymmetrical, probability distributions. It is shown that the distributions of the phase-components become Gaussian in the limiting case, and that three parameters are sufficient to specify the resulting distribution. Several distributions, including the Rayleigh and Rice forms, are derived as special cases of this distribution. A tentative comparison is made between the distribution derived and that of the amplitude of a VHF radio signal received beyond the horizon. *S. A. Bowhill* (University Park, Pa.)

13050:

Pivovonsky, Mark; Nagel, Max R. ★Tables of blackbody radiation functions. Macmillan Monographs in

Applied Optics. The Macmillan Co., New York, 1961. xlii+481 pp. $12.50.

The volume contains exactly what is promised on the dust cover: "... A tabulation of Planck's radiation law and a number of related functions describing the radiant and luminous properties of ideal thermal radiators. The tables cover a broad range of wavelengths and temperatures, saving the researcher time-consuming interpolations and other calculations. Conversion data have been included to prevent obsolescence in case of changes of the atomic constants on which the tables are based." In addition, the introduction includes a useful attempt to describe the various and often conflicting terms which are now currently being used to describe radiation concepts. Any worker in this field knows how deplorable the position is and will welcome this attempt at standardization which gives preference to the terminology recommended by the Optical Society of America. It is, however, inexplicable that the discussion of the International Temperature Scale stops at 1948 and ignores the Tenth Conference on Weights and Measures held in 1954.

J. Kestin (Providence, R.I.)

13051:

Heckl, Manfred. Wave propagation on beam-plate systems. J. Acoust. Soc. Amer. **33** (1961), 640–651.

13052:

Karczewski, B. Fraunhofer diffraction of an electromagnetic wave. Bull. Acad. Polon. Sci. Sér. Sci. Math. Astr. Phys. **7** (1959), 633–638. (Russian summary, unbound insert)

F. Kottler gave in 1923 a formulation of Huygens' principle for electromagnetic waves, not as a boundary value problem, but as a saltus problem across the thin diffracting screen [Ann. Physik **71** (1923), 457–508], and derived formulae for the Fresnel type of diffraction. The present author deduces from Kottler's results formulae for Fraunhofer diffraction. *E. T. Copson* (St. Andrews)

13053:

Faulkner, E. A. Calculation of stored energy from broadening of x-ray diffraction lines. Philos. Mag. (8) **5** (1960), 519–521.

The author calculates, from the broadening of x-ray reflections in deformed metals, the mean-square variation $\overline{(\Delta d/d)^2}$ in the separation d of the lattice planes contributing to the reflection. This leads, under suitable assumptions concerning the distribution of the stress components, to expressions for the energy stored in the material. The author corrects an error in a similar, earlier calculation by Stibitz [Haworth, Phys. Rev. (2) **52** (1937), 613–620; p. 619]. *H. A. Hauptman* (Washington, D.C.)

13054:

Vinogradov, V. S. The effect of departures from periodicity of the charge distribution in a crystal lattice on dielectric losses in the ultrahigh-frequency region. Fiz. Tverd. Tela **2** (1960), 2622–2628 (Russian); translated as Soviet Physics. Solid State **2** (1961), 2338–2344.

Earlier treatments of the problem described by the title are extended to take account of the effect of the interaction of the impurity charges with the regular lattice ions and with each other. The results are found to be comparable in magnitude to the "direct" effect of the impurities on the dielectric losses. All three quantities are linear in both the frequency and the concentration of impurities. The author finds the new effects insufficiently large to explain data on NaCl at 85° C, but feels that they might be observable at low enough temperatures.

H. B. Rosenstock (Washington, D.C.)

13055:

Bowhill, S. A. Statistics of a radio wave diffracted by a random ionosphere. J. Res. Nat. Bur. Standards Sect. D **65D** (1961), 275–292.

A solution of the wave equation in the half space $z>0$ is completely determined by its distribution $A(x, y)$ over the boundary plane $z=0$ (the sources being in the other half space $z<0$). This enables the derivation of statistical properties of the solution in any plane $z=\text{const}$ from known statistical properties of $A(x, y)$. The author considers an amplitude modulated distribution in $z=0$; this involves a real function $A(x, y)$. The splitting of the solution $E(x, y)$ in a plane $z=\text{const}$, after division by the phase factor $\exp(2\pi iz/\lambda)$ into its real and imaginary parts $E_1(x, y)$ and $E_2(x, y)$, corresponds to the splitting into contributions in phase and out of phase with the distribution $A(x, y)$ in $z=0$. The two-dimensional auto-correlation functions

$$\overline{E_1(x, y)E_1(x+\alpha, y+\beta)}, \quad \overline{E_2(x, y)E_2(x+\alpha, y+\beta)}$$

of α and β, and also cross-modulation variances such as $\overline{E_1(x, y)E_2(x, y)}$, are derived explicitly while assuming a Gaussian auto-correlation function for $A(x, y)$. The necessary integrations reduce to integrals of the Fresnel and Poisson type. Various limiting cases are discussed numerically. The extensions in the sections 5 and 6 concern an extra phase factor in the random distribution in $z=0$ that accounts for a source at a finite distance from $z=0$, and for an obliquely incident primary wave respectively. The computations are made in view of their applicability to radio-wave diffraction patterns at the earth's surface which are due to irregularities in the ionosphere (the lower boundary of which is idealized by the plane $z=0$). The quantities E_1 and E_2 are marked throughout as $E_1(x, y)$ and $E_2(x, y)$, but the reader should bear in mind the z dependence of these functions.

H. Bremmer (Eindhoven)

13056:

Hagfors, Tor. Some properties of radio waves reflected from the moon and their relation to the lunar surface. J. Geophys. Res. **66** (1961), 777–785.

Author's summary: "This report presents a theoretical discussion of the statistical properties of radio waves reflected from the moon. The discussion is based on the assumption of a large number of scattering areas simultaneously contributing to the signal. The properties of the echoes are usually described in terms of pulse broadening or by means of an average-power pulse response when very short pulses are transmitted. Here it is shown that the same type of information can be obtained by studying the correlation of complex amplitudes of two sine waves reflected from the moon at frequencies separated by $\Delta\omega$;

it is possible to compute the power pulse response of the earth-moon-earth propagation circuit. It is suggested that this method will prove particularly useful in the study of the surface properties of more distant targets such as the planets. It is shown how the correlation technique can be extended to a two-dimensional mapping of a rotating rough body. The properties of the echoes returned from the moon are related here to a crude statistical model of the lunar surface roughness. This model is shown to lead to a satisfactory account for the semispecular component of the return from the moon if a large-scale structure with rms slopes of 1/20 to 1/10 are assumed." *S. A. Bowhill* (University Park, Pa.)

13057:

Wait, James R.; Conda, Alyce M. On the computation of diffraction fields for grazing angles. Electromagnetic wave propagation, pp. 661–670. Academic Press, London 1960.

This paper presents certain results which have been obtained by the authors for the diffraction of electromagnetic waves by a convex cylindrical surface. The formula for diffraction loss is cast in the form of that for an idealized knife-edge plus a correction term. The paper presents only working formulas without detailed derivations. Some numerical results for diffraction loss, together with curves and diffraction patterns, are included which show the effects of surface curvature and polarization.
L. O. Goldstone (Brooklyn, N.Y.)

13058:

Agranovič, V. M. Dispersion of electromagnetic waves in crystals. Ž. Èksper. Teoret. Fiz. **37** (1959), 430–441 (Russian); translated as Soviet Physics. JETP **10** (1960), 307–313.

The relationship between the electromagnetic (optical) properties of condensed systems and those of their molecular constituents has been much studied by physicists. Recently U. Fano [Phys. Rev. (2) **103** (1956), 1202–1218] presented a systematic approach to the theory based on quantum mechanical methods. The author has applied this method to the study of optical dispersion in crystals. The discussion is concerned with the formalism of the theory rather than with the derivation of physically new results. *E. L. Hill* (Minneapolis, Minn.)

13059:

Nussenzveig, H. M. Diffraction theory in the k-representation. An. Acad. Brasil Ci. **31** (1959), 515–521.

The results of this paper are essentially known. Via Rayleigh's formulae, a monochromatic wave function defined in a half-space is represented by a superposition of travelling and evanescent waves (k-representation). Starting from Kirchhoff-like formulation of the scattering problem, the author applies the principle of stationary phase for evaluating Fraunhofer and Fresnel diffraction patterns in the k-representation. The author stresses that the classical patterns under optical conditions depend on a very small spectral region, and this leads to a qualitative explanation of the success of classical diffraction theory.
C. J. Bouwkamp (Eindhoven)

13060:

Seshadri, S. R. Scattering by a narrow unidirectionally conducting infinite strip. Canad. J. Phys. **38** (1960), 1623–1631.

The author's unidirectionally conducting strip is perfectly conducting in a given direction and is insulating in the perpendicular direction. The incident wave is plane polarized and its direction of incidence is arbitrary. The scattering problem is formulated in terms of an integral equation, the solution of which is obtained by iteration in the form of a series in powers of ka (k wavenumber, $2a$ width of strip) and log ka. Expressions for the scattering cross-section and the far-zone field are obtained.
C. J. Bouwkamp (Eindhoven)

13061:

van der Pol, Balth; Levelt, A. H. M. On the propagation of a discontinuous electromagnetic wave. Nederl. Akad. Wetensch. Proc. Ser. A **63** = Indag. Math. **22** (1960), 254–265.

This article gives a derivation of the electromagnetic field produced by an electrical dipole on the plane interface between two infinite dielectrics, when the dipole current jumps at $t=0$ from zero to a stationary value. This problem has been discussed before by the first author [IRE Trans. **AP-4** (1956), 288–293] for the special case of observation points in the interface itself. In the present paper the general situation, which is much more complicated, is treated in a very elegant way. In both half-spaces the solution constitutes (as a function of time) the inverse Laplace transform of Sommerfeld's classical solution (considered as a function of the frequency) for a time-harmonic dipole current. In this paper the Bessel function entering in Sommerfeld's solution is replaced by an elementary integral which leads to a double integral. Changing over to proper new variables, the latter can be transformed into the Laplace transform of an elliptic integral which therefore represents the solution for the above mentioned discontinuous dipole current. The analysis is rather intricate due to the branch points connected with the square roots occurring in Sommerfeld's solution. In the half-space of lowest dielectric constant ε_1 the discontinuous solution can be represented by a single elliptic integral, at least after the time of arrival, viz., $(R/c)\varepsilon_1^{1/2}$, of a wave propagating directly towards the observation point (the solution being zero before this moment). The representation in the other half-space, also in terms of elliptic integrals, becomes different in various regions. These regions are characterized by the property that a wave, propagating directly to the observation point, does arrive either earlier or later than the other wave which, after travelling some distance along the interface, penetrates into the half-space in question in the direction of the "critical angle" (angle of refraction for grazing incidence). The rays propagating in the latter direction are parallel to a cone which has been termed erroneously as a "Brewster cone". *H. Bremmer* (Eindhoven)

13062:

Briggs, B. H. Diffraction by an irregular screen of limited extent. Proc. Phys. Soc. **77** (1961), 305–317.

An examination is made of the extent to which experimental measurements of the diffraction pattern produced by an irregular sheet may be used to obtain information

about the physical nature of the sheet. It is assumed that the sheet varies irregularly in detail with time but preserves its statistical properties. A quantity of ${}_{(a)}\rho_g$ is defined as the time average correlation function between two fixed points in the diffraction pattern. The quantity ${}_{(a)}\rho_g$ is measured in experiments and a relationship is derived between ${}_{(a)}\rho_g$, the amplitude of the wave function leaving the screen, and the angular spectrum produced by the screen. The implications of this relationship are examined for the two cases when ${}_{(a)}\rho_g$ is measured near and far from the screen. Near the screen it is shown that ${}_{(a)}\rho_g$ is the same as the auto-correlation function of the small scale structure of the screen. Far from the screen ${}_{(a)}\rho_g$ is equal to the Fourier transform of the angular power distribution over the screen.

Particular examples of the measurement of the angular diameter of stars and the problem of reflection from irregularly ionized trails produced by meteors are examined. *W. E. Williams* (Liverpool)

13063:

Kuehl, Hans H. Current on an infinitely long cylindrical antenna. J. Math. and Phys. **39** (1960/61), 121–125.

An expression for the asymptotic form of the current distribution on the infinitely-long cylindrical antenna, with delta-function excitation, has been derived by Papas [J. Appl. Phys. **20** (1949), 437–440; MR **11**, 293] based on a saddle-point integration of a contour-integral representation of the field under the assumption $\rho \gg a$. To obtain the current on the cylinder the field must be evaluated at $\rho = a$, which contradicts the assumption $\rho \gg a$. The author shows that, at least for small radii, the correct z-dependence of the current is identical to that derived by Papas. *G. Sinclair* (Toronto)

13064:

Hu, Ming-Kuei. Fresnel region fields of circular aperture antennas. J. Res. Nat. Bur. Standards Sect. D **65D** (1961), 137–147.

An approximate expression is derived for the fields in the Fresnel zone of a circular aperture. The expression is obtained by the use of Newton's iteration formula for approximating a square root rather than by the usual truncated series approach. This results in a simplified expression for the integrand which is directly integrable for circularly symmetric aperture excitations, with arbitrary radial variations. The integrals are expressed in terms of Lommel functions and their derivatives. The author computes the electric fields (amplitude and phase) at various distances from the aperture (from $D^2/4\lambda$ to ∞) and for various observation angles. The aperture illuminations are circularly symmetric and of various degrees of taper in the radial direction. The expression derived in the paper is simpler than the one given by Silver or Hansen and Bailin and is readily integrable for various aperture excitations, but its accuracy is degraded for large observation angles. *A. Ksienski* (Culver City, Calif.)

13065:

Ryvkin, M. S. Rectangular waveguide of variable cross-section. Radiotehn. i Èlektron. **4** (1959), 1465–1474 (Russian); translated as Radio Engrg. and Electronics **4**, no. 9, 78–92.

The propagation of electromagnetic waves is considered for a waveguide having a rectangular cross-section which varies along the waveguide axes. A coordinate system is used which fits the walls of the waveguide and which is approximately orthogonal. The resulting wave equation for the electric vector is solved using the UKB approximation. *G. Sinclair* (Toronto)

13066:

Bedrosian, S. D. Elliptic functions in network synthesis. J. Franklin Inst. **271** (1961), 12–30.

The role of elliptic functions in the approximation problem, particularly in connection with Cauer image parameter filters, filters on an insertion-loss basis and wide band 90° phase-difference networks is discussed. Useful design charts and tables are included.

J. Blackman (Syracuse, N.Y.)

13067:

Ksienski, A. Equivalence between continuous and discrete radiating arrays. Canad. J. Phys. **39** (1961), 335–349.

Author's summary: "The radiation patterns produced by continuous excitation distributions and discrete arrays are compared and the conditions are derived under which one type of source may be substituted for the other with negligible errors. It is shown that the aperture lengths in both cases should be the same but the element spacing is dependent on the type of pattern desired. Examples are computed to demonstrate these relations for both directive patterns and shaped beams." *G. Sinclair* (Toronto)

13068:

Bickart, T. A. Flowgraphs for the representation of nonlinear systems. IRE Trans **CT-8** (1961), 49–58.

Non-linear systems are represented by signal flowgraphs. The following branch operators are used: power and exponential and their inverses (analytic) and switching and limiting (non-analytic). It is shown that by their combination and such manipulations as reduction or equivalence and inversion a great variety of non-linear systems including hysteretic ones can be thus represented. Stability is not investigated though the problem is hinted.

H. G. Baerwald (Albuquerque, N.M.)

13069:

Karasev, M. D. Some general properties of nonlinear reactive elements. Uspehi Fiz. Nauk **69** (1959), 217–267 (Russian); translated as Soviet Physics. Uspekhi **2**, 719–748.

A fine expository account of theory and application of non-linear reactive elements in circuits. After a brief review of non-linear theory of resonance, parametric excitation, and conversion, essentially following the classical treatment of the Mandelstam-Papalexi school, the general power relations are discussed, leading to the Manly-Rowe equations. Conventional analysis of the reactive modulator by the small-signal method leads to the modern microwave applications of parametric amplification. Recent developments (to 1958) of solid-state and beams devices and a brief discussion of noise properties conclude the outline.

H. G. Baerwald (Albuquerque, N.M.)

13070:

Lovass-Nagy, V. Investigation of polyphase electrical systems by means of the matrix calculus. Les mathématiques de l'ingénieur, pp. 369–374. Mém. Publ. Soc. Sci. Arts Lett. Hainaut, vol. hors série, 1958.

13071:

Szendy, Charles. Matrix-calculus to transient analysis of electrical networks. Les mathématiques de l'ingénieur, pp. 402–408. Mém. Publ. Soc. Sci. Arts Lett. Hainaut, vol. hors série, 1958.

13072:

Landee, Robert W.; Davis, Donovan C.; Albrecht, Albert P. ★Electronic designers' handbook. McGraw-Hill Handbooks. McGraw-Hill Book Company, Inc., New York-Toronto-London, 1957. xi + 1029 pp. (not consecutively paged) $16.50.

This is a standard text in electronic design, mainly dealing with the characteristics of vacuum tubes, transistors, oscillators, etc. The book is in twenty-three sections, and a few of these are of mathematical interest; in particular, there are sections on the principles of feedback, on computer and servomechanism techniques, on transmission lines, on waveform analysis, and on network analysis which will be of some interest to applied mathematicians working on these problems.

H. Kolsky (Providence, R.I.)

13073:

Fazekas, Francis. Quelques formules algébriques pour la distribution des tensions dans la chaine d'isolateur produites à l'aide du calcul matriciel. Les mathématiques de l'ingénieur, pp. 342–347. Mém. Publ. Soc. Sci. Arts Lett. Hainaut, vol. hors série, 1958.

13074:

Layton, J. M. Eddy currents in cores of rectangular cross-section sinusoidally magnetised. Les mathématiques de l'ingénieur, pp. 356–360. Mém. Publ. Soc. Sci. Arts Lett. Hainaut, vol. hors série, 1958.

CLASSICAL THERMODYNAMICS, HEAT TRANSFER

See also 12612, 12968.

13075:

Obert, Edward F. ★Concepts of thermodynamics. McGraw-Hill Book Co., Inc., New York-Toronto-London, 1960. xxi + 528 pp. (7 inserts) $11.00.

The author of this book finds that engineering thermodynamics is confusing to students and suggests that part of the confusion is due to misunderstanding of the concepts: function, and exact and inexact differentials. He devotes attention to these and many other matters of definition and logic in a book which covers more or less the conventional territory of macroscopic thermodynamics.

The reviewer believes that many readers will find the book stimulating. The author expresses original views on many matters, provides numerous illustrations and examples, and has thoroughly mastered the mechanics of book writing. His frequent shifts of viewpoint cannot fail to assist students, and some of his presentations will surely appeal to teachers.

Personally, however, the reviewer was irritated by the book's restless and imprecise style. The author's definitions reminded him of the repeated jabs of an inexpert nurse searching for the artery. A piece of particularly bad markmanship is contained in the third sentence of Chapter 1, which runs: "A concept, if fundamental, cannot be clearly explained"; to judge from this book, the concepts of thermodynamics must be very fundamental indeed.

D. B. Spalding (London)

13076:

Minasyan, R. S. On plane stationary propagation of heat in a prismatic body with hollow rectangular cross-section in the presence of heat-exchange on the sides. Akad. Nauk Armyan. SSR. Dokl. **28** (1959), 159–169. (Russian. Armenian summary)

This appears to be a competent analysis of the problem of an infinite prismatic bar whose cross-section is a rectangle provided with a rectangular hole. The cross-section is symmetric with respect to the two axes parallel to the sides of the rectangles. The bar is provided with distributed sources of heat and exchanges heat at the boundaries. Problems of this kind occur, for example, in nuclear fuel elements and transformers. The solution is given in terms of orthonormal series. The complexity of the problem arises from the complexity of the shape of the cross-section which involves a multiply connected region and non-analytic boundaries.

J. Kestin (Providence, R.I.)

13077:

Godunov, S. K. Thermodynamics of gases and differential equations. Uspehi Mat. Nauk **14** (1959), no. 5 (89), 97–116. (Russian)

The foundations of classical thermodynamics (or "thermostatics", as some people now prefer to call it) exert a magnetic fascination on many scientists. In particular, mathematicians find it odd that a mathematically precise statement about the existence of a universal integrating denominator (the so-called First Part of the Second Law) can be derived from such mathematically imprecise and ill-defined concepts, as the impossibility of constructing a perpetual-motion engine of the second kind. The present paper addresses itself to the study of the relation between the First Part of the Second Law (i.e., its mathematical formulation cited above) and the stability of systems. The study is not a general one, but is based on the stability of various coupled systems, each of which contains a gas, with respect to small oscillations performed by a piston. As the author recognizes, the examples chosen are somewhat artificial, and, consequently the paper contains a sketch of a study, rather than a complete investigation. However, there seems to be contained in it the germ of an important idea. The value of the paper lies in showing that the stability of gaseous systems, except at the critical point, is mathematically equivalent to the First Part of the Second Law.

J. Kestin (Providence, R.I.)

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Godunov, S. K. Thermodynamics of gases and differential equations. Acad. R. P. Romîne. An. Romîno-Soviet. Ser. Mat.-Fiz. (3) **14** (1960), no. 2 (33), 38–59. (Romanian)

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The author replaces known statements of the second law of thermodynamics by three axioms and claims that he has thereby achieved a more operational and direct approach to the second law.

{The work appears to the reviewer misleading and largely trivial. The first two axioms in fact amount directly to this: "there exists a universal function $g(t)(=d \ln T(t)/dt)$ of the empirical temperature t alone such that the integrability conditions for the linear differential form

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Für den Fall konstanter Umgebungstemperatur finden sich ausführlichere Ergebnisse in einer Arbeit von F. Berger [Z. Angew. Math. Mech. **11** (1931), 45–58; S.45]. *H. Parkus* (Vienna)

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Author's summary: "In questa ricerca si deducono le analogie fra i campi di velocità, temperatura, concentrazione e fra gli scambi di quantità di moto, calore, massa per correnti di miscela gas-vapore che lambiscono superfici sulle quali il vapore può subire cambiamenti di stato. L'analogia degli scambi simultanei estende la classica analogia di Colburn in quanto tiene conto: di elevate velocità, di concentrazioni di vapore comunque elevate, della differenza tra i calori specifici del gas e del vapore, di tutti i regimi di moto dall'ordinatissimo moto laminare di Couette al disordinatissimo moto completamente turbolento."

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Authors' summary: "A hypothetical gas mixture composed of A_2, B_2, and AB which reacts bimolecularly, $2AB \rightleftharpoons A_2+B_2$, is placed in a thermal conductivity cell. The heat flux, as well as the chemical and thermal profiles, are calculated numerically for fast, slow, and intermediate reaction rates. Since the chemical composition is not in equilibrium with the local temperature, considerable calculational difficulties are encountered. An approximation scheme is given for estimating the behavior of such systems. It is found that for fast reactions, local equilibrium applies. If the rate of the chemical reactions is slow compared to the rate of diffusion (an exact criterion is given), the chemical composition within the cell becomes homogeneous throughout the cell. This composition is determined by the set of equations $\int_{T_c}^{T_s} \lambda \Re_i \, dT = 0$, where λ is the coefficient of heat conductivity and $\Re_i$ is the net rate of production of molecules of the ith species. Conversely, from a knowledge of the steady-state chemical composition we may determine the chemical reaction rates. Calculations are made for a mixture of H_2, I_2, and HI. This system should be easy to study experimentally since the I_2 concentration can be determined from optical absorption and the HI concentration can be determined from infrared absorption."
H. L. Frisch (Murray Hill, N.J.)

QUANTUM MECHANICS

See also A12018, A12019, A12032a–b, A12411, 12792, 12910a–b, 13058, 13238, 13243.

13090:

Bates, D. R. (Editor). ★Quantum theory. I. Elements. Pure and Applied Physics, Vol. 10-I. Academic Press, New York-London, 1961. xv+447 pp. $10.00.

13091:

Powell, John L.; Crasemann, Bernd. ★Quantum mechanics. Addison-Wesley Publishing Co., Inc., Reading, Mass.-London, 1961. x+495 pp. $9.75.

This book is an excellent introduction to quantum mechanics at the senior and first-year graduate level. It contains a very clear and complete account of the physical and mathematical aspects of quantum mechanics. These two aspects are integrated in the best possible manner, with the physical reasons for developing the mathematical formalism being clearly discussed first. The book has the feeling of being "tested under fire" since small but important steps in many discussions, often causing trouble to students when left out, are carefully given in the book. This is indeed true, since the book is based on lectures taught for several years by the authors. The book opens with a historical discussion of the difficulties of classical physics and "old" quantum theory. The wave nature of free particles is considered in the next two chapters, where a careful discussion of waves and Fourier transforms leads to the free particle Schrödinger equation, the Born probability interpretation of the wave function, and the uncertainty principle. In Chapter 4 forces are introduced into the Schrödinger equation by means of the analogies between wave mechanics and optics, and one-dimensional examples are considered in Chapter 5. The ideas of linear operator, eigenvalue, and commutator brackets in quantum mechanics are then discussed in Chapter 6. The motion of a particle in a spherically symmetric force is considered in Chapter 7, and the angular momentum of such systems considered. An excellent discussion of scattering is contained in the next chapter, including an account of phase shifts, the S matrix and its relation to bound states, and the Born approximation. Matrix mechanics is then introduced in Chapter 9 and applied to angular momentum in Chapter 10. This latter chapter also discusses the electron spin in a non-relativistic fashion. Time-independent and time-dependent perturbation theory are then discussed, for both degenerate and non-degenerate systems, and applied to the Zeeman effect. A final chapter considers briefly

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13089:

Secrest, Don; Hirschfelder, Joseph O. Slowly reacting gas mixture in a heat conductivity cell. Phys. Fluids 4 (1961), 61–73.

Authors' summary: "A hypothetical gas mixture composed of A_2, B_2, and AB which reacts bimolecularly, $2AB \rightleftharpoons A_2+B_2$, is placed in a thermal conductivity cell. The heat flux, as well as the chemical and thermal profiles, are calculated numerically for fast, slow, and intermediate reaction rates. Since the chemical composition is not in equilibrium with the local temperature, considerable calculational difficulties are encountered. An approximation scheme is given for estimating the behavior of such systems. It is found that for fast reactions, local equilibrium applies. If the rate of the chemical reactions is slow compared to the rate of diffusion (an exact criterion is given), the chemical composition within the cell becomes homogeneous throughout the cell. This composition is determined by the set of equations $\int_{T_c}^{T_x} \lambda \Re_i \, dT=0$, where λ is the coefficient of heat conductivity and $\Re_i$ is the net rate of production of molecules of the ith species. Conversely, from a knowledge of the steady-state chemical composition we may determine the chemical reaction rates. Calculations are made for a mixture of H_2, I_2, and HI. This system should be easy to study experimentally since the I_2 concentration can be determined from optical absorption and the HI concentration can be determined from infrared absorption."

H. L. Frisch (Murray Hill, N.J.)

QUANTUM MECHANICS

See also A12018, A12019, A12032a–b, A12411, 12792, 12910a–b, 13058, 13238, 13243.

13090:

Bates, D. R. (Editor). ★Quantum theory. I. Elements. Pure and Applied Physics, Vol. 10-I. Academic Press, New York-London, 1961. xv+447 pp. $10.00.

13091:

Powell, John L.; Crasemann, Bernd. ★Quantum mechanics. Addison-Wesley Publishing Co., Inc., Reading, Mass.-London, 1961. x+495 pp. $9.75.

This book is an excellent introduction to quantum mechanics at the senior and first-year graduate level. It contains a very clear and complete account of the physical and mathematical aspects of quantum mechanics. These two aspects are integrated in the best possible manner, with the physical reasons for developing the mathematical formalism being clearly discussed first. The book has the feeling of being "tested under fire" since small but important steps in many discussions, often causing trouble to students when left out, are carefully given in the book. This is indeed true, since the book is based on lectures taught for several years by the authors. The book opens with a historical discussion of the difficulties of classical physics and "old" quantum theory. The wave nature of free particles is considered in the next two chapters, where a careful discussion of waves and Fourier transforms leads to the free particle Schrödinger equation, the Born probability interpretation of the wave function, and the uncertainty principle. In Chapter 4 forces are introduced into the Schrödinger equation by means of the analogies between wave mechanics and optics, and one-dimensional examples are considered in Chapter 5. The ideas of linear operator, eigenvalue, and commutator brackets in quantum mechanics are then discussed in Chapter 6. The motion of a particle in a spherically symmetric force is considered in Chapter 7, and the angular momentum of such systems considered. An excellent discussion of scattering is contained in the next chapter, including an account of phase shifts, the S matrix and its relation to bound states, and the Born approximation. Matrix mechanics is then introduced in Chapter 9 and applied to angular momentum in Chapter 10. This latter chapter also discusses the electron spin in a non-relativistic fashion. Time-independent and time-dependent perturbation theory are then discussed, for both degenerate and non-degenerate systems, and applied to the Zeeman effect. A final chapter considers briefly

identical particles and the exclusion principle. It is a pity that the relativistic equation of Dirac could not be included in this already quite lengthy book. However, recent material discussed in the text, such as the relation between the S matrix and bound states, makes the book more complete in the topics considered than would have been the case in a broader discussion.

There are eight short appendices and a good index. At the end of each chapter is a list of text books for further reading, together with a brief and very useful assessment of them. There is also a set of examples at the end of each chapter which are a very searching addition to the material of the text.

The style of writing is very enjoyable to read, and the printing and format of the book are excellent. In all this is an exceedingly useful addition to the literature of quantum mechanics. *John G. Taylor* (Baltimore, Md.)

13092:

Fantappiè, Luigi. Sui fondamenti gruppali della fisica. Collect. Math. **11** (1959), 77–136.

Edited by G. Arcidiacono, M. Carafa and D. Del Pasqua from a posthumous manuscript, this work should have served as introduction to a wider investigation, left incomplete by the untimely death of the author, having the purpose of providing a unified and mathematically rigorous description of the physical world, starting from general principles of invariance and using consistently the properties of linear functionals.

It states, in clear and rather simple mathematical language, definitions and concepts on groups, invariance, quantum theory and relativity, which are familiar to physicists. *E. R. Caianiello* (Naples)

13093:

Bertotti, B. Quantum mechanics and the uniqueness of the world. Nuovo Cimento (10) **17** (1960), supplemento, 1–7.

It is not feasible to discuss a paper of this kind properly in a brief review. As the title suggests, the author is uneasy as to the adequacy of present-day quantum mechanics. For example, the probability of one of a pair of counters being activated by a particle is $\frac{1}{2}$. Once one of them, A say, has been activated, why does every observer in the laboratory agree that this is in fact the case? What is meant by an "observer"? Does this example not suggest the need for a non-probabilistic link between the particle and each observer? Yet, is not the observer himself a "probabilistic", i.e., quantum mechanical system? This is the kind of question the author discusses somewhat cursorily; but he incidentally gives the reviewer the impression of wanting both more and less operationalism at the same time. *H. A. Buchdahl* (Hobart)

13094:

Landé, Alfred. Ableitung der Quantenregeln auf nichtquantenmässiger Grundlage. Z. Physik **162** (1961), 410–412. (English summary)

Author's summary: "The general law of probability interference is only the first step to quantum mechanics; it does not yet contain wave-like periodic traits. The latter enter the theory only through additional dynamical rules for the connection between coordinates and momenta, typified by the wave function $\Psi(p, q) = \exp(2i\pi qp/h)$. This quantum-dynamical rule is shown to be derivable from a non-quantal, non-periodic requirement in invariance of certain quantities with respect to displacement of the zero point in q- and p-space."

F. H. Brownell (Seattle, Wash.)

13095:

Wesley, James Paul. Classical interpretation of quantum mechanics. Phys. Rev. (2) **122** (1961), 1932–1941.

Following de Broglie, Bohm and others, it is assumed that quantum mechanics may be interpreted causally and that the ψ function plays the role of a generating function for particle trajectories. The four-momentum of a scalar particle is assumed to be given as the gradient of an unspecified function $F(\psi)$, where ψ is a pure real solution of the Klein-Gordon equation. Since the location of a particle is determined solely by its trajectory, the probability distribution differs from $\psi\psi^*$. Particle motion and trajectories are discussed for three examples: a free particle, a particle in a box, and the double slit.

{In many respects, the reviewer cannot follow the reasonings of the author.} *H. Wakita* (Hiroshima)

13096:

Aron, Jean-Claude. Sur la représentation hydrodynamique de l'équation de Feymann et Gell-Mann. C. R. Acad. Sci. Paris **251** (1960), 921–923.

13097:

Aron, Jean-Claude. Sur un modèle physique de l'onde quantique. C. R. Acad. Sci. Paris **251** (1960), 992–994.

13098:

Aron, Jean-Claude. Sur les relations de la théorie de la diffusion avec la mécanique quantique non relativiste. C. R. Acad. Sci. Sci. Paris, **251** (1960), 1059–1061.

13099:

Aron, Jean-Claude. Sur l'introduction des idées statistiques dans la mécanique quantique des particules à spin. C. R. Acad. Sci. Paris **251** (1960), 1117–1118.

13100:

Aron, Jean-Claude. Sur la représentation hydrodynamique de l'équation de Feynman et Gell-Mann et Sur un modèle physique de l'onde quantique. C. R. Acad. Sci. Paris **251** (1960), 1723.

13101:

Bohm, D.; Hillion, P.; Vigier, J. P. Internal quantum states of hyperspherical (Nakano) relativistic rotators. Progr. Theoret. Phys. **24** (1960), 761–782.

In this paper the authors review the results achieved by themselves and others on the use of conventional methods of quantizing the classical internal motion of the hyperspherical relativistic rotor. The discussion involves the consideration of representations of the complex three-

dimensional orthogonal group and of vector spaces irreducible under complex transformations isomorphic to the full four-dimensional Lorentz group; each such space is associated with a "level" of the system. Individual vectors in the space are said to be fine-structure states. Every vector is characterized by a set of quantum members which are used to classify families of rotator levels. In the section of the paper labeled "conclusion" the authors discuss three lines of research on the application of the theory of relativistic rotators to quantum theory. *A. H. Taub* (Urbana, Ill.)

13102:

Halbwachs, F.; Vigier, J. P. Lie relations associated with relativistic rotators and bilocal theory. Nuovo Cimento (10) **16** (1960), 576–578.

It is shown that the classical Poisson brackets satisfied by the linear momenta and the angular momenta, derived from a Lagrangian describing a classical model of a relativistic rotating fluid droplet, satisfy the Lie commutation relations of the infinitesimal Lorentz group. It is also shown that the "proper" momenta and the "orbital" angular momenta, defined in terms of the linear momenta and angular momenta mentioned above and the motion of the center of gravity of the droplet, also satisfy these relations. *A. H. Taub* (Urbana, Ill.)

13103:

Araki, Huzihiro; Yanase, Mutsuo M. Measurement of quantum mechanical operators. Phys. Rev. (2) **120** (1960), 622–626.

This paper purports to give general proofs of the assertions, already noted in special cases by Wigner, that for a physical quantity whose quantum mechanical operator fails to commute with that of a universally conserved one, there is no possibility of exact measurement, but that approximate measurement is possible. Mathematical precision is lacking in this paper, particularly concerning approximate measurement; presumably this could be corrected. *F. H. Brownell* (Seattle, Wash.)

13104:

Kuhn, Harold W. Linear inequalities and the Pauli principle. Proc. Sympos. Appl. Math., Vol. 10, pp. 141–147. American Mathematical Society, Providence, R.I., 1960.

A purely mathematical discussion of the energy eigenvalue problem of an n-fermion system in which there are r-body interactions. The problem discussed is the formulation of the Pauli Exclusion Principle in terms of an r-body density matrix for $r=1$ with a brief mention of the case $r=2$. *D. ter Haar* (Oxford)

13105:

des Cloizeaux, Jacques. Extension d'une formule de Lagrange à des problèmes de valeurs propres. Nuclear Phys. **20** (1960), 321–346. (English summary)

The essential result derived in this paper is to show that a large number of eigenvalue problems that appear in quantum mechanics can be related to the calculation of the spectrum of a hermitian operator K independent of the energy. The first step is to transform such a problem, which generally depends on an implicit eigenvalue equation, into an ordinary eigenvalue problem by generalizing the Lagrange formula of complex variable theory to operators. This method enables the building up of a constant operator h which has the same eigenvalues and eigenvectors as the original equation. But, in general, h need not be hermitian, so that the eigenvalue equation involving it may not be physically significant. It is shown that this difficulty can be removed if h be replaced by the operator $K=ghg^{-1}$, where the operator g can always be so chosen as to make K hermitian. Obviously K will have the same eigenvalues as the original equation, and the eigenfunctions which are solutions of the equation with K in place of h now form an orthonormal system, the operator K thus playing the role of a veritable Hamiltonian. The first part of the paper deals with the law of formation of the operators h, g and K in terms of contour integrals in a complex plane.

The second part deals with the application of the above results to the calculation of perturbation expansions for bound states using the Brillouin-Wigner formula. In this case, the eigenfunctions associated with the operator K can be considered as good unperturbed wave functions, corresponding to the bound states of the complete system, while K itself represents the interaction between the bound states. Besides, these eigenfunctions also possess a simple geometrical interpretation.

The Bethe-Salpeter, and the Bloch-Horowitz, equations are mentioned in the introduction as belonging to the type to which the formalism of the present paper can be applied. *B. S. Madhavarao* (Poona)

13106:

Primas, H. Über quantenmechanische Systeme mit einem stochastischen Hamiltonoperator. Helv. Phys. Acta **34** (1961), 36–57. (English summary)

Author's summary: "This paper deals with the theory of quantum mechanical systems with a stochastic Hamiltonian which are of importance in the theory of dissipative systems and in experimental investigations of the response of physical systems by means of electronic devices. A new formal development of quantum mechanical density matrices is given that is valid even for strong stochastic perturbations. If the stochastic part of the Hamiltonian has a Gaussian distribution and an almost constant spectral density the given solution reduces to an expansion in terms of Hermite functionals which are orthonormal respective to the Wiener measure (Cameron-Martin development). This expansion is operationally meaningful and characterized by good convergence and simple properties. As an example of the application of the theory a new foundation of Bloch's relaxation theory is sketched." *O. Frink* (Dublin)

13107:

George, Cl. Mouvement brownien d'un oscillateur quantique. Physica **26** (1960), 453–477.

The methods of Prigogine and Toda [Molecular Phys. 1 (1958), 48–62; MR **19**, 1208], Prigogine and Ono [Physica **25** (1959), 171–178: MR **23** #B932] are applied to the relaxation of a quantum mechanical oscillator weakly coupled to a temperature bath in thermal equilibrium. An equation for the temporal evolution of the density matrix is obtained and solved by a generating function

procedure. The time dependence of the expected values of the coordinate and the momentum, as well as of their various powers and products, is shown to be simply related to that obtained for the corresponding classical calculation. *S. Prager* (Minneapolis, Minn.)

13108:

de-Shalit, A.; Weisskopf, V. F. The wave function of nuclear matter. Ann. Physics **5** (1958), 282–298.

This paper concerns itself with the construction of an approximate N-body wave function ψ which takes into account strong, short-range two-body correlation. This is done by writing ψ as the product of a Slater determinant and $\frac{1}{2}N(N-1)$ correlation amplitudes, one for each pair. These latter functions are found by solving the Bethe-Goldstone equation. It is then shown that the ground state energy computed with ψ is correct in the low-density limit. However, the significance of the approximate ψ itself is not too clear, since it is known that the wave function, being neither extensive nor intensive, is not a simple function of the volume of the system, which makes any estimate of the accuracy of ψ a very difficult and hazardous task. *K. Gottfried* (Cambridge, Mass.)

13109:

Dąbrowski, J.; Sawicki, J. The medium and high energy optical model and the correlations in the nuclear wave function. Nuclear Phys. **13** (1959), 621–641.

The influence of ground-state correlations on the optical potential for nucleon-nucleus scattering is estimated. The treatment is based on Watson's multiple scattering series, but the scattering amplitudes appearing in this series are amended in order to take the exclusion principle approximately into account. The correlations in the ground-state are described both by Jastrow's well-known trial function and by a wave function which is due to de-Shalit and Weisskopf [#13108]. The authors then find that the correlations yield corrections of order 10% to the earlier single-scattering calculation of Verlet and Gavoret [Nuovo Cimento (10) **10** (1958), 505–519] and that double-scattering corrections are somewhat larger than this. From the purely theoretical point of view the method has the drawback of depending on approximate wave functions, instead of evaluating the optical potential directly without the intermediary of a wave function. The use of wave functions in this problem is open to some doubt because the optical potential is not stationary, and (convergent) perturbation methods for many-particle wave functions remain to be developed.

K. Gottfried (Cambridge, Mass.)

13110:

Bowcock, J.; Walecka, D. Analytic properties of the Schrödinger amplitude at a fixed angle. Nuclear Phys. **12** (1959), 371–388.

Authors' summary: "The scattering from a given class of potentials including exchange forces is considered. For a fixed angle the scattering amplitude can be analytically continued into the complex energy plane with the exception of the usual cut along the positive real axis and certain poles and cuts on the negative real axis depending on the nature of the potential. Our results are a partial confirmation of the fixed angle dispersion relations recently proposed within the framework of field theory."

O. Hara (Minneapolis, Minn.)

13111:

Omnès, R. Démonstration des relations de dispersion. Relations de dispersion et particules élémentaires (Grenoble, 1960), pp. 317–385. Hermann, Paris; Wiley, New York; 1960.

This is a set of notes of a course given at Les Houches 1960, based on the work of Bros, Froissart, Omnès and Stora. It deals with the proof of dispersion relations for the vertex function and for the process $A+B \to C+D$ where the masses are all different. Although the results have been mostly obtained before using, say, the method of H. Lehmann suitably modified, the work under review is the most elegant and probably the most rigorous demonstration published. The article contains a clear discussion of the variational derivative of the S-matrix, and of the vertex function in perturbation theory, and outlines the usual method of attack in the problem of proving dispersion relations. The rest of the article, the major part, is an original approach in terms of distribution theory and analytic functions. A number of small points, omitted in previous proofs, are tidied up. The main point of the "new" proof, however, is a method for finding the singularities of the Dyson representation using complex geometry in three dimensions, rather than the usual pedestrian way using algebra. The proof of the Dyson representation is omitted, since it was given in the chapter by A. S. Wightman in the same book. The article concludes with a summary of rules for determining whether dispersion relations can be proved or not; a list of provable relations is not given.

R. F. Streater (Upton, N.Y.)

13112:

Cehmistrenko, Yu. V. On the structure of the S matrix in the theory of elastic and inelastic scattering of non-relativistic particles. Ž. Èksper. Teoret. Fiz. **38** (1960), 1237–1244 (Russian. English summary); translated as Soviet Physics. JETP **11**, 894–898.

Author's summary: "Integral relations for the components of the S matrix describing a nuclear reaction with two channels (one channel is the elastic scattering of a nonrelativistic particle and the other is inelastic scattering with excitation of the nucleus) are derived from general principles of causality, unitarity, and symmetry, and the analytic properties of some components of the S matrix are established. For simplicity the treatment is confined to the case of spherically symmetrical scattering. In agreement with the results of Wigner [Phys. Rev. (2) **73** (1948), 1002–1009] and of Baz' [Ž. Èksper. Teoret. Fiz. **33** (1957), 923–928], the excitation function of the elastic scattering has a break at the threshold of the inelastic process. The form of the excitation function of the inelastic process near threshold is found in the general case."

13113:

Otokozawa, Jun. New analyses of anomalous imaginary part. Progr. Theoret. Phys. **25** (1961), 277–289.

Author's summary: "Using a new method of analytic continuation and the knowledge of parameter integration, anomalous imaginary part is written explicitly with the absorptive part (concerning t) of matrix element which connects initial or final state and intermediate one. The meaning of intermediate state is more clarified. In the case of π-meson deuteron scattering, all the singularities

are determined. It is confirmed that these singularities agree with those obtained by performing the direct parameter integration of fourth order diagram."

John G. Taylor (Baltimore, Md.)

13114:

Aaron, Ronald; Amado, Ralph D.; Lee, Benjamin W. Divergence of the Green's function series for rearrangement collisions. Phys. Rev. (2) **121** (1961), 319–323.

Authors' summary: "The convergence of the Born series for rearrangement collisions is investigated in a potential model. For a certain class of potentials it is shown that the iterated series for the full two-particle Green's function, $(\mathbf{k}_1'\mathbf{k}_2'|G(E)|\mathbf{k}_1\mathbf{k}_2)$, in terms of either the free-particle Green's function, the initial state Green's function, or the final state Green's function, diverges for some continuous range of the variables $\mathbf{k}_1$, $\mathbf{k}_2$, $\mathbf{k}_1'$, and $\mathbf{k}_2'$ independent of the energy, E, of the incident particle. It is suggested that the usual Born series, which is an integral over this Green's function series, therefore, also diverges for rearrangement collisions independent of the incident energy."

13115:

Barut, A. O.; Ruei, K. H. Kinematical and dynamical resonances. Phys. Rev. (2) **122** (1961), 1340–1342.

The authors suggested [Nuclear Phys. **21** (1960), 300–309; MR **22** #9118] that there is a distinction between kinematical resonances due to unstable particles, and dynamical resonances due to the nature of the forces between the initial particles. Whether the kinematical resonances are there or not cannot be determined theoretically, but must be put into the theory; for example, there are extra resonant solutions which satisfy the system of dispersion relations. Thus the question "which particles are elementary" is generalized to "which resonances are kinematical". It is suggested that the two types of resonance may be distinguished by a different dependence of the energy of resonance on the coupling constant in the two cases; this is illustrated for resonant scattering in a potential, and scattering through an unstable intermediate state. In the former case it is found that the number and energy of the resonances increases with the coupling constant, in analogy to bound states, while for kinematical resonances quite a different behaviour is found.

R. F. Streater (Upton, N.Y.)

13116:

Dalitz, R. H.; Tuan, S. F. The phenomenological representation of $\bar{K}$-nucleon scattering and reaction amplitudes. Ann. Physics **10** (1960), 307–351.

From the authors' summary: "An explicit parametrization of the $\bar{K}$-N scattering and reaction amplitudes, satisfying the unitarity conditions and including the implications of time-reversal invariance, is given in terms of the elements of a suitably-defined reaction (or K-) matrix. Plausible approximations then lead to the formulas of a 'zero-range' theory, an extension of that introduced by Jackson et al. [J. D. Jackson, D. G. Ravenhall and H. W. Wyld, Nuovo Cimento (10) **9** (1958), 834–841], in which all the low-energy $\bar{K}N$ cross sections are expressed in terms of two complex scattering amplitudes (A_0, A_1), a phase angle ϕ_t and a real parameter ε. The zero-energy and 175 Mev/c K^--p data allow four possible sets $(a^\pm)$, $(b^\pm)$ of these parameters; the Coulomb-nuclear interference in K^--p scattering favors (a^+) and (b^+). The most important qualitative conclusion based on this formalism is that the data indicates that the KYN coupling is so strong that perturbative methods are in general quite inadequate for the calculation of K-particle processes. The relationship of pion-hyperon (πY) scattering cross sections with the $\bar{K}N$ data is discussed in detail and it is shown that the parameter sets (a^-) or (b^-) would require the existence of a πY resonant state not far below the K^--p threshold. After determination of expressions for the $\bar{K}N\to\pi Y$ amplitudes, the energy dependence of the Σ^-/Σ^+ and Λ/Σ^0 ratios are discussed, together with some remarks on the $K^-p\to\Lambda\pi\pi$ cross section. The Coulomb corrections to the nuclear K-matrix elements are derived, as well as the modification of the corresponding T-matrix elements by the K^--$\bar{K}^0$ mass difference through the unitarity condition. These corrections are far from negligible and lead to appreciable modification of the parameter sets $(a^\pm)$, $(b^\pm)$. Each of these sets fits the Σ^-/Σ^+ ratios at zero energy and in the range 100-200 Mev/c; the (a^+) and (b^-) sets require a strong upward cusp in this ratio at the $\bar{K}^0$-n threshold, the (a^-) and (b^+) sets a downward S-shaped cusp. Taking all the present data together, the (a^+) set appears favored. A brief discussion of the advantages and disadvantages of the K-matrix formalism is given in the appendix."

G. Källén (Lund)

13117:

Bowcock, J.; Cottingham, W. N.; Lurié, D. Effect of a pion-pion scattering resonance on low energy pion-nucleon scattering. Nuovo Cimento (10) **16** (1960), 918–938. (Italian summary)

By the Cini-Fubini approximation [Ann. Physics **10** (1960), 352–389; MR **22** #11882] of the Mandelstam representation, the effect of a $J=1$, $T=1$ two-pion resonance on low energy pion-nucleon scattering is discussed. Approximation is made to the invariant amplitudes $A^{(\pm)}(s, t)$ and $B^{(\pm)}(s, t)$ in the region of low energy and low-momentum transfer. This adds to the fixed-momentum transfer relations in the representation used by Chew, Goldberger, Low, and Nambu [Phys. Rev. (2) **106** (1957), 1337–1344; MR **19**, 920] a strongly t-dependent term representing the contribution of inelastic pion-nucleon scattering. The paper is mainly analytical and approximative. The fit with experimental pion-nucleon phase shifts and nucleon electromagnetic form factors is discussed.

C. Strachan (Aberdeen)

13118:

Capps, Richard H.; Nauenberg, Michael. Relativistic pion-hyperon dispersion relations. Phys. Rev. (2) **118** (1960), 593–602.

This is a derivation (without proof) of the fixed momentum transfer dispersion relations for pion scattering by hyperons similar to the classic conjecture of M. L. Goldberger [Phys. Rev. (2) **99** (1955), 979–985] making use of "causality" and "crossing symmetry". Approximate equations are written down for s and p waves; leading to coupled singular integral equations. A brief discussion of the behaviour of the p-wave amplitudes is given and it is concluded that the K-N states may play an important role, a conjecture which derives additional support from

the theory worked out by R. Warnock [Thesis, Harvard Univ., 1959] within a Lagrangian frame-work.

E. C. G. Sudarshan (Rochester, N.Y.)

13119:

Berestetsky, V. B.; Pomeranchuk, I. Ya. On the asymptotic behaviour of cross sections at high energies. Nuclear Phys. **22** (1961), 629–639.

It is usual to assume that high-energy total and elastic cross-sections tend to constant limits as the energy is increased. This paper presents arguments which suggest a different asymptotic behaviour. Certain contributions to the inelastic scattering of a high-energy particle by a target nucleon are evaluated. The first contribution discussed is that which arises from elastic scattering on a quasi-free pion in the long-range part of the pion cloud of the nucleon. Provided attention is limited to sufficiently long-range effects, this process can be properly treated by breaking the incoming state into partial waves, taking for each partial wave just the contribution of the one-pion exchange term in the pole approximation, and summing over all partial waves whose angular momentum exceeds some suitable value l. For l the value p/μ is chosen (p being the c.m.s. momentum and μ the pion mass). The authors state the useful rule that the result of a calculation performed in this way may be reproduced more directly by taking the pole term in momentum representation, and summing over all final states such that the spacelike momentum t carried by the intermediary pion satisfies the inequality $|t| \lesssim \mu$. The latter calculation is then performed explicitly, and the resultant contribution to the inelastic scattering is found to be equal to the cross-section for elastic scattering on a pion, multiplied by a factor f^2/π (f being the usual pion-nucleon coupling constant). Because the partial-wave cross-sections are positive and additive, this sets a lower bound to the total inelastic cross-section. This bound is evidently compatible with constant asymptotic behaviour.

A similar method is then used to analyse the contribution of a "two-jet process". In such a process, the incoming high-energy particle emits two pions. One of these materializes in the final state, along with the incoming particle. The other pion, carrying a spacelike momentum t, scatters elastically from the nucleon target, and materializes along with the latter. Again treating the intermediate pion in the pole approximation, and summing over all final states such that $|t| \lesssim \mu$, a cross-section contribution is found, proportional to the product of the two elastic cross-sections concerned, multiplied by the logarithm of the incoming energy. The authors expect that the remaining contributions to the cross-section are additive to this one. Provided this unproved expectation turns out to be correct, the calculations thus prove a lower bound which contradicts a constant asymptotic inelastic cross section. It then becomes necessary to suppose that both total and elastic cross-sections tend to zero somewhat faster than a reciprocal logarithm at sufficiently high energies. [Cf. V. N. Gribov, Nuclear Phys. **22** (1961), 249–261; MR **22** #9114.]

G. R. Allcock (Liverpool)

13120:

Dennery, Philippe; Kroll, Norman M. On conservation of probability in the Lee model. Nuclear Phys. **21** (1960), 276–299.

The field-theoretical model suggested by T. D. Lee [Phys. Rev. (2) **95** (1954), 1329–1334; MR **16**, 317] has been the subject of many investigations. It has been shown that the original formulation of the model violates conservation of probability when all renormalizations have been performed [G. Källén and W. Pauli, Mat.-Fys. Medd. Danske Vid. Selsk. **30** (1955), no. 7; MR **17**, 927]. Afterwards, it was suggested by Heisenberg [Nuclear Phys. **4** (1957), 532] that one can overcome this difficulty by a suitable choice of the parameters in the model. In the terminology of Heisenberg, one introduces a "dipole ghost". It is characteristic for Heisenberg's technique that one adjusts the parameters of the model in such a way that certain simple scattering processes do conserve probability. However, the model contains only a small number of parameters and one has to hope that more complicated scattering problems involving several particles automatically conserve probability when the necessary adjustments are made for the simpler processes. It was conjectured by Heisenberg in the paper mentioned above that this would actually be the case. The authors of the paper under review here undertake to investigate this conjecture in some detail. To this purpose they have to consider scattering problems involving several heavy particles. They first show that when one has two heavy particles at a fixed distance from each other, one always obtains states with negative norms if the parameters are adjusted so as to introduce the dipole ghost. One might possibly object to this result that it is unrealistic to consider the particles as infinitely heavy and as having a fixed distance when one wants to consider scattering problems. To overcome this objection the authors devote the rest of their paper to a modification of the model where also the heavy particles have a kinetic energy. Their treatment is based on the adiabatic approximation (the Born-Oppenheimer method) where one first computes a potential between particles at a fixed distance and afterwards solves a Schrödinger equation for particles moving in this potential. This approximation is good when the particles are moving slowly, i.e., when they are heavy. Using this technique, the authors first prove that the model (with the parameters adjusted as above) always contains a bound state between two heavy particles. Further, this state has a negative norm. Once this result is established it is nearly self-evident that probability conservation fails in a scattering process where this bound state can appear among the outgoing particles. One appendix is devoted to an argument pointing out in detail where the equations on which Heisenberg based his conjecture have to be modified.

G. Källén (Lund)

13121:

Weidlich, W. On the axioms of quantum field theory. Nuovo Cimento (10) **19** (1961), 277–291. (German and Italian summaries)

The author proposes a new set of axioms for quantum field theory. The canonical commutation relations constitute an important portion of the axioms. As a result of this, the conventional assumptions about the transformation property and the local commutativity of the field are sacrificed and replaced by weaker conditions: "σ-local" and "quasi-local" natures of the field. The formalism is illustrated by the non-relativistic case but the author's main interest lies, of course, in the relativistic case.

H. Araki (Kyoto)

13122:

Rollnik, Horst. Operatoreichtransformationen in der Quantenelektrodynamik. Z. Physik **161** (1960/61), 370–379. (English summary)

This paper discusses carefully and systematically the so-called operator gauge transformations in quantum electrodynamics and gives the transformation properties of the renormalized field operators, of the renormalization constants and of the renormalized propagators. These results are not completely new [see, e.g., B. Zumino, J. Math. Phys. **1** (1960), 1–7; MR **22** #6505]; the main virtue of the paper being reviewed is that the discussion is given within the framework of the familiar Gupta-Bleuler operator formalism. (The previous discussions made use of functional methods or of the Coulomb gauge formulation of the theory; this reviewer still feels that the Coulomb gauge provides the most natural operator description of quantum electrodynamics.) The advantage of formulating the operator gauge transformations as done here is that they can be used to establish the equivalence between different forms given for the asymptotic condition in the Gupta-Bleuler formalism [H. Rollnik, B. Stech, and E. Nunnemann, Z. Physik **159** (1960), 482–494; MR **22** #2359]. In this way the author has considerably classified the formal aspect of the asymptotic condition in quantum electrodynamics.

B. Zumino (New York)

13123:

Bogoliubov, N. N.; Medvedev, B. V.; Tavkhelidze, A. N. The application of the methods of N. I. Muskhelishvili to the solution of singular integral equations in quantum field theory. Problems of continuum mechanics (Muskhelishvili anniversary volume), pp. 39–55. SIAM, Philadelphia, Pa., 1961.

This is a translation of an expository article, written for mathematicians rather than physicists. The authors point out that the solution of the integral equations of dispersion theory can be effected by using the techniques of Muskhelishvili [*Singular integral equations*, Noordhoff, Groningen, 1953; MR **15**, 434]. The methods are illustrated by short sketches of the solution of the problem of pion photo-production and the decay of the neutral K-meson.

C. A. Hurst (Adelaide)

13124:

Yamamoto, Kunio. Integral representation of absorptive part of vertex function. Progr. Theoret. Phys. **25** (1961), 361–368.

Recently R. Oehme has given an integral representation of the vertex function with the aid of the Jost-Lehmann-Dyson formula for a retarded, causal commutator [R. Oehme, Phys. Rev. (2) **117** (1960), 1151–1159; MR **22** #5384; F. J. Dyson, ibid. **110** (1958), 1460–1464; MR **20** #1537]. The result of Oehme can be expressed in the following way:

$$F(z_1, z_2, z_3) = \frac{1}{\pi}\int_{c^2}^{\infty} d\sigma^2 \frac{A(z_1, z_2, \sigma^2)}{\sigma^2 - z_3},$$

$$A(z_1, z_2, \sigma^2) = \int du \int d\kappa \frac{\chi(\kappa, u_0, |\bar{u}|, \sigma^2)}{\kappa^2 - (q-u)^2}.$$

Here, $F(z_1, z_2, z_3)$ is the vertex function (in p-space) expressed as an analytic function of three complex variables corresponding to the three scalar products which can be formed out of two vectors. The quantity u is a four-vector and κ is a scalar variable of integration. Finally, q is another four-vector related to the two variables z_1 and z_2 by

$$q_0 = (z_1 - z_2)/2\sigma,$$

$$\bar{q}^2 = (z_1^2 + z_2^2 + \sigma^4 - 2z_1z_2 - 2\sigma^2 z_1 - 2\sigma^2 z_2)/4\sigma^2.$$

This representation is derived under the assumption that both z_1 and z_2 are real and negative. In the paper reviewed here, the author remarks that if one performs the integration over the angles of the vector $\bar{u}$, one gets a result which is identical with an expression found in perturbation theory, viz.,

$$A^P(z_1, z_2, \sigma^2; m_1, m_2, m_3) = \int_0^1 d\alpha\, d\beta\, d\gamma\, \delta(1-\alpha-\beta-\gamma)$$
$$\times\, \delta(\alpha m_1^2 + \beta m_2^2 + \gamma m_3^2 - z_1\beta\gamma - z_2\gamma\alpha - \sigma^2\alpha\beta).$$

Also this result is true for z_1 and z_2 real and negative. Consequently, the Oehme representation can be written

$$A(z_1, z_2, \sigma^2) = \int d^3m\, \varphi(\sigma; m_i) A^P(z_1, z_2, \sigma^2; m_i).$$

The weight function φ is essentially the same function as χ above. It has certain support properties in the space of the masses m_i and the variable σ. The author shows that these support properties correspond exactly to what one expects from a direct analogy with perturbation theory.

An interesting example of an analytic function with regularity properties corresponding to those of the vertex function has been given by R. Jost [Helv. Phys. Acta **31** (1958), 263–272]. The author of the present paper seems to be under the impression that this example of Jost's cannot be represented by the Oehme formula. The reviewer cannot understand this point, as the Jost example has analyticity properties sufficient to guarantee that it is a possible vertex function. Further, the analyticity properties of the Oehme formula were discussed in some detail by Oehme in the paper mentioned above. There is certainly no contradiction between this representation and the analyticity necessary for the general vertex function [G. Källén and A. Wightman, Mat.-Fys. Skr. Danske Vid. Selsk. **1** (1958), no. 6, 1–58; MR **22** #3470].

G. Källén (Lund)

13125:

Konisi, Gaku; Yamamoto, Kunio. Transition amplitudes in perturbation theory and Dyson's integral representation. Progr. Theoret. Phys. **25** (1961), 461–466.

This paper is an elaboration of the technique earlier used by one of the authors to rewrite the Oehme representation of the vertex function [cf. the review above and the reference given there]. In particular, the authors show that the Dyson representation for a retarded commutator can be interpreted as a superposition of lowest-order diagrams corresponding to a two-particle scattering process.

G. Källén (Lund)

13126:

Lukierski, J. On the interpretation of isovector components. Bull. Acad. Polon. Sci. Sér. Sci. Math. Astr. Phys. **7** (1959), 577–581. (Russian summary, unbound insert)

The author considers the six-parameter group, Λ', formed from the operations of charge conjugation (C) and mass conjugation ($M=\gamma^5$). This group was first discussed by W. Pauli [Nuovo Cimento (10) **6** (1957), 204–215; MR **19**, 612] for the neutrino field and by F. Gürsey [ibid., letter, **7** (1958), 411–415] for the massed nucleon field. The group commutes with the proper Lorentz group Λ. One may form six real scalar quantities Y_A ($A=1, \cdots, 6$) from the quantities $\bar\psi\psi$, $\bar\psi\gamma^5\psi$, $\bar\psi C\psi$, etc., where ψ represents a four-component Dirac field. Similar vector quantities $Y_{\mu A}$ may be constructed from $\bar\psi\gamma^\mu\psi$, etc. In the classical theory, two of the $Y_{\mu A}$ vanish and so A may be viewed as an index representing a four-dimensional Minkowskian "isotopic" space. Transformations in Λ affect the vector index μ while transformations in Λ' only affect A.

If one assumes that ψ represents a charged field coupled to the electromagnetic field, then the author shows that the field equations are invariant only for rotations in the (1, 2) "isotopic" plane. For a chargeless particle, the field equations are invariant under rotations in the (1, 2, 4) subspace of the Minkowski "isotopic" space. For a massless charged particle, the field equations are invariant under rotations in the (3, 4) "isotopic" plane. Finally, a classical neutrino field would be invariant under the full group Λ'. Only couplings to the electromagnetic field are considered. *R. Arnowitt* (Syracuse, N.Y.)

13127:

Kessler, Paul. La méthode des processus quasi réels en électrodynamique quantique. C. R. Acad. Sci. Paris **249** (1959), 2162–2164.

In previous work [D. Kessler and P. Kessler, Nuovo Cimento (10) **4** (1956), 601–609] a justification of the Weizsacker-Williams method of treating the electromagnetic scattering of a charged particle off a complex system was given in terms of Feynman diagrams. Here the initial and final charged particles were real (and ultra-relativistic) with a virtual photon connecting them to the complex system. The cross-section was then related to the cross-section for an incident photon on the complex system. In this paper, the author considers two further situations: (a) a real incident charge, a real emitted photon, a virtual final charge connecting with the complex system; (b) a real final charge, a real emitted photon and an incident virtual charge (into the electromagnetic vertex) emitted by the complex system. Process (a) is related to the cross-section of a real charge going into the complex system while (b) is related to the cross-section for a real charge emitted by the complex system. The analysis is again performed by examining the Feynman diagrams of the related processes and depends only on the charged particles being ultra-relativistic. *R. Arnowitt* (Syracuse, N.Y.)

13128:

Kessler, Paul. La méthode des processus quasi-réels en électrodynamique quantique. Cahiers de Phys. **14** (1960), 41–54. (English and German summaries)

The semiclassical method of Williams and Weizsacker is generalized to the study of relativistic processes in quantum electrodynamics. Its field-theoretical generalization consists in defining probabilities for partial virtual ("quasi-real" in the author's terminology) processes, such as scattering of relativistic particles with emission of a real or virtual photon, or relativistic pair creation by a photon.

The method proposed amounts to a criterion for suppressing well-defined classes of Feynman diagrams. Examples of application are discussed in detail. *E. R. Caianiello* (Naples)

13129:

Källén, G.; Wilhelmsson, H. Generalized singular functions. Mat.-Fys. Skr. Danske Vid. Selsk. **1**, no. 9, 27 pp. (1959).

This paper gives one-parameter integral representations for the so-called generalized singular functions defined by

$$\Delta_{n+1}^{(+)}(x_1, \cdots, x_n; a_{kl}) = \frac{(-i)^n}{(2\pi)^{3n}} \int \cdots \int dp_1 \cdots dp_n \times \exp\left(i \sum_{k=1}^{n} p_k x_k\right) \prod_{k \leq l} \delta(p_k p_l + a_{kl}) \theta(p_k).$$

Here $x_1, \cdots, x_n$ and $p_1, \cdots, p_n$ are real four-vectors, $p_k p_l = \mathbf{p}_k \cdot \mathbf{p}_l - p_{k0} p_{l0}$ and a_{kl} is a real symmetric matrix satisfying certain positivity conditions. $\theta(p) = \frac{1}{2}(1 + \operatorname{sgn} p^0)$. It was previously known that all $\Delta_{n+1}^{(+)}$ for $n > 5$ could be expressed in terms of $\Delta_5^{(+)}$ and further that $\Delta_2^{(+)}$ and $\Delta_3^{(+)}$ are certain combinations of Hankel functions and algebraic functions. The authors here begin by giving an explicit formula for $\Delta_{n+1}^{(+)}$ in terms of $\Delta_5^{(+)}$ and expressions for $\Delta_4^{(+)}$ and $\Delta_3^{(+)}$ in terms of $\Delta_5^{(+)}$. The rest of the paper is a rather fancy piece of integration leading to the representation

$$\Delta_5^{(+)} = (2\pi)^{-9}(-D)^{-1/2} \tfrac{1}{4} \int_{-\infty}^{\infty} dt \cdot t H_0^{(1)}(t) \{[\Xi_1]^{-1/2} + [\Xi_2]^{-1/2}\} \times \theta(-D)\theta(D_0)\theta(a_{12}^2 - a_{11}a_{22}) \prod_{k=2}^{4} \theta(a_{1k}).$$

Here

$$D = \det(a_{jk}) \quad (j, k = 1, 2, 3, 4),$$
$$D_0 = \det(a_{jk}) \quad (j, k = 1, 2, 3),$$
$$\Xi_{1,2} = [(t^2 - Q)^2 - R \pm T^{1/2}]^2 - t^2 S \mp 8 T^{1/2} t^2 (t^2 - Q),$$

where, up to constant factors, Q, R, S, T are the elementary symmetric functions of the eigenvalues of the matrix $\sum_{l=1}^{4} a_{jl} x_l x_k$. These integrals can be understood as $\int \exp(i \operatorname{tr} \Lambda M) d\Lambda$, where M is a 4×4 matrix, Λ is a restricted Lorentz transformation, and $d\Lambda$ is Haar measure on the Lorentz group. Thus, the paper makes a nontrivial contribution to the theory of a certain class of special functions of many complex variables, analogous to the Bessel functions of matrix argument. *A. S. Wightman* (Princeton, N.J.)

13130:

Dell'Antonio, G. F.; Gulmanelli, P. Asymptotic conditions in quantum field theories. Nuovo Cimento (10) **12** (1959), 38–53. (Italian summary)

Axiomatic formulations of quantum field theory such as that of H. Lehmann, K. Symanzik and W. Zimmermann [Nuovo Cimento (10) **1** (1955), 205–225; MR **17**, 219] have assumed the existence of "asymptotic states", i.e., states in which all the individual particles are widely separated in the remote past and remote future. The authors generalize a result of Haag [Phys. Rev. (2) **112** (1958), 669–673; MR **20** #6296] that the existence of asymptotic states is equivalent to the statement

$$\lim_{R\to\infty} R^m\langle 0|C_1(x_1)\cdots C_n(x_n)|0\rangle_T = 0$$
$$(x_1^0 = x_2^0 = \cdots = x_n^0).$$

R is the radius of the smallest sphere enclosing all the points $x_1, \cdots, x_n$, $C_i(x)=\int g(x-y)A_i(y)dy$, $g(x-y)$ is an infinitely differentiable Schwartz function and $A_i(y)$ is a field operator.

The spatial asymptotic behaviour can be estimated from an integral representation of vacuum expectation values due to Källén and Wilhelmsson [#13129 above]. It is shown that if the usual assumptions of Lorentz invariance, causality, etc., are made, then the Haag condition is satisfied. A further condition that no particles of zero rest mass may be created is also imposed, but the authors comment in a footnote that only a weaker condition is necessary. The first condition implies the absence of long-range forces, and is physically reasonable. The analysis depends on a detailed discussion of the structure of the integral representations. *C. A. Hurst* (Adelaide)

13131:

Montaldi, E. Connections between generalized singular functions and Bessel functions. Nuovo Cimento (10) **12** (1959), 571–592. (Italian summary)

In the theory of vacuum expectation values of products of field operators, the so-called "generalized singular functions" or integrals of the form

$$\int\cdots\int dp_1\cdots dp_r\,\delta(p_1^2+1)\prod_{j=2}^{r}\delta(p_j^2-1)$$
$$\times\prod_{h<k=1}^{r}\delta(p_hp_k)\theta(p_1)\exp\Big[\sum_{s=1}^{r}\alpha_sp_sx_s\Big]$$

are of a certain interest. So far, these integrals have been studied for the physically interesting case that one integrates over four-dimensional vectors in a Lorentz space [#13129 above]. The author of the present paper studies the more general mathematical problem of investigating the properties of similar integrals in a space with Lorentz metric but with an arbitrary number of space dimensions.

These integrals have also been investigated earlier in a space with Euclidean metric and an arbitrary number of dimensions by S. Bochner, Med. Lunds Univ. Mat. Sem. Suppl. (1952), 12–20 [MR **15**, 422].

Montaldi gives explicit formulae for the cases $r=1$ and 2, i.e., for one and two external vectors. The result can be expressed with the aid of simple integrals over Hankel functions. In the more general case, the author does not give explicit results, but writes down reduction formulae which show the relation between these integrals and Bessel functions. It is stated that these formulae can be used to derive new integral relations between various Bessel functions. *G. Källén* (Lund)

13132:

Deser, Stanley; Gilbert, Walter; Sudarshan, E. C. G. Structure of the vertex function. Phys. Rev. (2) **115** (1959), 731–735.

The authors consider the vertex function in quantum field theory, and try to find an integral representation which gives the most general function satisfying the usual properties of Lorentz invariance, causality and positive mass-spectrum. The representation they obtain is equivalent to those of F. Dyson [Phys. Rev. (2) **111** (1958), 1717–1718; MR **20** #7535], M. Ida [#13136] and V. Ya. Faĭnberg [#13133], and does not give the most general form for a function satisfying the given conditions, being wrong in perturbation theory. This is discussed by A. Minguzzi and the reviewer [#13135].

In the first derivation given by the authors, they incorporate causality by putting on sufficient, but not necessary, conditions on the spectral function; examples from perturbation theory do not satisfy these conditions. In the appendix where a proof is attempted, the authors find it necessary to impose extra conditions on the spectral function to enable them to invert the order of certain integrations. It is here that the loss in generality arises.

The authors were able to incorporate the details of the mass spectrum, but the Jacobi identity is not satisfied. The representation leads to a domain of holomorphy in two invariants, the other being fixed at the physical mass. Although this domain has little physical significance, the authors were able to reproduce some well-known limits on the validity of present proofs of dispersion relations, and also obtained the anomalous thresholds.

R. F. Streater (Princeton, N.J.)

13133:

Faĭnberg, V. Ya. On analytic properties of causal commutators. Ž. Èksper. Teoret. Fiz. **36** (1959), 1503–1508 (Russian); translated as Soviet Physics. JETP **9**, 1066–1069.

The author studies integral representations of distributions in a four-vector variable, q, whose Fourier transforms $f(x)$ vanish for space-like x (i.e., $(x^0)^2-(x^1)^2-(x^2)^2-(x^3)^2<0$). His first result is a simple formal derivation of Dyson's integral representation [F. J. Dyson, Phys. Rev. (2) **110** (1958), 1460–1464; MR **20** #1537]

$$f(q) = \int_0^\infty d\kappa^2\int d\sigma_\alpha\left[\psi(\kappa^2, u), \frac{\partial}{\partial u_\alpha}\right]$$
$$\times\operatorname{sgn}(q^0-u^0)\delta((q-u)^2-\kappa^2),$$

where the $\int d\sigma_\alpha$ integral is over an arbitrary space-like surface of element $d\sigma_\alpha$ and ψ is an arbitrary weight. The author expresses the view that Dyson did not prove the necessity of his statements on the weight function $\psi(\kappa^2, u)$. {The reviewer disagrees.} Next, the author obtains a second formula of Dyson using the Lorentz invariance of the amplitude

$$f(q) = \int_0^\infty d\kappa^2\int d^4u\Phi(\kappa^2, u)\operatorname{sgn}(q^0-u^0)\delta((q-u)^2-\kappa^2),$$

where Φ is a certain weight. For the case in which $f(x)$ depends only on x^2 and px, where p is a time-like vector, or on x^2, px and $p'x$, where p and p' are time-like, he obtains simpler representations, e.g., in the former case

$$f(q) = \int_0^\infty d\kappa^2\int_{-\infty}^{\infty} d\alpha\rho(\kappa^2, \alpha, p^2)$$
$$\times\operatorname{sgn}(q^0-\alpha p^0)\delta((q-\alpha p)^2-\kappa^2).$$

In the remainder of the paper support properties of the weight functions in these representations are stated and the results applied to matrix elements of the form

$$\langle 0|[A(x/2), B(-x/2)]|p\rangle,\quad \langle p|[A(x/2), B(-x/2)]|p'\rangle.$$

(The simplicity of the derivations illustrates the power of the author's method; the second part of the paper, its weakness, because the most important results obtained there are wrong; in simple practical cases functions occur which, while representable by his formulae, do not have as small supports as the author asserts. For a discussion of this point see A. Minguzzi and R. Streater [#13135].)

A. S. Wightman (Princeton, N.J.)

13134:

Parasyuk, O. S. A theorem on the analytical continuation of generalized functions. Ukrain. Mat. Ž. **11** (1959), 328–330. (Russian)

This theorem is concerned with the continuation of a Lorentz-invariant function of a four-vector variable $q=(q_0, q)$, which is analytic in the timelike tube $(|\mathrm{Im}\, q_0| > |\mathrm{Im}\, q|)$, and also at certain real points. Such a theorem is important in extending the validity of the dispersion relations in quantum field theory, and is similar to that used by Deser, Gilbert and Sudarshan [#13132]. It also suffers from the defect that it is not valid for all functions satisfying the conditions of the theorem, and, in particular, counterexamples can be constructed from perturbation theory, as discussed by Minguzzi and Streater [#13135].

John G. Taylor (Orsay)

13135:

Minguzzi, A.; Streater, R. F. Some remarks on the vertex function. Phys. Rev. (2) **119** (1960), 1127–1128.

Authors' summary: "It is noted that the integral representations for matrix elements derived in three papers, (1) Dyson [Phys. Rev. (2) **111** (1958), 1717–1718; MR **20** #7535], (2) Deser, Gilbert, and Sudarshan [#13132], and (3) Faĭnberg [#13133], do not give the most general functions satisfying the conditions stated. It is shown that in (1) and (2) the convergence of certain integrals was not treated rigorously, while in (3) there was a misunderstanding of the Jost-Lehmann-Dyson representation."

John G. Taylor (Paris)

13136:

Ida, Masakuni. Integral representations of Bethe-Salpeter amplitudes. Progr. Theoret. Phys. **23** (1960), 1151–1156.

An integral representation for the matrix element of the retarded commutator of two field operators between vacuum and a one-particle state is derived. The representation and its support agree with that obtained earlier by Deser, Gilbert and Sudarshan [#13132]. Both derivations make use of the Dyson representation of double retarded commutators. As stated in the paper referred to, the representation is then, strictly speaking, valid only for spectral functions which are Schwartz distributions. However, it is interesting to note that perturbation theory does satisfy the representation in all orders [N. Nakanishi, Progr. Theoret. Phys. **24** (1960), 1275–1295].

S. Deser (Waltham, Mass.)

13137:

Eftimiu, C.; Klarsfeld, S. On the integral representation of the double commutator. Nuovo Cimento (10) **17** (1960), 902–907. (Italian summary)

By formal manipulations the authors arrive at Dyson's representation for the double commutator [Phys. Rev. (2) **111** (1958), 1717–1718; MR **20** #7535] which is not the most general form for functions with the given properties [see A. Minguzzi and R. F. Streater, #13135]. Loss of generality occurs in restricting the range of an integration to be bounded (Eq. (2.12)) which is not justified because the weight-function in the Jost-Lehmann-Dyson representation is not unique. The method of incorporating the Jacobi identity is probably not the most general, since it leads to W-functions depending on two complex variables instead of three.

R. F. Streater (Princeton, N.J.)

13138:

Deser, S.; Gilbert, W.; Sudarshan, E. C. G. Structure of the forward scattering amplitude. Phys. Rev. (2) **117** (1960), 266–272.

Authors' summary: "The matrix elements of various products of two currents between states of equal energy-momentum are studied. Use of the axioms of local field theory leads to an integral representation for the Fourier transform of matrix elements of the retarded commutator in terms of two invariant momentum parameters. Further restriction on the class of allowed functions permits explicit incorporation of the mass restrictions as support conditions on the weight function. The 'bound-state' term is separated off and related to the vertex functions. As a simple application, forward-scattering dispersion relations are derived by specialization of the Green's function. In particular, these can be obtained for nucleon-nucleon and K-meson-nucleon scattering."

R. Arnowitt (Syracuse, N.Y.)

13139:

Fairlie, D. B. On the Ruijgrok-Van Hove model. Physica **27** (1961), 95–98.

The model by Ruijgrok and Van Hove [Physica **22** (1956), 880–886; MR **18**, 626] of an exactly soluble quantum field theory was studied in further detail by Ruijgrok [ibid. **24** (1958), 185–204, 205–213; **25** (1959), 357–364; MR **20** #693, 694; **21** #4779]. It consists of n heavy fermions V_q (extreme non-relativistic limit) coupled with bare coupling constants g_q^0 $(q=1, 2, \cdots, n$; g_q^0 periodic with period $n)$ to a relativistic neutral boson θ (spin 0). There are mass renormalizations δm_q of the V_q masses, wave function renormalizations $Z_2(q)$, and charge renormalizations $Z_1(q)$ which can all be computed exactly. $Z_3=1$ trivially. In particular, $Z_1(q)=Z_2(q+1)$. The present author succeeds in extracting new information from this model by specializing its (unrenormalized) coupling in a clever way,

$$((g_q^0)^2 = [1-(q-1)/n](g_q^0)^2).$$

The condition for the appearance of additional bound states (ghosts) can then be given very simply, and it is found to be identical with that for the Lee model. If the cut-off is left finite, but n approaches infinity (limit of "realistic" field theory) the wave function renormalization becomes non-negative and the ghost states disappear. Then the renormalized coupling constants $g_q=g_q^0$.

F. Rohrlich (Iowa City, Iowa)

13140:

Kaiser, Hans Jürgen. Zur neuen Tamm-Dancoff-Methode. Wiss. Z. Friedrich-Schiller-Univ. Jena **8** (1958/59), 377.

13141:

Kaiser, Hans Jürgen. Untersuchungen zur neuen Tamm-Dancoff-Methode. Ann. Physik (7) **6** (1960), 131–149.

The spectrum of masses expected from non-linear field equations is discussed in this paper, by means of the new Tamm-Dancoff method. After a brief discussion of the anharmonic oscillator, the mass spectrum of the ϕ^4 non-linear base field and the Heisenberg non-linear equation are determined in the first approximation.

John G. Taylor (Baltimore, Md.)

13142:

DeCelles, Paul C. Renormalization constants in quantum electrodynamics. Phys. Rev. (2) **121** (1961), 304–311.

Author's summary: "The electron self-energy and wave-function renormalization constant are calculated with the use of the technique of dispersion relations. In order to do this, exact representations of these renormalization constants are obtained in terms of matrix elements of Heisenberg field operators. The matrix element $\langle e\gamma|\bar{\psi}|0\rangle$ is studied in detail and its contribution to the renormalization constants determined. In retaining only the contributions due to $\langle e|\bar{\psi}|0\rangle$ and $\langle e\gamma|\bar{\psi}|0\rangle$ an infinity of other matrix elements is ignored. A test of the validity of this approximation is discussed. Within the limits of the approximations, the self-energy and wave-function renormalization constants are shown to have a logarithmic, ultraviolet divergence. The wave-function renormalization constant also has an infrared divergence."

K. Johnson (Cambridge, Mass.)

13143:

Lomsadze, Ju. M. Concerning a "strong" coupling method in the quantized field theory. Nuclear Phys. **24** (1961), 143–150.

The Schrödinger operators of pseudoscalar meson theory are written down in a representation in which the basis vectors are simultaneous eigenvectors of the meson field and the nucleon occupation numbers in each space cell. The interaction Hamiltonian is fully diagonal in this representation, which is therefore put forward as a first step towards a strong coupling formalism, including nucleon recoil and nucleon pair effects. It is suggested that an expansion in powers of the free Hamiltonian might be useful, and to obtain such an expansion the Schrödinger equation is transcribed into a free field picture, which differs from the usual interaction picture by interchange of the respective rôles of the free and interaction parts of the Hamiltonian. No procedure is given for obtaining the correct asymptotic behaviour of scattering wave functions from the equations of the free field picture.

G. R. Allcock (Liverpool)

13144:

Shimodaira, Hajime. Some remarks on fields with negative propagators in quantum field theory. Nuclear Phys. **17** (1960), 486–498.

13145:

Károlyházy, F. Über Phasenoperatoren in der Strahlungstheorie. Acta Phys. Acad. Sci. Hungar. **12**, 321–328 (1960). (Russian summary)

Der bekannte Widerspruch zwischen dem Prinzip, dass ein Photon nur mit sich selbst interferiert und der experimentellen Tatsache, dass auch Wellen von zwei voneinander unabhängigen Radiosender miteinander interferieren, wird besprochen. Der Verfasser ordnet dem klassischen Phasenoperator $e^{i\phi}(=e^{-i\omega t})$ einfach einen Operator φ zu und gibt ausserdem auch die Matrizendarstellung von φ an. Es wird dann gezeigt, dass man jetzt aus dem mathematischen Formalismus der Quantenelektrodynamik das klassische Interferenzbild auch in dem besprochenen Fall herleiten kann. Jedoch muss dabei angenommen werden, dass zwei (voneinander geographisch weit entfernte) Sender Photonen gemeinsam aussenden.

T. Neugebauer (Budapest)

13146:

Senitzky, I. R. Induced and spontaneous emission in a coherent field. Phys. Rev. (2) **111** (1958), 3–11.

Author's summary: "The interaction between a coherently oscillating radiation field, such as is generally encountered in microwave experiments, and a number of similar atomic systems coupled to the field through their electric dipole moments is analyzed for the case of resonance between atomic system and field, with both the field and molecules treated quantum-mechanically. Expressions are derived for the expectation value of field strength and the energy, and comparison between the two makes it possible to distinguish between coherent and incoherent parts of the energy, the former having its counterpart in the expectation value of the field strength, while the latter has no such counterpart. Terms corresponding to induced and spontaneous emission are identified, and it is shown that the latter includes both coherent and incoherent components. Special situations related to masers and to the coherence of spontaneous radiation are discussed. The distinction between the behavior of correlated and uncorrelated states is examined, and it is shown that both the coherent emission from uncorrelated states and the incoherent emission from correlated states may be proportional to the square of the number of molecules. The phase of the field is predictable for the coherent emission but unpredictable for the incoherent emission."

K. Gottfried (Cambridge, Mass.)

13147:

Senitzky, I. R. Induced and spontaneous emission in a coherent field. II. Phys. Rev. (2) **115** (1959), 227–237.

Author's summary: "The interaction between the electromagnetic field and a number of identical atomic systems, individually characterized by an electric dipole moment and two energy levels, is analyzed for the case where the atomic systems are inside a lossy cavity and exposed to a coherent driving field, resonance being assumed between atomic system, cavity, and driving field.

"The problem of introducing loss into a quantum-mechanical formalism is treated first. Formal operator expressions are obtained for the field variables which include the absorption and the fluctuation (both thermal and quantum-mechanical) effects of the loss mechanism. Expectation values are then obtained for the field strength and the field energy which are valid for times short compared to the lifetime of the excited state. It is shown that the spontaneous-emission energy in the field increases initially as the square of the time and approaches a steady-

state value after a transient period which is of the order of the cavity relaxation time. The induced emission contains two parts: incoherent emission induced by the thermal field, and coherent emission induced by the driving field. The incoherent induced-emission energy has the same time dependence as the spontaneous-emission energy, and the ratio of the former to the latter is the number of photons in the thermal (inducing) field. The coherent induced-emission energy does not approach a steady-state value, but, after a transient period, increases linearly with the time. The ratio of the coherent induced-emission energy to the spontaneous-emission energy is equal, initially, to the number of photons in the driving-field energy, but becomes βt times as large after the transient period, where β is the reciprocal of the cavity relaxation time.

"The expectation value of the rate of energy emission by the atomic systems is also obtained. It is shown that the ratio of the downward to the upward transition probabilities has the well-known value of $(n+1)/n$, where n is the field energy in units of the photon energy, only in the absence of a coherent field."

K. Gottfried (Cambridge, Mass.)

13148:

Sinanoğlu, Oktay. Variation-perturbation method for excited states. Phys. Rev. (2) **122** (1961), 491–492.

A variational principle, based on the usual first-order perturbation expansion, is written down for the energy of an excited state of a system. It is shown that by suitably limiting the domain of variation of the trial function, one can guarantee that the energy so obtained is an upper limit to the exact energy of the excited state.

H. W. Lewis (Madison, Wis.)

13149:

Sinanoğlu, Oktay. Perturbation theory of many-electron atoms and molecules. Phys. Rev. (2) **122** (1961), 493–499.

The perturbation expansion for a many-electron system is rewritten, treating the electron-electron interactions as the perturbation. Advantage is taken of the fact that electrons interact only in pairs, to construct the first order wave-functions of two-electron systems. The example chosen is the lithium atom, but the treatment is not carried past the formal construction.

H. W. Lewis (Madison, Wis.)

13150:

Stephen, M. J. The interaction of nuclear multipole moments with an external charge distribution in elliptic coordinates. Proc. Cambridge Philos. Soc. **57** (1961), 348–352.

The interaction between a point-multipole moment and an external charge distribution is calculated using confocal elliptic coordinates (λ, μ, ϕ). A series expansion of $(\lambda+\mu)^{-n-1}P_n((1+\lambda\mu)/(\lambda+\mu))$ in Legendre polynomials is derived.

A. Dalgarno (Belfast)

13151:

Geltman, Sydney. Variational treatment of electron-hydrogen atom elastic scattering. Phys. Rev. (2) **119** (1960), 1283–1290.

The Hulthen and Kohn variational methods are used to calculate the s, p and d phase shifts appropriate to the elastic scattering of low-energy electrons by atomic hydrogen. The trial function allows for the virtual excitation of the $2s$- and $3s$-states. The derived scattering length is larger than the upper bound obtained by Rosenberg, Spruch and O'Malley [#13152], and the author remarks that the origin of the discrepancy remains unexplained. The reviewer believes it is due to neglect of the long-range polarization effects.

A. Dalgarno (Belfast)

13152:

Rosenberg, Leonard; Spruch, Larry; O'Malley, Thomas F. Upper bounds on electron-atomic hydrogen scattering lengths. Phys. Rev. (2) **119** (1960), 164–170.

Variational methods are applied to obtain upper bounds to the scattering lengths for the singlet and triplet scattering of zero-energy electrons by atomic hydrogen, it being assumed that there is no bound triplet state and that there is just one bound singlet state. The triplet scattering length is less than or equal to 1.91 atomic units and the singlet scattering length is less than or equal to 6.23 atomic units.

A. Dalgarno (Belfast)

13153:

Vizbaraîte, Ya. I.; Èringis, K. K.; Yucis, A. P. Extended Hartree-Fock methods. Dokl. Akad. Nauk SSSR **135** (1960), 809–810 (Russian); translated as Soviet Physics. Dokl. **5** (1961), 1251–1252.

Reports a calculation for the $(1s)^2(2p)^2$ configuration of beryllium, using the non-restricted approximation which allows different wave functions for the two $(2p)$ electrons.

D. F. Mayers (Oxford)

13154:

Vizbaraîte, Ya. I.; Strockite, T. D.; Yucis, A. P. The generalized Hartree-Fock methods. Dokl. Akad. Nauk SSSR **135** (1960), 1358–1360 (Russian); translated as Soviet Physics. Dokl. **5** (1961), 1300–1302.

An investigation of the effect of exchange terms, and of the use of different wave functions inside the same group, in the multiconfiguration approximation.

D. F. Mayers (Oxford)

13155:

Cohen, M.; Dalgarno, A. An expansion method for calculating atomic properties. Proc. Roy. Soc. Ser. A **261** (1961), 565–576.

Authors' summary: "An expansion method is described which provides a simple and rapid means of calculating for any atom the expectation values of sums of one-electron operators. All the equations which arise can be solved analytically and the results are obtained as functions of the nuclear charge. For inner shells the accuracy is comparable with that of the Hartree-Fock approximation. The method gives a quantitative description of the effects of direct and exchange interactions between electron shells. Results are given for all members of the helium and beryllium iso-electronic sequences."

D. F. Mayers (Oxford)

13156:

Grosjean, C. C.; Van de Walle, R. T. On the calculation of radial wave functions corresponding to energies in the

continuum part of the helium spectrum. Nuovo Cimento (10) **19** (1961), 696–722. (Italian summary)

An extensive development of the theory of the Schrödinger-type equation satisfied by the radial part of continuum wave functions in He. The numerical methods used in solving the equation are described in detail.

D. F. Mayers (Oxford)

13157:

Machacek, Milos; Scherr, Charles W. Simple configuration interaction wave functions. III. Three-electron atomic system: an analytic study. J. Chem. Phys. **33** (1960), 242–247.

For part II see C. W. Scherr and J. N. Silverman, same J. **32** (1960), 1407–1409.

13158:

Ghosh, Arundhati; Sil, N. C. Neutron energy levels in a diffuse potential. Nuclear Phys. **17** (1960), 264–270.

A method of calculating energy levels in a spherically symmetric potential well. The infinite range is divided into a number of regions in each of which the wave function is approximated by a power series in an appropriate variable, using the Lanczos τ-method.

D. F. Mayers (Oxford)

13159:

Sood, P. C.; Moszkowski, S. A. Energy of a low density neutron gas. Nuclear Phys. **21** (1960), 582–594.

An interesting problem, related to nucleon-nucleon interactions, is the determination of the energy of a pure neutron gas as a function of its density, and the subsequent derivation of the possibility or otherwise of the existence of bound states. Brueckner, Gammel and Kubis, who dealt with this problem, concluded that a neutron gas is not bound at any density. In the present paper, the low-density region is examined in greater detail, including not only the correction due to the Pauli exclusion principle, but also the correction due to the pairing effect, i.e., the depression of the ground-state energy due to pair correlations, and it is shown that one still gets an unbounded system even at quite low densities. The determination of the energy of the neutron gas in the low-density limit is based on the phenomenological Gammel-Thaler potential, since this facilitates the carrying out of numerical computations, and the corrections due to the Pauli effect and the pairing effect are considered separately by assuming an interaction of the factorable (or separable) type. Two tables are given indicating numerical values (corresponding to different values of the Fermi momentum) of the energy, and the corrections to it due to the two effects.

B. S. Madhavarao (Poona)

13160:

Chun, K. W. Charged-scalar strong-coupling theory for two-nucleon system. Phys. Rev. (2) **122** (1961), 973–983.

The charged-scalar strong-coupling theory by Pais and Serber [Phys. Rev. (2) **105** (1957), 1636–1652; MR **19**, 503] is extended to the two-nucleon system. It is shown that the nuclear force expressed in terms of the renormalized coupling constant is explicitly independent of the source size. However, as the two nucleons come close together, the dependence of the nuclear force on the unrenormalized coupling constant alone becomes increasingly pronounced. This situation is analogous to the case of the quantum electrodynamics.

The nuclear forces discussed here are subject to the usual criticism that they are independent of the isotopic spin at small separation and the singlet state is lower than the triplet state at large separation.

T. Sasakawa (Cambridge, Mass.)

13161:

Gupta, Suraj N. Spin-orbit interaction in two-pion exchange nuclear potential. Phys. Rev. (2) **122** (1961), 1923–1926.

Author's summary: "The spin-orbit interaction term in the two-pion exchange nuclear potential is derived. It is found that while the spin-orbit force is quite small in the triplet odd states, it is large and repulsive in the triplet even states. The relationship of the pion theoretical result with the phenomenological spin-orbit interaction is discussed."

13162:

Namiki, Mikio. Semi-phenomenological interpretation of the optical model in nuclear reactions from the point of view of fluctuation-dissipation theorem. Progr. Theoret. Phys. **22** (1959), 843–856.

Author's summary: "The behaviors of the nuclear optical model for the elastic scattering of neutrons at low energies are investigated on the theoretical basis of the fluctuation-dissipation theorem. The theory starts from a Schrödinger equation with the optical potential and the fluctuating source function. The source function, which is a representative of motions of the compound nucleus, is subject to the fluctuation-dissipation theorem, in which the imaginary part of the optical potential is proportional to the correlation function of the fluctuating source function. From this it is found that the strength function characterizing nuclear reactions is represented by the Fourier transform of the correlation function of the fluctuating wave function, and that the average rate of energy dissipation of the compound nucleus is proportional to the strength function and temperature of the compound nucleus."

13163:

Namiki, Mikio. One-particle motions in many-particle systems and the optical model in nuclear reactions. Progr. Theoret. Phys. **23** (1960), 629–661.

Let $\psi(\mathbf{x}, t)$ be the destruction operator for a nucleon, $|A\rangle$ the ground state of the nucleus A, and $|E^+, A+1\rangle$ the scattering state determined by the initial condition that a nucleon is incident upon the nucleus $|A\rangle$. The amplitude $\chi_E(\mathbf{x}, t) = \langle A|\psi(\mathbf{x}, t)|E^+, A+1\rangle$ then satisfies a Schrödinger-like equation in which the effective interaction, which is non-local and energy dependent, may be identified as a generalized optical potential. Formally it is the self-energy operator of the present theory and can be constructed by studying the equation for Green's function $G(\mathbf{x}t, \mathbf{x}'t') = i\langle A|T(\psi(\mathbf{x}, t)\psi^\dagger(\mathbf{x}'t'))|A\rangle$. The optical potential is decomposed into two parts, the first or Hartree-Fock terms being identified with the conventional real part of the optical potential, though in fact this can only be accurately true for a weak long-range interaction. The

remaining terms describe absorption (imaginary part) and the associated diffraction scattering. The amplitude $\chi_E(x, t)$ may itself be decomposed into a part describing the elastic scattering (obeying a Schrödinger equation with the conventional optical potential) and a fluctuation part governed by a Langevin-like equation with the same optical potential and by the fluctuation-dissipation theorem. This justifies assumptions made in previous work [#13162]. *A. Klein* (Philadelphia, Pa.)

13164:

Tauber, G. E.; Wu, Ta-You. Nuclear shell model; Hartree-Fock approximation with Gammel-Thaler two-nucleon potential. Nuclear Phys. **16** (1960), 545–567; erratum, **22** (1961), 339.

The many-particle wave function for the system is built up from Slater determinants of harmonic oscillator single-particle functions, and a correlation function. The two-nucleon potential used is a recent one of Gammel-Thaler, containing a repulsive core, a central, a tensor, and an L·S interaction. The parameters in the wave functions and the correlation function are adjusted to minimise the total energy and the multiplicative constant in the potential chosen to fit the empirical energy of O^{16}. These parameters are then used to calculate the energy of O^{15}, giving a result which is a considerable improvement on previous calculations which omitted the repulsive core. The doublet splitting appears to be too large; the reasons for this are discussed in some detail. *D. F. Mayers* (Oxford)

13165:

Verhaar, B. J. A method for the elimination of spurious states in the nuclear harmonic oscillator shell model. Nuclear. Phys. **21** (1960), 508–525.

It has been recognized for some time that models of nuclear structure which have been constructed from one-particle models offer difficulties in the interpretation of their energy levels. The trouble arises from the necessity of separating the motion of the center of mass of the nucleus from the internal motions of the nucleons. The author offers a solution of this problem for the shell model based on individual nucleons moving in harmonic oscillator potentials, making use of group-theoretical considerations. The theory involves the reduction of the space of eigenfunctions under the rotation group, the permutation group of the nucleons, and a dynamical group involving both position and momentum operators which arises in the harmonic oscillator model, and which has been applied in this connection by J. P. Elliott [Proc. Roy. Soc. London Ser. A **245** (1958), 562–581; MR **20** #697]. The theory is illustrated on some particular nuclei. {For a similarly oriented, but somewhat differently formulated, group-theoretical treatment of this problem, see M. Kretschmar [Z. Physik **158** (1960), 284–303; MR **22** #5410b].} *E. L. Hill* (Minneapolis, Minn.)

13166:

da Providencia, J. Perturbation theory in finite nuclei. Proc. Phys. Soc. **77** (1961), 81–92.

The aim of this paper is to test the validity of perturbation theory for small nuclei, in which surface effects might be important. O^{16} is used as a test case. The force law used has no hard core, and no tensor or L·S force, in line with the theoretical aim of the study. Comparison is made with the pre-war results of Euler for infinite nuclear matter. It is found that two-body correlations are stronger in the surface region than in infinite nuclear matter; but the binding energy per particle is not altered much. This may be because the effect of the finite excitation energy of a small nucleus offsets the reduction in the quenching due to the Pauli exclusion principle.

J. M. Blatt (Murray Hill, N.J.)

13167:

Brown, G. E.; Castillejo, L.; Evans, J. A. The dipole state in nuclei. Nuclear Phys. **22** (1961), 1–13.

It is shown how the particle-hole interaction in a closed shell nucleus raises the energy of one of the particle-hole states with angular momentum unity. This dipole state contains most of the electric dipole matrix element from the ground state, and accounts for the giant dipole resonance. Calculations in j-j coupling for O^{16} and Ca^{40} are made with a zero-range force, and give good agreement with finite-range calculations; it is suggested that the results are not sensitive to the details of the interaction. The suggestion that the dipole state may be split in two by spin-orbit coupling is confirmed. An explanation of the narrow width of the dipole state in closed shell nuclei is attempted. *D. J. Thouless* (Birmingham)

13168:

Baranger, Michel. Self-consistent field theory of nuclear shapes. Phys. Rev. (2) **122** (1961), 992–996.

The generalization of the Hartree-Fock equations that takes pairing into account is described in its general form; in addition to a Hartree potential there is also a pairing potential. The method is applied to a nucleus in which there is a pairing force and a quadrupole-quadrupole interaction between particles; there may be deformed solutions as well as spherically symmetric solutions. An equation for the collective modes is derived; one of the frequencies goes to zero when the spherical solution becomes unstable. It is observed that this approximation may not be very good for a system such as a nucleus with rather few particles. *D. J. Thouless* (Birmingham)

13169:

Kumar, Kailash; Bhaduri, Rajat K. Fermi-Thomas type approximation for nuclei. Phys. Rev. (2) **122** (1961), 1926–1929.

Authors' summary: "The Hartree-Fock to Fermi-Thomas reduction is carried through for finite nuclei, starting with the *K*-matrix formulation. The resulting expression represents the nuclear energy to good accuracy in terms of the density and its first derivatives only; it differs in detail from the semiempirical expressions previously proposed for this purpose. This improved expression shows the inadequacy of the 'semi-infinite' approximation used often in earlier studies."

13170:

Greuling, Eugene; Whitten, R. C. Lepton conservation and double beta-decay. Ann. Physics **11** (1960), 510–533.

Authors' summary: "Double beta-decay is investigated

without assuming lepton conservation. If the lepton part of the universal V–A current operator is linear in the massless Majorana neutrino field, leptons are conserved. The intermediate state neutrino is completely polarized and may not be reabsorbed to produce a neutrinoless double beta-decay final state. Two slight modifications of the interaction, (1) finite neutrino mass, and/or (2) a deviation from exact equality of the effective V and A lepton currents, result in the creation of an incompletely polarized intermediate state neutrino which may then be reabsorbed. Neutrinoless double beta-decay, in violation of lepton conservation, is then possible. These neutrinoless modes of double beta-decay are computed for several nuclei and are compared with the usual lepton conserving two-neutrino mode. Measurable competition between these decay modes may exist. The abandonment of lepton conservation does not alter significantly any of the predictions of the universal V–A Fermi interaction for all first order beta-processes." *P. Chevallier* (Strasbourg)

13171:

Brunner, Witlof; Paul, Harry. Zur Theorie der Kernspaltung. I. Die Kernkraftwechselwirkung zwischen den Bruchstücken. Ann. Physik (7) **6** (1960), 267–278.

Earlier theories of nuclear fission, due to Fong and Newton, failed to account completely for the large preference for asymmetrical fission of U^{235}. Newton's theory is modified here by allowing for some residual nuclear interactions between the two separating fragments, rather than assuming a pure Coulomb interaction for $r > R$, the channel radius. This residual nuclear interaction is treated as a parameter, to be fitted to fission data, and it is claimed that the fit is not unreasonable from first principles. *J. M. Blatt* (Murray Hill, N.J.)

13172:

Takayanagi, Kazuo; Kaneko, Shobu. Vibrational transition in molecular collision. Sci. Rep. Saitama Univ. Ser. A **1**, 111–123 (1954).

The method of modified wave number, which is a simplification of the distorted wave approximation, is used to calculate the vibrational deactivation of oxygen molecules by collision with helium atoms. The resulting cross section is about five times too small. However, this may well be due to inaccuracies in the intermolecular potential function assumed in the calculations.

A. C. Hurley (Melbourne)

13173:

Takayanagi, Kazuo. Vibrational transition in molecular collision. II. Influence of the Van der Waals force. Sci. Rep. Saitama Univ. Ser. A **3**, 1–19 (1958).

The vibrational transition probabilities of N_2 and Cl_2 are calculated. It is shown that, if the intermolecular potential is expressed in the form $A \exp(-\alpha R) - B/R^6$, the first term, i.e., the short range repulsive force, plays the major role in the transition. However, the Van der Waals attraction given by the second term cannot be omitted entirely because it affects the slope of the repulsive part of the potential. The depth of the potential valley is of minor importance.

A. C. Hurley (Melbourne)

13174:

Takayanagi, Kazuo; Miyamoto, Yoshiko. Vibrational transition in molecular collision. III. A short table of a functional useful for calculating the vibrational transition probability. Sci. Rep. Saitama Univ. Ser. A **3**, 101–114 (1959).

Some numerical tables and figures are presented which are useful for estimating vibrational transition probabilities in molecular collisions from an assumed potential function. The matrix elements of the interaction potential are assumed to vary exponentially with the nuclear separation. *A. C. Hurley* (Melbourne)

13175:

Takayanagi, Kazuo; Kaneko, Shobu. Vibrational transition in molecular collision. IV. Another short table of functions useful for calculating the vibrational transition probability. I. Sci. Rep. Saitama Univ. Ser. A **3**, 167–178 (1960).

Authors' summary: "The energy-dependence of the vibrational transition cross section comes mainly from the translational matrix element. This energy-dependent factor, averaged over the Maxwellian velocity distribution, is tabulated here, the matrix elements of the intermolecular potential with respect to the vibrational eigenfunctions being assumed to be of the Morse type."

A. C. Hurley (Melbourne)

13176:

Fišer, Ya. [Fischer, Jan]; Čulli, S. [Ciulli, S.]. Recurrent construction of angular operators. II. Introduction of an integral spin and an arbitrary orbital momentum. Ž. Èksper. Teoret. Fiz. **39** (1960), 1349–1356 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 941–945.

The method previously proposed by the authors [Soviet Physics JETP **11** (1960), 1256–1262; MR **22** #7701] for the practical construction of angular momentum operators, in which one uses only differential operations, is applied to the case when the form of the operator is changed by the addition of a unit of orbital angular momentum or an integer spin to the process, or by the addition of a new boson. The method is recursive and so makes use of the solution of a simpler process. In this way it involves less work than the usual method with Clebsch-Gordan coefficients. *R. P. Streater* (Upton, N.Y.)

13177:

Marshak, R. E.; Sudarshan, E. C. G. ★Introduction to elementary particle physics. Interscience Tracts on Physics and Astronomy, No. 11. Interscience Publishers, New York-London, 1961. viii+231 pp. $4.50.

This book is a welcome new member of the still scarcely populated family of systematic texts on the theory of elementary particles. In the reviewer's opinion, the most appealing feature of this compact monograph is that, while emphasizing the systematics and internal relationships of elementary particles, only a minimum amount of quantum field theory is used or preassumed. In the present stage of development it indeed appears as though our knowledge on the symmetry properties of elementary particles and their interactions were the "permanent" feature of the theory, while the methods and even the fundamental

concepts of field theory may have to be drastically altered in the future.

After a short introductory discussion, Chapter two surveys the quantum theoretical formalism and contains a brief summary of the junior author's rather original researches along these lines. Chapter three discusses the familiar non-continuous transformation groups and the associated selection rules. It includes a review of the research results of both authors and others on the universal Fermi interaction. The next chapter, on additive conservation laws, is perhaps the most stimulating one. The last chapter is devoted to the familiar topic of isospin selection rules. The authors exercized here commendable restraint.

This book will be a useful introduction to this exciting field of research. *P. Roman* (Boston, Mass.)

13178:

Tzou, Kuo-Hsien. Un modèle des leptons. Interaction électromagnétique et interaction faible. C. R. Acad. Sci. Paris **251** (1960), 2895–2897.

Author's summary: "On introduit les interactions électromagnétique et faible dans le modèle des leptons proposé dans la note précédente [mêmes C. R. **251** (1960), 2659–2661; MR **22** #11930]. Le courant leptonique de l'interaction faible ressemble exactement au courant électrique de l'électron-muon. La théorie à quatre composantes du neutrino est nécessaire pour la symétrie M et les neutrinos associés à ϵ et à μ sont deux particules d'hélicité opposée."

13179:

Novožilov, Yu. V.; Terent'ev, I. A. The two-nucleon LS potential in nonrelativistic meson theory. Ž. Èksper. Teoret. Fiz. **36** (1959), 129–139 (Russian. English summary); translated as Soviet Physics JETP **9**, 89–96.

The two-nucleon meson-theoretic potential is calculated by treating the nucleons non-relativistically (terms up to linear in the velocity of the nucleons being included). The expansion carried out is in terms of the rate of fall-off of the internucleon distance (in units of meson Compton wavelength). The formalism of an earlier paper [Yu. V. Novožilov, same Ž. **35** (1959), 742–749; MR **21** #6227] is used to describe the scattering of "dressed" nucleons, each nucleon being accompanied by its meson cloud in the absence of interaction. It is found that in the static limit the potential derived agrees with the potential previously obtained by Klein and McCormick [Phys. Rev. (2) **104** (1956), 1747–1757; MR **18**, 856], and the terms linear in the nucleon velocity yield a spin-orbit potential with a range half the meson Compton wavelength. This potential is semiphenomenological since it contains, besides the term earlier obtained by Klein [ibid. (2) **90** (1953), 1101–1115; MR **15**, 844] in the perturbation theory, terms which depend on the total pion-nucleon scattering cross-section. The sign of the complete spin-orbit potential can be determined by numerical integration. However, asymptotic expansions are given in the paper.

B. P. Nigam (Rochester, N.Y.)

13180:

Lubkin, E. Angular distributions. Nuovo Cimento (10) **16** (1960), 1098–1108. (Italian summary)

This is a general discussion of the allowed angular distributions in a reaction (governed by rotationally invariant "transition matricee") where the initial state is specified in a (J, M)-representation. Extending previous work [C. N. Yang, Phys. Rev. (2) **74** (1948), 764–772; M. Jacob and G. C. Wick, Ann. Phys. **7** (1959), 404–428; MR **22** #2396] the complete angular distributions in all three Euler angles is discussed. Particular attention is paid to the restrictions imposed on the dependence on the Euler angles by the restrictions on the initial state as regards the angular momentum and for the particular case of two particles of definite helicities. A discussion of the conditions under which the phase change of π suffered by a spinor amplitude under a rotation through 2π becomes measurable is also presented.

E. C. G. Sudarshan (Rochester, N.Y.)

13181:

Bollini, G. C. [misprint for C. G.]. Irreducibility constraints and field equations for the elementary particles. II. Fermions. Nuovo Cimento (10) **13** (1959), 46–56. (Italian summary)

This is a continuation of an earlier paper [C. G. Bollini, Nuovo Cimento **11** (1959), 342–350; MR **21** #4797], extending the method described there to the case of fermions. A projection operator is defined by which the independent spin components are selected from an arbitrary spin tensor of the order appropriate to the spin of the particle. The quantization of the free field is carried through and leads to a slightly different expression for the S_F function. *C. A. Hurst* (Edinburgh)

13182:

Bollini, C. G. On the coupling of the elementary particles with the electromagnetic field. Nuovo Cimento (10) **14** (1959), 560–570. (Italian summary)

In two previous papers [#13181 and reference there] the author has developed a method for deriving wave equations representing particles with higher spin. The method depended on the determination of projection operators which would select irreducible tensors out of a general tensor. In the previous papers the main emphasis was on the free field case, with some reference to interaction schemes. [Similar ideas were also put forward by R. E. Behrends and C. Fronsdal, Phys. Rev. (2) **106** (1957), 345–353; MR **19**, 506.]

In this paper the author makes some preliminary comments on the problem of determining the correct interaction with the electromagnetic field. The classic papers of Dirac, Pauli and Fierz were only able to give a partial answer to this question, for the proposed interactions either were inconsistent, or were rather complicated and did not satisfy the constraints. It is assumed that ψ^a (a mixed spinor tensor) satisfies the equation $(p^2-m^2)\psi=PI'\psi$, where P is the projection operator compatible with the irreducibility conditions, and I' is the interaction term, which is determined by successive approximations. At each stage, the approximation is chosen to ensure that the S-matrix is gauge invariant. If ψ transforms according to $\psi'^a=\frac{1}{4}T_{\mu\nu}{}^{ab}a_{\nu\mu}\psi^b$ under a Lorentz transformation $x_\mu'=a_{\mu\nu}x_\nu$, suitable first approximation to I' is assumed to be $I^{(1)}=eT_{\mu\nu}(p_\mu A_\nu+A_\mu p_\nu)$. In this presentation the constraints are claimed to be automatically satisfied and gauge invariance is assured.

{However, if I' and P do not commute, ψ could contain terms of the form $\delta(p^2-m^2)Q\Phi$ where Q is the complementary projector to P. This would violate the constraints, unless it can be shown that no such solutions exist.}
C. A. Hurst (Edinburgh)

13183:

Bollini, C. G. Total Compton cross section for arbitrary spin. Phys. Rev. (2) **121** (1961), 314–318.

The theory proposed in an earlier paper [#13182] on the coupling of elementary particles, of arbitrary spin, with the electromagnetic field is applied to the calculation of the total Compton scattering cross-section. The relation between the forward scattering amplitude and the total cross-section is used to obtain the total cross-section in the fourth order in the charge.

The main results are that in the low-energy approximation the cross-section tends to the Thomson cross-section, independently of spin, and the derivative of the cross-section with respect to energy at zero energy is also spin independent. In the extreme relativistic region the cross-section increases no faster than the first power of the energy, in contrast with the Fierz-Pauli theory.
C. A. Hurst (Edinburgh)

13184:

Królikowski, W. On the intrinsic Pauli principle. Nuclear Phys. **17** (1960), 421–432.

Absence of higher spins and multiple charges in elementary particles is ascribed to an "intrinsic Pauli principle" [Królikowski, Nuclear Phys. **10** (1959), 213–214; **11** (1959), 687–690] which would imply that the present list of baryons and leptons is closed. A Fock representation in a three-dimensional iso-space of three spinor fields and one pseudoscalar field leads to relations of the Gell-Mann-Nishijima type among charges, strangeness, baryonic number, etc. *C. Strachan* (Aberdeen)

13185:

Ikeda, Mineo; Miyachi, Yoshihiko; Ogawa, Shuzo. Symmetry in Sakata's model and weak interactions. II. Progr. Theoret. Phys. **25** (1961), 121–152.

This paper continues the investigation of non-leptonic hyperon and K-meson decays in the Sakata model that was initiated by these authors earlier [Progr. Theoret. Phys. **24** (1960), 569–587; MR **22** #9188]. The decay amplitude relations that were previously derived by assigning the particles to particular configurations are now shown to follow from the quantum number assignments alone. K-3π decay is investigated assuming that various 3π states of different permutation symmetry dominate. The possibility of anomolous decays involving new mesons $\pi^{0'}$ or $\pi^{0''}$ in place of π^0 is considered.
S. Bludman (Berkeley, Calif.)

13186:

Hormann, Elizabeth. Applications of the Sakata model for elementary particles. Nuclear Phys. **24** (1961), 514–518.

Within the framework of the composite particle theory, investigations are carried out to utilize simple subsymmetries of the three-dimensional unitary group for the description and classification of all interactions. It is shown that all known types of interaction, and only these, can occur. The scheme can be easily extended to the leptons and yields interesting results concerning the properties of muons. *P. Roman* (Boston, Mass.)

13187:

Murai, Yasuhisa. Discrimination between strong and weak interactions. Nuclear Phys. **17** (1960), 529–547.

In an earlier paper [Nuclear Phys. **6** (1958), 489] the author developed a scheme of relativistic wave equations based on a Lagrangian formalism in a space described by six homogeneous coordinates; the fundamental group of invariance of this five-dimensional space is such that its restriction to the hypersurface $r=0$ (where r is the fifth coordinate, the other four being the usual space-time coordinates) is the conformal group. The spin 0 wave equation describes not a simple Klein-Gordon field but a generalized free-field [unpublished results of O. W. Greenberg and G. F. Dell'Antonio] with an arbitrary mass spread. The spin $\frac{1}{2}$ wave equation describes an eight-component generalized spinor field with arbitrary mass-spreads. The doubling of the spinor components can be utilized to define operators of chirality [S. Watanabe, Phys. Rev. (2) **106** (1957), 1306–1315; MR **19**, 925] and hypercharge [J. Schwinger, ibid. **104** (1956), 1164–1172; MR **19**, 1235]; d'Espagnat and Prentki [Nuclear Phys. **1** (1956), 33–53; MR **19**, 223]. Specific choices are then made for the amplitudes describing the baryons and mesons; the internal wave-functions (i.e., the dependence on the fifth coordinate) correspond to a small but finite mass-spread of the generalized free-fields described by the small parameter ε. By an appropriate choice of the coupling matrices it is shown that one can select out a subclass of Yukawa interactions which are "observed". The remarkable result now appears that only the "stable vertices" (associated with strong interactions) are finite in the limit of small ε but the "unstable vertices" (associated with weak decay interactions) are proportional to ε for small ε. The strong kaon interactions are parity-violating. It appears that while the details of the interaction scheme and even the specific choice of the generalized free-fields made in this paper are quite arbitrary, the basic ideas underlying the formulation deserve further study and investigation. *E. C. G. Sudarshan* (Rochester, N.Y.)

13188:

Pais, A. Relative parity of charged and neutral K-particles. Phys. Rev. (2) **112** (1958), 624–641.

Most of this paper is directed to remedying some difficulties of the doublet approximation theory assigning independent parity to K^+ relative to Λ-nucleon and relative to K^0 (the isotopic spin of K-meson being taken as 0). In section V the author provides a formal basis for classifying the symmetries of strong interactions by the direct product representations of the rotation group of four-dimensional isospin space. *N. Kumasawa* (Tokyo)

13189:

Shimamoto, Yoshio. Internal symmetries of strong baryon-meson interactions. Phys. Rev. (2) **122** (1961), 289–297.

This paper develops the symmetry theory of A. Pais [#13188], when K-mesons are taken as $\mathscr{K}$-spin doublets. The author considers that the situation in which, assuming

bare Σ and Λ masses equal, the strong baryon-pion interaction is regarded as a direct product representation of the 4-dimensional rotation group by two 3-dimensional spin operators $\mathscr{J}$ and $\mathscr{K}$, also holds in baryon-K-meson interactions; he examines conceivable types of coupling schemes and concludes that there is a possibility of constructing strong baryon-meson interactions requiring at most four coupling constants and two bare baryon masses. Production and scattering amplitude are also discussed.
N. Kumasawa (Tokyo)

13190:

Renson, P. Particules inconnues. I, II. Acad. Roy. Belg. Bull. Cl. Sci. (5) **46** (1960), 468–474, 609–620. (English summary)

Some speculations are made as to the possible existence and decay modes of a meson, a baryon, and an anti-baryon, each having zero isospin and unit electric charge.
G. R. Allcock (Liverpool)

13191:

Dallaporta, N.; Pandit, L. K. Baryon mass-differences and symmetries of strong interactions. Nuovo Cimento (10) **16** (1960), 135–167. (Italian summary)

The authors describe the interaction of baryons with pions in terms of a doublet scheme with an associated isobaric spin, and the interaction with K-mesons in terms of a four-dimensional scheme whose associated three-dimensional spins are called hypernumber spin and hypercharge spin. When the actual mass differences between the baryons are taken into account none of these spins are separately conserved but the sum of isobaric and hypernumber spin is in fact conserved and is therefore identified with conventional isotopic spin. The resulting theory has only four independent constants.
J. C. Polkinghorne (Cambridge, England)

13192:

Jacob, R.; Sachs, R. G. Mass and lifetime of unstable particles. Phys. Rev. (2) **121** (1961), 350–356.

It is assumed in this paper that complex poles may occur in the second sheet of the propagator for an unstable particle. The authors show, by a detailed discussion of the production and decay mechanism of the particle, that the mass and lifetime of the unstable particle determined by a time plot are given by the real and imaginary parts of the pole position, respectively. To achieve this result, wave packets are used to describe the initial production states and the final decay states. For times large compared to the mass uncertainty the decay amplitude can be written as the sum of three terms. The first term contains the expected exponential time decay coming from an unstable particle with mass and lifetime given by the pole position. The second and third terms give rise to a non-exponential time decay, and can be understood in terms of the uncertainty principle applied to the measuring apparatus (for the second term) and to the decay of the particle (for the third term). These non-exponential terms are negligible in comparison to the exponential one for weak decays, though they may not be so for some nuclear reactions.
John G. Taylor (Baltimore, Md.)

13193:

Résibois, P. Three body collisions in quantum mechanics. Physica **27** (1961), 33–47.

It is shown that Prigogine's diagram technique for solving the Bloch equation can be applied to systems with only a few degrees of freedom. The transition probabilities occurring in three-body collisions are described in terms of irreducible connected diagrams.
D. ter Haar (Oxford)

13194:

Möbius, P. Introduction of many-particle variables for the treatment of special translationally invariant many-body problems. Nuclear Phys. **16** (1960), 278–303.

A method is described to introduce special combinations of the $A-1$ relative coordinate vectors of a system of A particles which are such that the exclusion principle is automatically satisfied (translational invariance is, of course, conserved by the introduction of these variables). The method is applied to a three-dimensional system of particles where the particles are coupled harmonically and to some one-dimensional systems.
D. ter Haar (Oxford)

13195:

Kohn, W.; Luttinger, J. M. Ground-state energy of a many-fermion system. Phys. Rev. (2) **118** (1960), 41–45.

The ground-state energy of an interacting gas of fermions is calculated by constructing the grand partition function at finite temperature using the method of Bloch and Dominicis [Nuclear Phys. **7** (1958), 459–479]. In the limit of vanishing temperature the series so derived differs from the Brueckner-Goldstone perturbation series. It is suggested that the discrepancy occurs because the Bruecker-Goldstone series does not in general (an exception occurs in the case of a spherically symmetric interaction) describe the ground state but some higher state.
A. Dalgarno (Belfast)

13196:

Luttinger, J. M.; Ward, J. C. Ground-state energy of a many-fermion system. II. Phys. Rev. (2) **118** (1960), 1417–1427.

Authors' summary: "The perturbation series for the ground-state energy of a many-fermion system is investigated to arbitrary order for the 'isotropic' case. This is the case of over-all spherical symmetry, both in the interaction and in the unperturbed single particle energies. It is shown that for spin one-half fermions the Brueckner-Goldstone perturbation series is valid to all orders in the perturbation. For spins greater than one-half it is in general incorrect even in the isotropic case, unless the interactions are spin dependent. The discussion to arbitrary order in the interaction is carried out by means of a Feynman-like propagator formalism, which is developed in detail."
A. Dalgarno (Belfast)

13197a:

Klein, Abraham. Perturbation theory for an infinite medium of fermions. II. Phys. Rev. (2) **121** (1961), 950–956.

13197b:

Klein, Abraham. Perturbation theory for an infinite medium of fermions. III. Derivation of the Landau theory of Fermi liquids. Phys. Rev. (2) **121** (1961), 957–961.

An infinite system of interacting particles subject to the Pauli exclusion principle (fermions) is studied by a perturbation formalism of quantized field theory. The major novelty of the method consists in the use of new forms for the linked cluster expansions in terms of single-particle propagators and self-energy operators. The analysis is a continuation of that of an earlier paper by A. Klein and R. E. Prange [Phys. Rev. (2) **112** (1958), 994–1007; MR **21** #5465], which will be referred to as paper I. {The reader may also be referred to #13196 above.}

In paper II the system is supposed to be at absolute zero temperature. A formula obtained in paper I for the ground state of the system is taken as the starting point. This expression is re-formulated in terms of the self-energy operator and propagator function of a particle. It is shown that when these latter quantities are related by an exact functional connection to each other, the energy expression is insensitive to variations made in the self-energy operator. This is interpreted to mean that the energy, considered as a functional of the self-energy operator, satisfies a variational principle. The existence of such a principle is made the basis of a new formulation of the energy operator for the ground state energy of the system. It explains also the fact that calculations of the ground state energy have been found to be insensitive to the assumed single-particle energy. Lastly, it is shown how these considerations can be extended to the case in which the Hamiltonian operator is not rotationally invariant.

In paper III the theory is applied to the study of a system at a finite temperature. The assumption is made that the density matrix can be derived by adiabatic transformation from the density matrix of a system of non-interacting particles at the same temperature, volume, and chemical potential. It is shown that the theory can be formulated in such a way as to lead to the semi-phenomenological theory of Fermi liquids proposed by L. Landau [Soviet Physics JETP **3** (1957), 920–925; MR **18**, 975]. *E. L. Hill* (Minneapolis, Minn.)

13198:

Daniel, E.; Vosko, S. H. Momentum distribution of an interacting electron gas. Phys. Rev. (2) **120** (1960), 2041–2044.

The momentum distribution for an interacting electron gas is computed by making use of the Gell-Mann-Bruckner technique of summation of divergent graphs. The plasmagraphs, corresponding to subsequent hole-particle creation and annihilation are considered both with and without exchange. A fictive momentum-projecting interaction reduces the problem to the calculation of the ground-state energy. Graphical results are given for metallic densities. *G. Kalman* (Haifa)

13199:

Bolsterli, M. Perturbative treatment of pairing forces in many-fermion systems. Phys. Rev. (2) **122** (1961), 1946–1948.

Author's summary: "A noncanonical transformation which allows perturbative techniques to be applied to the pairing force problem is introduced. The lowest-order eigenvalue equation gives the standard results for both strong and weak coupling."

13200:

Ahiezer, I. A.; Peletminskiĭ, S. V. On the theory of the magnetic properties of a nonideal Fermi gas at low temperatures. Ž. Èksper. Teoret. Fiz. **39** (1960), 1308–1316 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 913–918.

Authors' summary: "The methods of quantum field theory are used to study the effect of the interaction between particles on the oscillations of the magnetic moment of a Fermi gas. The forces between the particles are assumed to be of the short-range type, and the calculations are made in the gas approximation. Values are found for the changes of period and amplitude of the oscillations of the magnetic moment that are caused by the interaction between particles. At not too low temperatures the amplitude of the oscillations contains a factor that decreases exponentially with decrease of the magnetic field."

13201:

Bargmann, V.; Moshinsky, M. Group theory of harmonic oscillators. II. The integrals of motion for the quadrupole-quadrupole interaction. Nuclear Phys. **23** (1961), 177–199.

In this paper the discussion of a system of N spinless particles in a common oscillator potential, begun in the previous paper [Nuclear Phys. **18** (1960), 697–712; MR **22** #11942], is continued. The stationary-state problem for such a system, including a quadrupole-type interaction, is completely solved by constructing $3N$ commuting and independent integrals, then determining the eigenvalues of these integrals and expressing the energy as functions of them. *J. M. Jauch* (Geneva)

13202:

Chan, Hong-Mo; Valatin, J. G. On the density field description of a Boson system. Nuovo Cimento (10) **19** (1961), 118–130. (Italian summary)

An effort is made to give a rigorous construction of the density-field description of a system of bosons, a description which leads naturally to the hydrodynamics of such systems. The formal transformation from the conventional field theory to the density-field description involves singular difficulties which are circumvented by regarding the field as the limit of a lattice space. The resulting formalism is then related to the configuration space treatment of Bogolyubov, and the weight function necessary to relate the two descriptions is derived. The weight function is combinatoric, but non-trivial.

H. W. Lewis (Madison, Wis.)

13203:

Boys, S. F.; Cook, G. B. Mathematical problems in the complete quantum predictions of chemical phenomena. Rev. Mod. Phys. **32** (1960), 285–295.

This paper discusses at length the automatic programming of a computer to calculate the properties of many-electron atoms and molecules. The approach is based on the well-known expansion of the wave function as a linear combination of Slater determinants constructed from orthonormal single electron functions.

W. B. Brown (Manchester)

13204:

Fukuda, Nobuyuki. A new approach to the many body problem, in particular to the theory of the electron gas. Physica **26** (1960), supplement, S 162–S 169.

A canonical transformation to plasmon creation and annihilation operators leads to the Sawada-Wentzel Hamiltonian which is exact in the high-density limit. The present paper concentrates on a further canonical transformation which diagonalizes the Sawada-Wentzel Hamiltonian. After the transformation is found, matrix elements with respect to the exact states of the high-density problem can be calculated. Examples are given for ground-state matrix elements. *G. Kalman* (Haifa)

13205:

Azbel', M. Ya. Quasiclassical quantization in the neighborhood of singular classical trajectories. Ž. Èksper. Teoret. Fiz. **39** (1960), 1276–1285 (Russian. English summary); translated as Soviet Physics. JETP **12** (1961), 891–897.

In this paper the author indicates how to obtain the energy levels of self-intersecting trajectories of an electron in metals in a magnetic field for the momentum space. Two forms of non-convex Fermi surfaces leading to the case of self-intersection are discussed, and it is shown that near the point of self-intersection the separation between levels contains a part oscillating with the magnetic field. In forming the Hamiltonian operator the author seems to ignore the criticism of Zil'berman's work of which his own is an extension [Soviet Physics JETP **5** (1957), 208–215; MR **20** #1559]. *A. H. Klotz* (Newcastle upon Tyne)

13206:

Carlson, B. C.; Keller, Joseph M. Eigenvalues of density matrices. Phys. Rev. (2) **121** (1961), 659–661.

The eigenfunctions and eigenvalues of the generalized density matrix of order p [P. O. Löwdin, Phys. Rev. (2) **97** (1955), 1474–1489; MR **16**, 983],

$$\Gamma_p(x, x') = \binom{N}{p} \int \psi(x, y)\psi^*(x', y)\, dy,$$

of an N-particle system are investigated, and it is shown that Γ_p and Γ_{N-p} have the same non-zero eigenvalue with the same multiplicities. If the number of non-zero eigenvalues is finite, Γ_{N-p} and Γ_p are shown to be unitarily equivalent. The transpose $\tilde{\Gamma}_{N-p}$ is also unitarily (anti-unitarily?) equivalent. *C. A. Hurst* (Adelaide)

13207:

Osada, Jun'ichi; Takeda, Minoru. Nuclear potential in many-body problems. Progr. Theoret. Phys. **24** (1960), 755–780.

What the authors show is as follows: Let $f(\mathbf{k})$ be the k-component of the two-body potential and $f'(\mathbf{k})$ be the k-component of the potential in a many-body system which is assumed to be low enough to allow the perturbation calculation and is given by the symmetrical scalar meson field. Cancellation of many terms makes $f'(\mathbf{k})$ equal to $f(\mathbf{k})$ after the charge renormalization. Accordingly, the potential $F(\mathbf{r})$, the Fourier transform of $f(\mathbf{k})$, is equal to the two-body potential, in particular to the T.M.O.-type. As a result the potential energy of the many-body system is given by $\int_0^k r d\mathbf{k} \int_0^\infty e^{i\mathbf{k}\mathbf{r}} F(\mathbf{k}) d\mathbf{r}$, and nothing should be added to the T.M.O. potential. *T. Sasakawa* (Cambridge, Mass.)

13208:

Migdal, A. B. Superfluidity and the moments of inertia of nuclei. Ž. Èksper. Teoret. Fiz. **37** (1959), 249–263 (Russian); translated as Soviet Physics. JETP **10** (1960), 176–185.

A method is developed for the treatment of superfluidity of nuclei. A formula which agrees satisfactorily with experiment is obtained for the moment of inertia of a nucleus. An expression is found for the change in the energy of "pairing" in the transition from an even-even to an even-odd nucleus, and also for the change in the moment of inertia associated with this transition. *L. N. Cooper* (Providence, R.I.)

13209:

Abrikosov, A. A.; Khalatnikov, I. M. Theory of the Fermi fluid. (The properties of liquid He^3 at low temperatures.) Soviet Physics. Uspekhi **1** (**66**) (1958), 68–90 (177–212 Uspehi Fiz. Nauk).

The authors expound the theory of the Fermi liquid developed by Landau [Ž. Èksper. Teoret. Fiz. **30** (1956), 1058; MR **18**, 975] for the isotropic model. They apply this to Helium 3, calculating various properties of that fluid, and compare their results with experiment. *L. N. Cooper* (Providence, R.I.)

13210:

Nambu, Yoichiro. Quasi-particles and gauge invariance in the theory of superconductivity. Phys. Rev. (2) **117** (1960), 648–663.

This formulation of the theory of superconductivity is gauge invariant and also satisfies the continuity condition. The pairing approximation is reproduced by summing a subset of Feynman graphs and the electromagnetic interaction introduced in a gauge-invariant manner using generalized forms of Ward's identity. The vertex operator, as well as the electron propagator, is modified. *L. N. Cooper* (Providence, R.I.)

13211:

Oliphant, T. A.; Tobocman, W. Gauge invariant formulation of the Bardeen-Cooper-Schrieffer theory of superconductivity. Phys. Rev. (2) **119** (1960), 502–503.

The pairing condition of the theory of superconductivity is reformulated so that electrons of equal and opposite angular momenta, rather than of opposite linear momenta, are paired. This treatment should be applicable to very small particles (smaller than the penetration depth) in a constant magnetic field. The point is first to find the single-electron wave functions in the constant field and then pair them to take into account correlations. Such a treatment is gauge invariant. *L. N. Cooper* (Providence, R.I.)

13212:

Harper, P. G. The quasi-particle approximation in superconductivity. Proc. Phys. Soc. **77** (1961), 299–302.

13213:

Abrikosov, A. A.; Gor'kov, L. P. On the theory of

superconducting alloys. I. The electrodynamics of alloys at absolute zero. Soviet Physics. JETP **35** (8) (1959), 1090–1098 (1558–1571 Ž. Èksper. Teoret. Fiz.).

In this paper the authors give the theory of superconductors containing impurities at absolute zero. The dependence of the penetration depth on the impurity concentration is considered for small atomic concentrations. They obtain the electrodynamic equations in an alternating field for superconductors with a mean free path which is smaller than the correlation length.

L. N. Cooper (Providence, R.I.)

13214:

Blatt, J. M. The Meissner effect in the quasi-chemical equilibrium theory of superconductivity. Progr. Theoret. Phys. **24** (1960), 851–876.

The author gives an explicitly gauge-invariant proof of the occurrence of the Meissner effect, based on the quasi-chemical equilibrium theory of superconductivity. This proof has the merit that it is mathematically rigorous (in particular, it makes no use of a random phase approximation). Moreover, it is easy to see that the essential requirement for a Meissner effect is the Bose-Einstein condensation of electron pairs; other properties, such as the energy gap and plasma waves, are not necessary. It should be noted that in this paper the true (velocity dependent) interaction between electrons in a metal is replaced by a schematic interaction, namely, an ordinary potential $V(r-r')$. *R. M. May* (Cambridge, Mass.)

RELATIVITY

See also A12020, A12032a–b, 13007, 13101, 13249, 13258.

13215:

Faggiani, Dalberto. Possibilità di considerare un simbolo fondamentale in più nell'analisi dimensionale. Atti Accad. Naz. Lincei. Rend. Cl. Sci. Fis. Mat. Nat. (8) **28** (1960), 787–793.

If a physical system is considered in relation to two systems of reference such that the motion of one system in relation to the other is a pure translation with the constant velocity vector w of length w, a transformation from one system to the other will multiply certain fundamental measures of the system by some power of $(1-w^2c^{-2})^{-1/2}$. If a physical formula is invariant with respect to the Lorentz transform, this property can be used in a way analogous to the usual dimensional analysis, and it is more powerful in some respects: e.g., the ratio between a length parallel to w and a length perpendicular to w is not invariant by the Lorentz transform. The author uses this extended dimensional analysis to derive a number of known formulas ranging from the period of planetary motion and the maximal mass of a suspended drop of liquid to the resistance in viscous and in compressible fluid flow. *H. Tornehave* (Copenhagen)

13216a:

Swann, W. F. G. Certain matters in relation to the restricted theory of relativity, with special reference to the clock paradox and the paradox of the identical twins. I. Fundamentals. Amer. J. Phys. **28** (1960), 55–64.

13216b:

Swann, W. F. G. Certain matters in relation to the restricted theory of relativity, with special reference to the clock paradox and the paradox of the identical twins. II. Discussion of the problem of the identical twins. Amer. J. Phys. **28** (1960), 319–323.

Part I is an expository paper discussing the effect of acceleration on a measuring device. Part II uses the ideas of Part I to discuss the traveling twin paradox. The measuring device (say a clock) carried by the traveler undergoes acceleration, while the corresponding clock of the stay-at-home twin does not. The well-known result is obtained that the traveler, upon return, is younger than his brother. *Mary L. Boas* (Chicago, Ill.)

13217:

Abonyi, Ivan. Steady state solution of the relativistic Boltzmann transport equation. Z. Angew. Math. Phys. **11** (1960), 169–175. (French summary)

An expression for the distribution function in position and velocity for a gas of relativistic particles in an external field is obtained by first obtaining a relativistic Boltzmann equation and then seeking a time-independent solution which also makes the collision term vanish. This kinetic method yields the same solution, a generalization of the Maxwell velocity distribution, as was obtained by Juttner [Ann. Physik **34** (1911), 856–882] for the thermodynamic equilibrium distribution using Liouville's theorem and relativistic mechanics. *D. Falkoff* (Waltham, Mass.)

13218:

Rayski, Jerzy. Geometrization of meso-electrodynamics. Wiss. Z. Friedrich-Schiller-Univ. Jena **8** (1958/59), 349.

13219:

Cahen, Michel. Étude du champ électromagnétique singulier dans l'espace de Minkowski. Acad. Roy. Belg. Bull. Cl. Sci. (5) **46** (1960), 61–69.

Let $g_{\lambda\mu}$ be the metric tensor of the Lorentz space-time and $F^{\lambda\nu}$ the electromagnetic field of the third class

$$(1)\qquad \begin{aligned} \eta_{\omega\mu\lambda\nu}F^{\omega\mu}F^{\lambda\nu} &= 0,\\ g_{[\omega[\lambda}g_{\mu]\nu]}F^{\omega\mu}F^{\lambda\nu} &= 0 \end{aligned}$$

[see Hlavatý, *Geometry of Einstein's unified field theory*, Noordhoff, Groningen, 1957; MR **20** #842; p. 3]. The equations (1) lead to a pencil of quadratic complexes of characteristic [(111)(111)]. [See Hlavatý, *Differential line geometry*, Noordhoff, Groningen, 1953; MR **15**, 252; Chapter VI]. This enables the author to express $F^{\lambda\nu}$ in terms of a two-component spinor ξ_0, ξ_1. If $\tilde{F}^{\lambda\nu}$ is the dual to $F^{\lambda\nu}$ and $\mathscr{X}^{\lambda\nu} \overset{\text{def}}{=} F^{\lambda\nu}+i\tilde{F}^{\lambda\nu}$, then the case of Maxwell's equation for pure radiation can be condensed in (2) $\partial_\lambda\mathscr{X}^{\lambda\nu}=0$. The author solves (2) by substituting for $\mathscr{X}$ the spinor ξ and by considering all possible cases. {The generalized equations (2) for general relativity were solved by the reviewer by means of an algorithm applicable to all three classes: Math. Pures Appl. (19) **40** (1961), 1–41.}

V. Hlavatý (Bloomington, Ind.)

13220:

Swann, W. F. G. Can there be a shield for gravitation? J. Franklin Inst. **271** (1961), 355–360.

13221:

Burcev, P. On the mechanics of a mass point in the general theory of relativity. Czechoslovak J. Phys. **11** (1961), 122–127. (Russian summary)

The equations of motion for mass points, the energy-momentum tensor of which is a linear function of the δ-function, are discussed in connection with the principle of equivalence. The equations of motion for mass points subject to a supplementary non-gravitational force effect can, under certain conditions, be transformed to equations for free motion in a certain gravitational field, which is defined along the trajectories of the mass points.

G. L. Clark (London)

13222:

Soergel-Fabricius, Charlotte. Über den Ursprung von Coriolis- und Zentrifugalkräften in stationären Räumen. Z. Physik **161** (1960/61), 392–403.

The forces are defined in terms of geodesic motion in the rest-frame of an observer who is at rest in the stationary metric. The connection of fields with sources is established by drawing analogies between Maxwell theory and linearized Einstein theory. The relation of the results to Mach's principle (of relativity of inertia) is discussed.

F. A. E. Pirani (London)

13223:

Lipkin, Daniel M. Note on the modified gravitational equations $g^2R_{ik}=0$. Phys. Rev. (2) **122** (1961), 972.

The proposal of Einstein and Rosen [same Phys. **48** (1935), 73–77] to replace the field equations $R_{ik}=0$, by the equations $g^2R_{ik}=0$ arose from the aim to ensure that the transformed Schwarzschild metric

$$\text{(A)}\quad ds^2 = \frac{u^2}{A+u^2}\,dt^2 - 4(A+u^2)\,du^2 - (A+u^2)^2(d\theta^2+\sin^2\theta d\phi^2)$$

should satisfy the field equations across the locus $u=0$ on which g vanishes. It is shown by explicit computation that all components of the Riemann tensor are, in fact, single-valued and continuous (and differentiable) across $u=0$, so that the proposed modification is redundant as far as (A) is concerned. No generalization of this result is given.

H. A. Buchdahl (Hobart)

13224:

Bel, Louis. Sur les géodésiques isotropes en théorie de Jordan-Thiry. C. R. Acad. Sci. Paris **244** (1957), 3129–3131.

Author's summary: "Étude de la correspondance entre variétés caractéristiques et géodésiques isotropes dans les variétés V_5 et V_4 de la théorie."

13225:

Hennequin, Françoise. Le rôle des conditions d'isothermie en relativité générale. Cahiers de Phys. **12** (1958), 308–310.

L'auteur rappelle les propriétés fondamentales des coordineés isothermes, puis leur rôle dans la résolution approchée des équations d'Einstein: les équations approchées du mouvement entraînent les conditions d'isothermie approchées à tout ordre voulu (théorème global). [Cf. F. Hennequin, Bull. Sci. Com. Trav. Hist. Sci. **1** (1956), 2e partie, 73–154].

Y. Fourès-Bruhat (Marseille)

13226:

Pham Tan Hoang. Comparaison des deux méthodes d'obtention des équations du mouvement en relativité générale. Cahiers de Phys. **12** (1958), 399–406.

The Einstein-Infeld-Hoffmann method [Ann. of Math. (2) **39** (1938), 65–100] of obtaining the equations of motion of n bodies in general relativity is equivalent to the method based on the energy momentum tensor as expounded by Fock [Acad. Sci. USSR. J. Phys. **1** (1939), 81–116; MR **1**, 183] and Hennequin [Bull. Sci. Com. Trav. Hist. Sci. **1** (1956), 2e partie, 73–154].

A. J. Coleman (Toronto, Ont.)

13227:

Droz-Vincent, Philippe. Généralisation des équations d'Einstein correspondant à l'hypothèse d'une masse non nulle pour le graviton. C. R. Acad. Sci. Paris **249** (1959), 2290–2292.

L'auteur introduit, en plus de la métrique riemannienne, une métrique euclidienne d'abord quelconque, puis qui réalise une approche isotherme. Il les utilise pour introduire un potentiel symétrique. Il forme les équations aux variations du champ en fonction de la différence des deux connexions déduites de ces métriques.

J. Charles-Renaudie (Montpellier)

13228:

Nariai, Hidekazu; Ueno, Yoshio. On the concept of regraduation in general relativity. Progr. Theoret. Phys. **24** (1960), 593–613.

After discussing the general concept of scale regraduation in relativity, the authors consider the problem in relation to those theories of gravitation which are based on flat space-time. Turning to general relativity, they then study in detail conformal regraduation similar to those formulated by the reviewer [Proc. Roy. Soc. Edinburgh Sect. A **62** (1946), 164–174; MR **7**, 531], but now approached from a somewhat different point of view. A "field equation" for the conformal factor is obtained and is shown to be given by a variational equation. The invariance of Maxwell's and Dirac's equations under general and restricted conformal regraduations is examined.

A. G. Walker (Liverpool)

13229:

Cattaneo, Carlo. Moto di un fotone libero in un campo gravitazionale. Atti Accad. Naz. Lincei. Rend. Cl. Sci. Fis. Mat. Nat. (8) **27** (1959), 54–59.

Ausgehend von den allgemein angenommenen Definitionen der Relativitätstheorie behandelt der Verfasser das Problem der Bewegung eines freien Photons im Gravitationsfeld. Er betrachtet dabei das Photon wie üblich als ein Teilchen der Ruhmasse null. Die Bewegungsgleichungen im Raum-Zeit Kontinuum werden abgeleitet

und daraus folgt: (1) dass die raumzeitliche Bahn des Photons eine geodätische Linie der Länge null ist; und (2) dass diese Bewegungsgleichungen ein dem Energieintegral entsprechendes erstes Integral besitzen.

T. P. Anđelić (Belgrade)

13230:

Wigner, E. P. Geometry of light paths between two material bodies. J. Mathematical Phys. **2** (1961), 207–211.

Author's summary: "The pattern of light signals, which was proposed before for the measurement of the curvature, is investigated in a two-dimensional manifold of constant curvature (de Sitter space). The pattern consists of light signals between two freely moving bodies, each signal being emitted when the signal from the other body arrives. It is shown that the coordinates of the arrivals (or emissions) of the light signals can be obtained from the coordinates of the emission of the first signal by means of projective transformations which are iterates of a single such transformation. The same applies to the proper times at which these signals are received."

P. G. Bergmann (Syracuse, N.Y.)

13231:

Coxeter, H. S. M. On Wigner's problem of reflected light signals in de Sitter space. Proc. Roy. Soc. Ser. A **261** (1961), 435–442.

Author's summary: "Consider two material bodies, moving freely. One emits a light signal, the other reflects it back, the first reflects it again, and so on. When de Sitter's world is represented by the part of projective space outside a non-ruled quadric, the world lines of the material bodies are two secants and the world lines of the light signals form a zigzag of tangents between them. Let t_{2n} be the proper time for the nth event at the first body. If the two world lines are 'ultra-parallel', so that the bodies approach each other to a minimal distance ϕ (at time zero) and then recede, we have $t_{2n}=\log\tan(n\phi+\frac{1}{4}\pi)$ for $n<\pi/4\phi$. If the world lines intersect at time zero and form a hyperbolic angle ϕ (depending on the relative velocity), we have

$$t_{2n} = \log\coth(\tfrac{1}{2}\log\coth\tfrac{1}{2}t_0-n\phi)$$

or

$$-\log\coth(n\phi-\tfrac{1}{2}\log\tanh|\tfrac{1}{2}t_0|)$$

according as t_0 is positive or negative. If the world lines are parallel (in the sense of hyperbolic geometry), so that the bodies approach each other asymptotically, we have $t_{2n}=\log(2n+\varepsilon)$ where $\varepsilon=\exp t_0$. Finally, if the world lines are skew, t_{2n} is given implicitly by a recursion formula which does not seem to have an elementary solution. The first two results were obtained by Wigner another way; the third and fourth are apparently new."

P. G. Bergmann (Syracuse, N.Y.)

13232:

Papapetrou, Achille; Schoepf, Hans-Georg. Sur l'énergie des champs gravitationnels non stationnaires. C. R. Acad. Sci. Paris **251** (1960), 2889–2891.

Alternative forms (Einstein, Møller) for the energy pseudo-tensor, which yields the same result for the total energy of a bounded stationary system, are shown to yield results differing by factor 2 for the total energy of a gravitational wave-packet.

F. A. E. Pirani (London)

13233:

Sciama, D. W. Recurrent radiation in general relativity. Proc. Cambridge Philos. Soc. **57** (1961), 436–439.

Riemann spaces V_4 of hyperbolic normal signature which are recurrent in the sense of Ruse and Walker [cf. A. G. Walker, Proc. London Math. Soc. (2) **52** (1950), 36–64; MR **12**, 283] are discussed in terms of gravitational radiation theory.

F. A. E. Pirani (London)

13234:

Arnowitt, R.; Deser, S.; Misner, C. W. Heisenberg representation in classical general relativity. Nuovo Cimento (10) **19** (1961), 668–681. (Italian summary)

The authors define a classical "Heisenberg representation" for a dynamical system to be a canonical formalism such that the energy is the numerical value of the Hamiltonian: within the class of such representations time-independent canonical transformations of the dynamic variables are permitted, but Hamilton-Jacobi-like transformations are not. General relativity differs from merely Lorentz covariant theories in possessing a wider class of Heisenberg formalisms on account of the extra freedom of coordinate transformations. In every such formalism the measurable quantities may be constructed from the canonical variables at the same time without any explicit coordinate dependence; in each the Hamiltonian is a constant of the motion whose value, the energy, depends only on the state of the gravitational field and not on the particular representation. It is shown that two Heisenberg frames may differ by a coordinate transformation which depends only on the canonical variables and not explicitly on the coordinates, and which preserves the asymptotic Lorentz metric at spatial infinity. The Heisenberg representations discussed here include the canonical formalisms which the authors introduced in earlier papers [see Phys. Rev. (2) **117** (1960), 1595–1602; MR **22** #4505b].

P. W. Higgs (Edinburgh)

13235:

Mariot, Louis. Le champ électromagnétique pur en relativité générale. Rend. Mat. e Appl. (5) **18** (1959), 178–266.

Study of the algebraic and analytic properties of the electromagnetic field without sources in a Riemann-Einstein manifold. The chapter headings are: (1) The vacuum Maxwell equations in non-relativistic physics; (2) The Maxwell-Einstein equations in vacuum in general relativity; (3) Algebraic examination of the electromagnetic energy-momentum tensor; (4) Propagation of an electromagnetic perturbation; (5) Local properties of the singular electromagnetic field; (6) Polarization of the singular electromagnetic field. {Note: "Singular" fields are those in which both scalars $\phi^{\mu\nu}\phi_{\mu\nu}$ and $\phi^{\mu\nu}\phi_{\mu\nu}{}^*$ vanish; this case is also called in the literature "pure radiation".}

P. G. Bergmann (Syracuse, N.Y.)

13236:

Soergel-Fabricius, Charlotte. Thirring-Effekt im Einsteinkosmos. Z. Physik **159** (1960), 541–553.

Generalization of the classical empty-universe calculation [by H. Thirring, Physik. Z. **19** (1918), 33–39; modified by L. Bass and the reviewer, Philos. Mag. (7) **46** (1955), 850–856; MR **18**, 704] with a discussion of Mach's principle.

F. A. E. Pirani (London)

13237:

Raychaudhuri, A. A general deduction of two important relations in relativistic cosmology. Z. Astrophys. **51**, 88–90 (1961).

It is shown that previous expressions for the change of radiation density with time in isotropic universes [R. C. Tolman, *Relativity, thermodynamics and cosmology*, Clarendon Press, Oxford, 1934; pp. 417–419; R. A. Alpher, J. W. Follin and R. C. Herman, Phys. Rev. (2) **92** (1953), 1347–1361] hold in anisotropic universes as well.

D. W. Sciama (Ithaca, N.Y.)

13238:

Kimura, Toshiei. Coupling of Dirac particle and gravitational field. Progr. Theoret. Phys. **24** (1960), 386–406.

Dirac's equation is written in a generally covariant form. Following H. S. Green [Nuclear Phys. **7** (1958), 373–383] the spin-affine connection is related to rotations in a 6-dimensional pseudo-Euclidean space. These rotations are taken as independent field-variables in a variational principle. A tentative physical interpretation is given.

S. A. Wouthuysen (Amsterdam)

13239:

Tonnelat, Marie-Antoinette. Sur la représentation des contributions materielle et électromagnétique dans une théorie purement géométrique. C. R. Acad. Sci. Paris **251** (1960), 2892–2894.

13240:

Santaló, L. A. On Einstein's unified field equations. Univ. Nac. Tucumán. Rev. Ser. A **12**, 31–55 (1959). (Spanish. English summary)

Let ${}^a\Gamma_{jk}{}^i$ be six arbitrary connections ($a=1, \cdots, 6$). The author reduces this number to substantially three by means of the requirement that for an arbitrary vector U_i the following condition holds:

$$\nabla_k \underset{3}{\nabla_j} \underset{\substack{1\\2}}{U_i} - \nabla_j \underset{6}{\nabla_k} \underset{\substack{4\\5}}{U_i} = 0.$$

Moreover, this requirement implies that a certain "generalized" curvature tensor of three connections is zero. This leads to a "contracted curvature tensor" T_{ik}. Applying the variational principle to the Hamiltonian $\mathfrak{A}^{ik}T_{ik}$, one obtains a system of field equations which, under certain conditions, reduces to the "weak system" considered by Einstein in his unified theory [see the reviewer's book *Geometry of Einstein's unified field theory*, P. Noordhoff, Groningen, 1957; MR **20** #5067]. In the last part of this paper the same problem is considered for the "pseudo-hermitian" part of tensors considered in the first part (for pseudo-hermitian symmetry see the above-mentioned book, p. 54; the author uses the name "hermitian").

V. Hlavatý (Bloomington, Ind.)

13241:

Lindner, Jochen. Zur nichtlinearen Feldtheorie. Z. Naturforschg. **16a** (1961), 346–356.

The theory is essentially Bechert's version of the Einstein-Maxwell theory [K. Bechert, same Z. **11a** (1956), 177–182; MR **17**, 1145], with a charge-current vector source $S^\mu = CV^\mu$ for the Maxwell field, which also enters the energy tensor in the form $UV^\mu V^\nu$ ($V^\mu V_\mu = 1$). It is shown how to find solutions of the form

$$ds^2 = e^{2\sigma}dr^2 + e^{2\tau}dz^2 + e^{2\mu}r^2d\theta^2 + e^{2\nu}dt^2,$$

where σ, τ, μ, ν are functions of r, z only. Some interpretations, notably as classical models of elementary particles, are given.

C. W. Kilmister (London)

13242:

Nakamura, Hiroshi; Toyoda, Toshiyuki. Connection of spinor fields. Nuclear Phys. **22** (1961), 524–528.

The formulation of spinor relativity proposed by Green [Proc. Roy. Soc. Ser. A **245** (1958), 521–535; MR **20** #1571] is shown to be equivalent to the introduction of an electromagnetic and gravitational field only. By use of a suitable gauge transformation, all other extraneous fields can be eliminated.

C. A. Hurst (Adelaide)

13243:

Rayski, Jerzy. Interpretation of electrodynamics and baryon theory within a six-dimensional manifold. Nuclear Phys. **17** (1960), 289–316.

This paper represents an attempt to "geometrize" the electromagnetic field by a device that has some similarity with Kaluza's but is different in that two dimensions are added to the usual four, so that the over-all number of dimensions becomes six. It is assumed that the metric with respect to the two additional dimensions is conformally flat and singular at the origin. Gauge transformations are introduced as coordinate rotations in the two-dimensional "electric space", which may depend on the location in the four-dimensional ordinary space-time manifold. The electric charge is associated with the (invariant) "angular momentum" of a field with respect to the "electric coordinates", and is quantized. The magnitude of the elementary electric charge quantum is associated with the nature of the assumed singularity of the metric of the electric space. Baryon, lepton, and meson fields are associated with various geometric structures possible in six-dimensional space; their quantum numbers (iso-spin, etc.) are associated with various types of spinors possible in such spaces. The paper is preliminary in nature rather than definitive, and it appears that the author intends to indicate the range of problems that should be amenable to rational treatment from his point of view. There is no discussion of any possible relationship between his construction and the earlier theories by Kaluza-Klein, or the current ones by Gell-Mann and Pais.

P. G. Bergmann (Syracuse, N.Y.)

13244:

Kühnel, A.; Schmutzer, E. Exakte Lösung der Feldgleichungen einer einfachen Variante der projektiven Relativitätstheorie für eine geladene Punktsingularität. Ann. Physik (7) **7** (1961), 243–250.

The paper discusses static, spherically symmetric solutions, representing charged particles, of the field equations recently proposed by one of the authors [E. Schmutzer, Z. Physik **154** (1959), 312–318; MR **21** #2520] as a modification of Jordan's projective relativity theory. A particular solution, which the authors believe to be significant in nature, is examined in detail, and is claimed to yield a finite electrostatic self-energy. In the

case of neutral particles, the theory's predictions for the three Einstein effects agree with general relativity.

{The omission of a factor $e^{\nu/2}$ from the integrand of the authors' expression for the electrostatic field energy, $U^{el}=\frac{1}{2}\int_0^\infty EDr^2dr$, appears to be an oversight. The expression is in any case of dubious validity as it stands, since the integration is carried into a region where the coordinate r becomes physically meaningless.}

W. Israel (Edmonton, Alta.)

ASTRONOMY

See also 12988, 13055, 13056, 13236.

13245:

Yarov-Yarovoi, M. S. The application of Hansen's ideal coordinates. Astronom. Ž. **37** (1960), 908–917 (Russian); translated as Soviet Astronom. AJ 4 (1961), 850–858.

The first three sections contain a review of the introduction of Hansen's ideal coordinates, a derivation of the equations for the variation of the elements that determine the orientation of the orbital plane and the angle σ that determines the departure point, and a derivation of the equations in polar coordinates for the motion in the orbital plane. These equations are found in various treatises, either for the conservative case in which the components of the disturbing acceleration can be represented as derivatives of a disturbing function, or for the case in which the components are not, or cannot be, expressed in this form. In this article the equations are given explicitly for the case that the disturbing acceleration consists of a conservative and a non-conservative part. In addition, the author comments on various aspects of the equations: on Hansen's modification for orbits with small inclinations, on methods of integration by successive approximations, and on special circumstances in applications to artificial satellite motion.

D. Brouwer (New Haven, Conn.)

13246:

Kozai, Yoshihide. The gravitational field of the earth derived from motions of three satellites. Astronom. J. **66** (1961), 8–10.

Coefficients of the second, third, fourth and fifth harmonics of the earth's gravitational field are determined from the analysis of the motions of three artificial satellites, 1958 $\beta 2$, 1959 η and 1959 $\iota 1$.

D. Brouwer (New Haven, Conn.)

13247:

Eckert, W. J. Numerical determination of precise orbits. Proc. Sympos. Appl. Math., Vol. 9, pp. 145–151. American Mathematical Society, Providence, R.I., 1959.

A summary of applications of high-speed computers to the calculation of accurate orbits of bodies in the solar system. Particular attention is given to the numerical integration of the orbits of the outer planets and to the calculation of the improved lunar ephemeris.

D. Brouwer (New Haven, Conn.)

13248:

Huang, Su-Shu. Some dynamical properties of natural and artificial satellites. Astronom. J. **66** (1961), 157–159.

Some qualitative properties of satellites in the solar system are discussed in terms of the restricted three-body problem. Under certain initial conditions a satellite will remain inside the zero-velocity surface known as the inner contact surface, always revolving around the same body. Under other initial conditions the satellite may escape from its parent planet. The three outermost satellites of Jupiter are found to be in this category.

Artificial satellites of the earth, the moon, and the earth plus moon are also considered. An earth satellite initially in a near-circular orbit can never escape from the earth if its semi-major axis is less than 0.33 of the moon's distance from the earth.

G. E. Cook (Farnborough)

13249:

Tauber, G. E.; Weinberg, J. W. Internal state of a gravitating gas. Phys. Rev. (2) **122** (1961), 1342–1365.

The authors treat large masses of gas so heavily concentrated or rotating so rapidly, or both, that general-relativistic effects may become important. Such conditions are met with if the radius of the gas cloud is reduced to the order of magnitude of the Schwarzschild radius, and when the fluid near the surface is rotating at a speed approaching c. In order to keep the problem manageable, the authors restrict themselves to the treatment of rigid rotation, in which, through the introduction of co-moving coordinates, all macroscopic variables (and particularly the components of the metric tensor) can be rendered stationary. In general, rigid rotation with non-zero angular velocity excludes spherical symmetry. The work reported in this paper also includes the statistical mechanics in μ-space of relativistic gases, and the construction of equations of state. Section headings: (1) Introduction, (2) Covariant statistical mechanics, (3) Equations of state, (4) Thermal equilibrium, (5) Rotating fluids.

P. G. Bergmann (Syracuse, N.Y.)

13250:

Mestel, L. A note on the magnetic braking of a rotating star. Monthly Not. Roy. Astronom. Soc. **119** (1959), 249–254.

In this paper, the author criticizes the particular field chosen by Lüst and Schlüter [Z. Astrophys. **38** (1955), 190–211] and tries to choose a more plausible alternative. The magnetic field is poloidal and is highly twisted in the region between a rapidly rotating star and a slowly rotating envelope. Angular momentum is being transported across the intermediate region. The author shows that there is a large variety of magnetic fields satisfying the condition that there is a flux of angular momentum across a certain sphere of radius r_K, and not necessarily a force-free field. The method of solution is given but involves a considerable amount of numerical integration. No numerical estimate is given of the rate of exchange of angular momentum for the new solution.

E. Schatzman (Paris)

13251:

Cowling, T. G. Note on magnetic instabilities in stellar structure. Monthly Not. Roy. Astronom. Soc. **121** (1960), 393–397.

The instability of a uniform liquid star with an appropriate magnetic field was considered by Prendergast [Astrophys. J. **128** (1958), 361–374; MR **20** #5080]. He showed that his particular model of a magnetic star is

unstable if the magnetic field is sufficiently large. The author of this article now shows that Prendergast's model is unstable for all magnitudes of the magnetic field. Furthermore, he suggests that such instabilities must relieve themselves by a readjustment of the field in the star. This may take place in the form of a redistribution of the lines of force through the mass. Although such a redistribution may be violent, the author does not think that it can cause an explosion of the star.

R. S. B. Ong (Leiden)

13252:

Rublev, S. V. The approximate solution of the transfer equation for a plane gray photosphere taking account of the blanketing effect. Astronom. Ž. **37** (1960), 677–685 (Russian); translated as Soviet Astronom. AJ **4** (1961), 645–652.

Author's summary: "A 'mean cosine' method is developed and applied to the solution of the transfer problem in a plane-parallel gray photosphere. A generalized linear approximation for the source function $B(\tau)$ is obtained, taking into account the blanketing effect produced by the reversing layer. A method of deriving the second approximation of $B(\tau)$ is given. In the case of the sun, a comparison is made with Przybylski's model [A. Przybylski, Monthly Not. Roy. Astronom. Soc. **117** (1957), 483]."

13253:

Ledoux, P.; Lambert, J.; Whitney, Ch. Détermination des premiers modes d'oscillations radiales d'une étoile gazeuse. Les mathématiques de l'ingénieur, pp. 361–368. Mém. Publ. Soc. Sci. Arts Lett. Hainaut, vol. hors série, 1958.

13254:

Whitrow, G. J. ★The structure and evolution of the universe: An introduction to cosmology. Hutchinson & Co., Ltd., London; Hillary House, Inc., New York; 1959. 212 pp. (8 plates) $4.50.

This book is a revised version of *The structure of the universe* [Hutchinson, London] which was published in 1949. It takes account of two major advances in cosmology, one observational and one theoretical. The observational advance is the coming into operation of the 200-inch telescope. The theoretical advance is the realization by Bondi, Gold and Hoyle that the universe may be in a steady state.

The present text gives a good non-technical account of cosmology up to the end of 1958, although some of the statements about the steady-state theory are incorrect.

D. W. Sciama (Ithaca, N.Y.)

13255:

Finzi, Arrigo. A possible test of the expansion of the universe from luminosity curves of distant supernovae. Ann. Astrophys. **24** (1961), 68–70. (French and Russian summaries)

13256:

Kraus, Karl. On condensation processes in the expanding universe. Z. Astrophys. **52** (1961), 18–21. (German summary)

The effect of radiation on galaxy formation processes in evolving universes is briefly examined.

D. W. Sciama (Ithaca, N.Y.)

13257:

Just, Kurt. The collapse of a contracting universe. Z. Astrophys. **52** (1961), 13–17. (German summary)

It is shown that a universe consisting of radiation and matter tends to contract at an infinite rate as its radius approaches its minimum value.

D. W. Sciama (Ithaca, N.Y.)

13258:

Rindler, W. Remarks on Schrödinger's model of de Sitter space. Phys. Rev. (2) **120** (1960), 1041–1044.

Schrödinger in his book *Expanding universes* [Cambridge Univ. Press, London, 1956; MR **17**, 1015] has studied a subspace of de Sitter space in terms of a hyperboloid in Euclidean 3-space. He has further shown that all time-like sections of the hyperboloid by planes through its center represent non-null geodesics in de Sitter space-time and that the generators of the hyperboloid represent light-paths. The main purpose of the paper being reviewed is to show that time-like sections of the hyperboloid by arbitrary planes represent world lines of constant curvature and zero torsion in the de Sitter space-time, and to give some simple properties of such paths.

A. H. Taub (Urbana, Ill.)

13259:

Zel'manov, A. L. The deformation of "co-traveling" (concomitant) space in Einstein's gravitational theory. Dokl. Akad. Nauk SSSR **135** (1960), 1367–1370 (Russian); translated as Soviet Physics. Dokl. **5** (1961), 1309–1312.

This paper treats some of the aspects of cosmological models of the universe that are not required to be isotropic and homogeneous. The most important result appears to be that even with vanishing or negative cosmological constant there are solutions that oscillate, and solutions in which some regions of the universe experience contraction at the same time that others expand.

P. G. Bergmann (Syracuse, N.Y.)

GEOPHYSICS

See also 12674, 12727, 12960, 12961, 13246.

13260:

Krylov, Yu. M. Statistical theory and calculation of wind waves on the sea. Trudy Gos. Okeanograf. Inst. **42** (1958), 3–88. (1 insert) (Russian)

This is the second part of a statistical theory of sea-waves, the first of which appeared in same Trudy **33** (**45**) (1956), 5–79 [MR **19**, 1240], but which unfortunately has not been available. In this part the theoretical results of the first part are applied to the calculation of the growth and decay of wind waves up to explicit numerical results. The methods developed for deep water here could be applied also to shallow water. In the last chapter of this part a further development of the theory is indicated in particular with reference to the energy spectrum of the wind waves.

H. A. Lauwerier (Amsterdam)

13261:

Longuet-Higgins, M. S.; Stewart, R. W. Changes in the form of short gravity waves on long waves and tidal currents. J. Fluid Mech. 8 (1960), 565–583.

Authors' summary: "Short gravity waves, when superposed on much longer waves of the same type, have a tendency to become both shorter and steeper at the crests of the longer waves, and correspondingly longer and lower in the troughs. In the present paper, by taking into account the non-linear interactions between the two wave trains, the changes in wavelength and amplitude of the shorter wave train are rigorously calculated. The results differ in some essentials from previous estimates by Unna [Nature **159** (1947), 239–242]. The variation in energy of the short waves is shown to correspond to work done by the longer waves against radiation stress of the short waves, which has previously been overlooked. The concept of the radiation stress is likely to be valuable in other problems." *D. C. Gilles* (Glasgow)

13262:

Abbot, M. R. L'influence de la force de Coriolis sur les courants de marée dans un estuaire exponentiel. Houille Blanche **15** (1960), 616–624. (Text also in English)

Author's summary: "This paper gives a simple approximate treatment of the tidal flow in an estuary, of constant mean depth and exponential width, taking the rotation of the earth into account. The frictional resistance is represented by Lorentz's linearisation of Chezy's law. It is shown that the maximum difference of level across the estuary for a progressive wave is given by the same formula as for a uniform frictionless channel, but, due to a difference of phase, the difference of amplitude can be considerably less. The theory is applied to a section of the Thames Estuary and it is shown that the amplitude difference is only about a third of the maximum difference of level." *M. H. Rogers* (Bristol)

13263:

Thompson, Philip D. ★Numerical weather analysis and prediction. The Macmillan Company, New York, 1961. xiv+170 pp. $6.50.

This is the first English language textbook on the prediction of atmospheric flow patterns from purely physical principles. The subject has been developed in the last 12 years, and is based on the numerical integration (with electronic computers) of the relevant equations of hydrodynamics and thermodynamics. The present book is written as a "primer", with the prerequisite mathematical background intentionally limited to one of calculus and simple differential equations.

The most important aspects of the subject are presented in a very lucid style; the effect of acoustic and gravity waves and their elimination from the equations by "filtering" assumptions, finite-difference techniques and computational instability, and a brief summary of the results obtained by the Joint Numerical Weather Prediction Unit in the United States. The principal defect of the book is a failure by the author to exploit consistently the powerful and physically instructive technique of scale analysis. *N. Phillips* (Cambridge, Mass.)

13264:

Kao, S.-K. Transfer of momentum vorticity and the maintenance of zonal circulation in the atmosphere. J. Meteorol. **17** (1960), 122–129.

13265:

Platzman, George W. The spectral form of the vorticity equation. J. Meteorol. **17** (1960), 635–644.

From the author's summary: "The non-linear aspects of the vorticity equation for two-dimensional planetary circulations of the earth's atmosphere may be studied by expansion of the solution in spherical surface harmonics. Some of the main mathematical problems are discussed here that arise in an examination of the 'spectral' form of the vorticity equation which results from such an expansion. The truncation of the spectral equations is discussed, and a proof is given of the invariance of mean square velocity and vorticity for truncated spectra." *M. H. Rogers* (Bristol)

13266:

Hazay, I. Statische Koordinatenausgleichung mit Richtungsmessungen, ohne Orientierungsrichtungen. Acta Tech. Acad. Sci. Hungar. **30** (1960), 207–229. (English, French, and Russian summaries)

The application of the principle of static adjustment of points [same Acta **23** (1959), 397–430, 431–472; MR **21** #6276, 6277] is extended to cover the case where an initial orientation from the given points is not fixed in advance. The bulk of the article is taken up by the complete working out of an example involving 6 points, with 20 measured directions. *B. Chovitz* (Washington, D.C.)

13267:

Ledersteger, Karl. Die geometrischen und physikalischen Daten des Normalsphäroides der Erde. Bayer. Akad. Wiss. Math.-Nat. Kl. S.-B. **1959**, 23–39 (1960).

This is one of an important current series of papers by the author expounding a particular solution for the figure of the Earth from the standpoint of potential theory. The determination of a "best" figure of the Earth is a central problem of geodesy; the problem becomes very deep, and involves extensive ramifications in mathematical techniques because of the many possible interpretations of the word "best". The author points out, with strong justification, that an ellipsoid of revolution, although a satisfactory approximation for geometric applications like trigonometric survey, has no physical meaning and leads to complications when used as a reference surface for gravity measurements and crustal density determinations. He therefore develops a purely physical theory resulting in a specific level surface bounding a figure of heterogeneous density which closely approximates the actual Earth. No assumptions are made concerning density distributions in the Earth's crust, but hydrostatic equilibrium is taken as a basic postulate.

The development begins from the Helmert formula for an equipotential spheroid of fourth order which describes level surfaces external to a regularized Earth (that is, the Earth with random density variations in the crust removed). From this there follow eight well-known relations connecting thirteen parameters which are needed to completely describe the physical and geometric properties

of a specific figure. These relations do not include an equilibrium condition. Since the author assumes hydrostatic equilibrium, four parameters would suffice to define his "normal spheroid"; he selects the Earth's velocity of rotation, ω, the Earth's mass, E, the Earth's principal moment of inertia about its axis, C, and the potential of the actual geoid, W_0. In this particular development, the additional equilibrium condition is not given, and therefore in lieu of it, a fifth parameter has to be assumed. The one chosen is β_4, the coefficient of the highest-order term in the standard gravity formula. Based on results of a preceding paper [Z. Vermessungswesen **84** (1959), 73–90], the author asserts that β_4 is a function principally of ω, which is an accurately known empirical quantity, and hence β_4 can be obtained without further information on density variations in the Earth, which are largely unknown. The parameters E, C, and W_0 are not obtained directly. Instead, values of the Earth's semi-major axis, the equatorial gravity, and the dynamical flattening, d, are taken instead. Then by an ingenious method of iteration which applies the simplified Helmert relations for a homogeneous figure as a limiting case and extends this to the heterogeneous figure with corresponding initial values of ω and C, the complete set of thirteen parameters defining the normal spheroid is derived.

{Reviewer's comments: (1) In a later paper [Deutsche Geodätische Kommission bei der Bayerischen Akad. Wiss. (A) **36** (1960), 1–18], the author derives a theoretical equilibrium condition, dispensing with the assumption of a fifth parameter. By this approach he obtains numerical results quite different from those above. In particular, β_4 changes by about 1 part in 10^5, which is 10 times the largest possible variation for fixed ω claimed in the paper under review. (2) The author utilizes d rather than the so-called static flattening (which is the coefficient J_2 of the spherical harmonic development of the Earth's potential) because he states that the latter is imperfectly known. However, observations of artificial satellites have greatly improved the accuracy of J_2, and their results differ from the value obtained from d under the assumption of hydrostatic equilibrium.} *B. Chovitz* (Washington, D.C.)

OPERATIONS RESEARCH, ECONOMETRICS, GAMES

See also A12108, 12339, 12572, 13380.

13268:

Baumol, William J. ★Economic theory and operations analysis. Prentice-Hall International Series in Management. Prentice-Hall, Inc., Englewood Cliffs, N.J., 1961. x+438 pp. $9.00.

This book is essentially a survey of economic theory from the point of view of the mathematics of maximization or optimization. The first chapter exemplifies the relevance of formal maximization to economic decisions. The next three chapters explain the classical theory of maximization, beginning with the notion of differentiable functions, ending with the method of Lagrange multipliers, and covering the graphical and formal analysis of the marginal curves beloved of economists. Then follow three chapters at a more advanced level dealing with linear programming, non-linear programming, and integer programming (i.e., programming problems in which the variables are restricted to integral values).

Resting on this technical background, the central part of the book (Chapters 8 through 14) presents a swift survey of economic theory proper, with emphasis on the theories of demand and of the behavior of firms, but covering also the doctrines of general equilibrium, economic welfare, and the distribution of income.

The final eight chapters introduce the reader to a variety of recent developments in economic analysis, including input-output analysis, activity analysis, game theory and recent advances in the theories of utility and decision making.

The entire book is accurate, lively and clear, and for the most part intelligently selective of the depth of treatment and wealth of detail appropriate to a review of a broad range of topics. The mathematical students and operations research workers to whom it is addressed will find it an instructive introduction to the use that economists make of mathematical techniques and to the economist's style of thinking, as well as to the current state of many economic doctrines. *R. Dorfman* (Stanford, Calif.)

13269:

Jecklin, H. Renditenbestimmung mittels hyperbolischer Interpolation. Mitt. Verein. Schweiz. Versich.-Math. **60** (1960), 229–238. (English, French and Italian summaries)

The yield rate i on a financial obligation purchased other than at par value satisfies a transcendental equation of the form $y(i)=0$. A reasonably accurate value is obtained with relatively little arithmetical effort on the assumption that the function $y(i)$ can be approximately represented by a rectangular hyperbola with asymptotes parallel to the axes of i and y, using three known points to determine the hyperbola. The method is applicable to loans repayable at the end of a fixed period, linearly decreasing loans, and loans repayable by annuities.

T. N. E. Greville (Kensington, Md.)

13270:

Klein, L. R. Some theoretical issues in the measurement of capacity. Econometrica **28** (1960), 272–286.

Author's summary: "Alternative approaches to the measurement of capacity are possible. Economic aspects of capacity measurement, as contrasted with pure engineering considerations, involve the introduction of cost considerations and limitations imposed by interdependence of different sectors of the economy. Some approaches to capacity measurement in terms of cost functions are explored, particularly as properties of a probit total cost function. A method of aggregating individual capacities, taking into account the interdependence of an input-output model, is outlined. Capacity measures, so obtained, are useful as descriptive statistics of economic efficiency and as explanatory variables in modern theories of investment behavior."

13271:

Morishima, Michio. The problem of intrinsic complementarity and separability of goods. Metroecon. **11** (1959), 188–202.

The author gives the mathematical formulation of Hicks' "intrinsic complementarity" by introducing the

Sono-Leontief separability conditions into the traditional utility function. N commodities are divided into m groups, and each group of goods is assumed to be separable from the others in the preference function. Each group represents the means to attain an objective: $y_i=f_i(x_{i1}, \cdots, x_{ik})$, and $u=u(y_1, \cdots, y_m)$. By assuming homogeneity of degree one for f_i, the author suceeds in deriving many interesting deductive laws. The last section presents a method of introducing linear programming formulation instead of the f_i functions above to tackle the problem of intrinsic complementarity. *S. Ichimura* (Osaka)

13272:

Georgescu-Roegen, Nicholas. Mathematical proofs of the breakdown of capitalism. Econometrica **28** (1960), 225–243.

P. M. Sweezy [*Theory of capitalist development*, New York, Oxford, 1942; pp. 186–189] set up a model of the general economy along Marxist lines and sought to show that it could not satisfy natural equilibrium conditions. This failure was interpreted as showing the essential tendency of capitalism to develop crises and eventually, in some sense, lead to a breakdown. The author criticizes both Sweezy's proof and the technical correctness of the model itself. He constructs a model free of the defects attributed to Sweezy's and shows that it can rise to a consistent solution with a rapidly growing economy.

K. J. Arrow (Stanford, Calif.)

13273:

Bertoletti, Mario. Planning continuous production by linear programming. Management technology, Monograph No. 1, 1960, pp. 75–80.

A linear programming model in which the activities are simultaneous production processes in technologically feasible combinations.

M. J. Beckmann (Providence, R.I.)

13274:

Uzawa, Hirofumi. Preference and rational choice in the theory of consumption. Mathematical methods in the social sciences, 1959, pp. 129–148. Stanford Univ. Press, Stanford, Calif., 1960.

Two systems of axioms are presented, one referring to overt choice (also known as "revealed preference") and the other to preference, and various equivalences between the wholes or parts of the two systems are proved. More particularly the author shows [following the reviewer in Economica (N.S.) **17** (1950), 159–174; MR **13**, 146] that if choice functions satisfy certain obvious conditions as well as the strong axiom of revealed preference, then a relation on the set of positive commodity bundles is generated which possesses the usual properties of a preference relation. The relation of indifference is recognized to be inessential; problems of continuity are more carefully considered than in the preceding literature. The author also shows that the weak axiom of revealed preference implies the strong axiom if the choice functions satisfy a certain regularity condition. Finally, the symmetry of the substitution effect is derived directly from properties of the choice functions.

H. S. Houthakker (Cambridge, Mass.)

13275:

Fleischer, Isidore. Numerical representation of utility. J. Soc. Indust. Appl. Math. **9** (1961), 48–50.

A note related to current literature on the axiomatic foundations of utility measurement.

T. Haavelmo (Oslo)

13276:

Shiskin, Julius. Zerlegung wirtschaftlicher Zeitreihen in saisonale, zyklische und irreguläre Faktoren mittels elektronischer Rechenmaschinen. Math.-Tech.-Wirtschaft **7** (1960), 112–118.

The analysis of economic time series into seasonal, cycle and trend, and irregular components, is accomplished by the utilisation of large scale digital electronic computers. Graphs of monthly data from the period 1948–1958 are exhibited. The method is compared to the procedures of the National Bureau of Economic Research.

G. Tintner (Ames, Iowa)

13277:

Simon, Herbert A. ★The new science of management decision. The Ford Distinguished Lectures, Vol. III. Harper & Brothers Publishers, New York, 1960. xii+50 pp. $2.50.

This monograph is concerned with some relatively new instruments of management for use in processes for making decisions, especially as related to the use of electronic digital computers. The book evolved from a sequence of three lectures given by the author at New York University during the early spring of 1960.

The first chapter treats of the executive as a maker of decisions; in fact, managing is equated to decision-making. The author discusses "programmed" and "non-programmed" decisions. Crudely, these refer to decisions that deal with routine, repetitive matters and one-shot, policy situations, respectively. The former are readily programmed onto conventional digital computers; the latter are usually conceived as involving some kind of human judgment. In any case the author does not interpret this as a difference of kind but only as a difference of degree. The second chapter relates to classical methods of decision-making. He considers the habit-taking of William James and C. S. Peirce as the most general scheme for making programmed decisions. That is followed by the technique of establishing standard operating procedures in the society (e.g., business organization) under consideration. He closely allies nonprogrammed decision making with "learning processes" of psychology. The author gives an argument to justify "Gresham's Law of Planning", i.e., that in a given situation programmed activity tends to drive out nonprogrammed activity. Chapter three deals cursorily with some new techniques for programmed decision-making as relate to scientific management, e.g., operations research, cybernetics, game theory and dynamic programming. {Perhaps this chapter would have been more useful had the author referred to the following: Stafford Beer, *Cybernetics and management* [Wiley, New York, 1959]; F. H. George, *Automation, cybernetics and society* [Philosophical Library, New York, 1959]; G. P. Shultz and T. L. Whisler, editors, *Management organization and the computer* [Free Press, Glencoe, Ill., 1960].} Chapter four is concerned with heuristic problem solving: systematic, but not necessarily algorithmic, procedures for

solving problems. [Cf. Simon and Newell, Operations Res. **6** (1958), 1–10; R. Bellman, ibid., 448–449.] He presents the state-of-the-art to defend the thesis that "There is now good reason to believe that the process of non-programmed decision making will soon undergo as fundamental a revolution as the one which is currently transforming programmed decision making in business organizations". For a more mathematically technical exposition of heuristic problem solving see, e.g., M. L. Minsky's paper [Proc. IRE **49** (1961), 8–30]. The last chapter considers organization design as refers to man-machine systems for the making of decisions. Here the author presents some arguments to demonstrate the reasonableness of the proposition that organizations will continue to be hierarchical in form and will be constructed in three layers: (i) dealing with processes of production and distribution, (ii) concerning routine processes of decision and (iii) treating processes of a nonprogrammed nature as carried out by man-machine systems.

The text of the book is very well written but unfortunately the book is unindexed.

A. A. Mullin (Urbana, Ill.)

13278:

Flood, Merrill M. System engineering. Management technology, Monograph No. 1, 1960, pp. 21–35.

An expository paper drawing on well-known examples.

M. J. Beckman (Providence, R.I.)

13279:

Ackoff, Russell L. The meaning, scope, and methods of operations research. Progress in Operations Research, Vol. I, pp. 1–34. Wiley, New York, 1961.

13280:

Magee, John F.; Ernst, Martin L. Progress in operations research: the challenge of the future. Progress in Operations Research, Vol. I, pp. 465–491. Wiley, New York, 1961.

13281:

Phillips, A. W.; Quenouille, M. H. Estimation, regulation, and prediction in interdependent dynamic systems. Bull. Inst. Internat. Statist. **37** (1960), no. 2, 335–343. (French summary)

Expository paper emphasizing the similarity between automatic control in engineering and policy problems in economics. The problems of optimizing the performance of an economic system is illustrated with reference to a model with two simple relations.

H. Wold (Uppsala)

13282:

Morgenthaler, George W. The theory and application of simulation in operations research. Progress in Operations Research, Vol. I, pp. 363–419. Wiley, New York, 1961.

A comprehensive review and critical appraisal of simulation and operational gaming as analytical tools of operations research. An extensive bibliography is included.

F. Edelman (Princeton, N.J.)

13283:

McDowell, Ian. The economical planning period for engineering works. Operations Res. **8** (1960), 533–542.

A public utility which meets a growing demand for service must undergo periodic expansions of capital equipment. The cost of providing n new units of equipment is assumed to be $an+b$. Owing to the fixed cost term b, it is cheaper to expand periodically at the beginning of each "planning period", rather than continuously. In this paper the optimum planning period is computed for various values of a/b and the interest rate on capital expenditure. The results, for both exponential and linear demand growth, are presented in graphs.

B. A. Chartres (Providence, R.I.)

13284:

Rashevsky, N. A contribution to the mathematical biology of automobile driving. II. Passing as a case of psychophysical discrimination. Bull. Math. Biophys. **22** (1960), 263–267.

[For part I see same Bull. **22** (1960), 257–262; MR **22** #5482.]

13285:

Rashevsky, N. Contribution to the mathematical biophysics of automobile driving. Bull. Math. Biophys. **23** (1961), 19–29.

It is assumed that the reaction time of drivers is increased by distracting effects of passing. The minimum stopping distance depends upon the reaction time and, according to the author's previous theories, so does the maximum speed at which a car will stay on the road when there are errors in steering. Formulas are derived relating these safe speeds to the number of passings and thus to the dispersion in the velocities of cars.

G. Newell (Providence, R.I.)

13286:

Bisi, Walter. Controllo del traffico agli incroci stradali. Statistica (Bologna) **19** (1959), 422–433.

The traffic at a fixed cycle traffic light is treated essentially as a bulk service queue with Poisson arrivals. Tables of average waiting times are given. An optimal distribution of red and green times is chosen to minimize the sum of waiting times for all lanes of traffic that feed the same intersection.

G. Newell (Providence, R.I.)

13287:

Petigny, B. Le calcul des probabilités et la circulation des véhicules sur les chaussées à deux ou trois voies. Ann. Ponts Chaussées **131** (1961), 145–223.

This paper describes an extensive theoretical and empirical investigation of simple mathematical models for low density traffic flow on two- and three-lane roads. The topics discussed are the distribution of space and time between cars, the number of passings, time lost in passing and related subjects. The theory is not limited to Poisson traffic but assumptions of statistical independence of velocities and spacings are used frequently. Passing models are based upon dividing traffic into only two speed groups. The paper is written primarily for traffic engineers and includes considerable experimental data and background material on probability distributions, but the

author also manages to extract some simple approximations to mathematically formidable problems.

G. Newell (Providence, R.I.)

13288:

Kudô, Akio; Furukawa, Nagata. A model in probit analysis. Bull. Math. Statist. 8, 1–7 (1958).

13289:

Ficken, F. A. ★The simplex method of linear programming. Holt, Rinehart and Winston, New York, 1961. vi+58 pp. $1.50.

The author has separated the mathematical aspects of linear programming from the applied and computational areas and has developed a rigorous approach to the why's of Dantzig's simplex method of linear programming. This is done via the concepts of duality, feasibility, etc., which leads to what the author has termed the "prepared problem", i.e., the given linear program imbedded in an equivalent feasible and bounded linear program. From this the author discusses the simplex tableau and the conquest of degeneracy by the perturbed problem of Charnes. This presentation is accomplished in 36 pages, with the remaining pages devoted to two appendices which contain mathematical prerequisites, e.g., matrix algebra and theorems on existence and duality. The reader will find in this volume most of the necessary mathematical discussions related to linear programming (integer programming and the simplex method is an important omission); the initiated will find it an excellent compendium while the novice will find it difficult.

S. I. Gass (Silver Spring, Md.)

13290:

Hanssmann, Fred; Hess, Sidney W. A linear programming approach to production and employment scheduling. Management technology, Monograph No. 1, 1960, pp. 46–51.

This problem was treated by S. M. Johnson and G. B. Dantzig [Proc. 2nd Sympos. Linear Programming, pp. 151–176, National Bureau of Standards, Washington, D.C., 1955; MR 17, 759].

M. J. Beckmann (Providence, R.I.)

13291:

Eisemann, Kurt; Young, William M. Study of a textile mill with the aid of linear programming. Management technology, Monograph No. 1, 1960, pp. 52–63.

13292:

Suzuki, Yukio. Note on linear programming. Ann. Inst. Statist. Math. Tokyo **10** (1959), 89–105.

Suppose that x^0 is an optimal solution of the following linear programming problem: Maximize $c'x$ subject to $Ax \leqq P_0$, $x \geqq 0$. The paper discusses whether it is possible to make any simplification in solving the new problem with changes in c', A or P_0 if we know x^0. It also discusses the cases in which new restrictions are added or some restrictions are eliminated. *S. Ichimura* (Osaka)

13293:

Bulavskiĭ, V. A. An iterative method of solving the problem of linear programming. Dokl. Akad. Nauk SSSR **137** (1961), 258–260 (Russian); translated as Soviet Math. Dokl. **2**, 250–252.

Let $a^{(i)}$, $i=1, \cdots, m$, be given (real) n-dimensional vectors, and $p^{(i)}$, $i=1, \cdots, m$, given real numbers. The nonnegative vectors h satisfying the inner product inequalities $(a^{(i)}, h) \geqq p^{(i)}$, $i=1, \cdots, m$, define a closed convex subset Q of the space R_n of all real n-dimensional vectors. The problem of linear programming, called here problem I, is formulated as follows: For a given vector $c \in R_n$, find a vector $h_0 \in Q_n$ such that $(c, h_0) \leqq (c, h)$ for all $h \in Q$. A problem II is posed: For a given c and positive number σ, and a given $n \times n$ matrix B, find a vector $h_\sigma \in Q$ such that $(c+\sigma Bh_\sigma, h_\sigma) \leqq (c+\sigma Bh_\sigma, h)$ for all $h \in Q$. The aim of the paper is to solve problem I by solving problem II. To this end the author shows that problem II has a unique solution h_σ, which is a continuous function of σ, that (c, h_σ) is a monotonic increasing function of σ, and that there exists $\sigma_0 > 0$ such that for all $\sigma < \sigma_0$, h_σ is a solution of problem I. Thus solutions h_σ to problem II for all sufficiently small σ will provide solutions of problem I. A convergent iterative process, too lengthy to describe here, is established for finding h_σ.

R. F. Rinehart (Cleveland, Ohio)

13294:

Dantzig, George B. On the status of multistage linear programming problems. Bull. Inst. Internat. Statist. **36** (1958), no. 3, 303–320. (French summary)

The author gives in this article a survey of different methods for solving multistage linear programming problems, without going into any details. The references are the most valuable part of the paper.

L. Törnqvist (Helsinki)

13295:

Brudno, A. L. Lur'e's method of differential rents for determining the optimum transportation plan. Dokl. Akad. Nauk SSSR **131** (1960), 1238–1241 (Russian); translated as Soviet Math. Dokl. **1**, 405–408.

An algorithm is stated for solving the transportation problem of linear programming. It is a variant of the Hungarian method of Kuhn [Naval Res. Logist. Quart. **2** (1955), 83–97; MR 17, 759]. Additions, called "differential rents", are made at each step only to the rows of the cost matrix. Only those elements minimal in their columns are used to form the trial set at each step. The six theorems on which the algorithm is based are only stated but are easily proved, as corollaries of familiar theorems.

M. M. Flood (Ann Arbor, Mich.)

13296:

Pollack, Maurice; Wiebenson, Walter. Solutions of the shortest-route problem—a review. Operations Res. **8** (1960), 224–230.

This paper reviews and compares a number of published and unpublished methods (using linear programming, dynamic programming, and other approaches) which have been proposed for solving the problem of finding the shortest route through a network. It also states the duality between this problem and the network capacity problem.

E. F. Moore (Murray Hill, N.J.)

13297:

Kalaba, Robert. On some communication network

problems. Proc. Sympos. Appl. Math., Vol. 10, pp. 261–280. American Mathematical Society, Providence, R.I., 1960.

The paper surveys certain extremal problems connected with networks, with emphasis on efficient computational algorithms. The problems discussed are: (1) the finding of a minimal spanning tree in a network; (2) finding the shortest and also the kth shortest route through a network; (3) a stochastic version of (2) in which the transit times of edges are given by independent random variables with known distribution and it is desired to maximize the probability that the time of transit between two given points is less than some value t; (4) the problem of optimal routing of messages in a network, which is perhaps more familiar in the equivalent form of a multicommodity maximum flow problem. The methods of solution include the purely combinatorial in (1) and (2), the functional equation method of dynamic programming in (2) and (3), and adaptations of the simplex method in (4). *D. Gale* (Providence, R.I.)

13298:

Chien, R. T. Synthesis of a communication net. IBM J. Res. Develop. 4 (1960), 311–320.

The author explains that the properties of a communication net can be described in terms of two matrices B and T. The element b_{ij} of B denotes the capacity of the edge joining the vertices v_i and v_j (or is 0 if there is no such edge). The element t_{ij} of T is the maximum possible communication capacity between v_i and v_j. The author considers the problem of finding B when T is given (and satisfies the known necessary and sufficient conditions for the existence of a B). He shows how this problem can be solved so as to obtain the least possible sum for the elements of B. *W. T. Tutte* (Toronto)

13299:

Wing, O.; Chien, R. T. Optimal synthesis of a communication net. IRE Trans. **CT-8** (1961), 44–49.

This paper gives solutions to the problem of realizing a communication network at minimum cost. The communication network is represented by a linear graph with nodes and branches. Associated with the branch connecting nodes i and j are two quantities: (1) the branch capacity b_{ij}, and (2) the cost c_{ij} of installing a unit capacity between the nodes. The network is assumed to be bilateral. The system requirement is given by the "terminal capacity matrix" T, with elements $t_{ij}=t_{ji}$ denoting flow capacity. The set of branch capacities can be put into the "branch capacity matrix" B.

The synthesis problem can be stated as follows: Given a terminal capacity matrix T and the set of unit costs c_{ij}, find a graph whose branch capacity matrix B realizes T and is such that the system cost is minimum.

Two cases are considered. (1) The unit costs of the branches are in general all different. The technique of linear programming (section III) is applied to find the minimum cost or optimal realization. (2) The unit costs are assumed to be all the same. Several minimal realizations are obtained without the use of linear programming. The realizations require fewer branches than previously reported (section IV).

The problem considered is related to the transportation problem and the authors suggest that simple algorithms merit investigation. They note also that "the determination of the minimum number of branches required in a minimal realization is as yet an unsolved problem". *Y. H. Ku* (Philadelphia, Pa.)

13300:

Tintner, Gerhard. A note on stochastic linear programming. Econometrica **28** (1960), 490–495.

Author's summary: "In linear programming we assume that all the parameters of the problem, i.e., the coefficients of the objective function, the inequalities and the availabilities are known numbers. This is frequently a not very realistic assumption. In stochastic linear programming the parameters become random variables, i.e., we know only their distribution. In the passive approach to stochastic linear programming the distribution of the objective function is approximated and decisions are based upon this distribution. In the active approach the decision variables are the amounts of resources to be devoted to the various activities."

13301:

Votaw, D. F., Jr. Statistical programming. Ann. Math. Statist. **31** (1960), 1077–1083.

The objective function is $\sum_{i=1}^n \sum_{j=1}^n c_{ij}x_{ij}$. This is to be maximized by choice of the x_{ij} under the conditions: $\sum_{i=1}^n x_{ij}=1$, $\sum_{j=1}^n x_{ij}=1$, x_{ij} is 0 or 1. Let $j_1, \cdots, j_n$ be a permutation of $1, \cdots, n$. Then the problem is solved by finding the permutation $j_1', \cdots, j_n'$ which maximizes the objective function $\sum_{i=1}^n c_{ij'}$.

Now let $H(w_{11}, \cdots, w_{nn})$ be a n^2-dimensional probability distribution with the c_{ij} as parameters. Each of the $n!$ subsets represents a permutation. If observations are available, the permutation corresponding to the largest estimate is selected. This is programming by estimation. If all $n!$ permutations are assumed to be equally probable we have a distribution $H(w_{11}, \cdots, w_{nn})$. This leads to random programming.

Under certain assumptions the performance function of programming by estimation is larger than or equal to the performance function under random programming. The same holds for the mathematical expectation of the objective function. The relationship to the transportation problem is pointed out, some examples based upon negative exponential distributions are included and the extension to the generalized optimum assignment problem indicated. *G. Tintner* (Ames, Iowa)

13302:

Zoutendijk, G. Maximizing a function in a convex region. J. Roy. Statist. Soc. Ser. B **21** (1959), 338–355.

The problem considered is that of maximizing a function f in a convex region. To solve this problem a "method of feasible directions" is developed. It is a method of steepest ascent. Points in the convex region will be called feasible. Starting with an initial feasible estimate of the solution, a sequence of feasible trial solutions with ever-increasing values of f is obtained. This sequence will converge to a (local) maximum in all cases of importance. To obtain a new trial solution one must determine (a) a direction in which f increases through feasible points and (b) the

length of step to be taken. The method of feasible directions is first developed with linear constraints and applied to special cases of a linear and a quadratic objective function f. In the latter two cases the number of steps is finite. An extension of the method to problems involving nonlinear constraints is also considered.

M. R. Hestenes (Los Angeles, Calif.)

13303:

Elmaghraby, Salah E. Allocation under uncertainty when the demand has continuous d.f. Management. Sci. **6** (1960), 270–294.

Let there be n products, each producible from each of m resources. Let r_{ij} be the output of product j from one unit of resource i, c_{ij} the cost of such production, C_{0j} the storage cost per unit of product j, C_{uj} the penalty cost per unit of unfilled demand, S_j the total output of product j, X_{ij} the amount of resource i used to produce product j, a_i the amount of resource i available, and $f_j(y)$ the density of demand for product j, assumed random. Then aggregate cost is given by

$$C = \sum_i \sum_j c_{ij} r_{ij} X_{ij} + \sum_j C_{0j} \int_0^{S_j} (S_j - y) f_j(y)\,dy + \sum_j C_{uj} \int_{S_j}^{\infty} (y - S_j) f_j(y)\,dy,$$

which is to be minimized subject to the resource constraints $\sum_j X_{ij} \leqq a_i$. Since C is a convex function of the variables X_{ij}, the Lagrangian conditions as modified by Kuhn and Tucker [Proc. 2nd Berkeley Sympos. Math. Statist. and Probability, 1950, pp. 481–492, Univ. of Calif. Press, Berkeley, Calif., 1951; MR **13**, 855] are applicable. The author suggests a procedure of successive adjustments to bring about the desired relations among derivatives for achieving a minimum. Each step requires the solution of a system of non-linear equations; for a proof that the minimum will be attained in a finite number of steps, the reader is referred to the author's unpublished doctoral dissertation.

K. J. Arrow (Stanford, Calif.)

13304:

Dreyfus, Stuart. Dynamic programming. Progress in Operations Research, Vol. I, pp. 211–242. Wiley, New York, 1961.

A lucid introduction to the field of dynamic programming, covering mathematical formulation, analytical considerations, and computational solution of a variety of typical problems in operations research.

R. Kalaba (Santa Monica, Calif.)

13305:

Bellman, Richard; Kalaba, Robert. On kth best policies. J. Soc. Indust. Appl. Math. **8** (1960), 582–588.

Consider a network of nodes and interconnecting links, and suppose that the time to travel along any connecting link is known. The problem is to find the best (i.e., shortest) path between two particular nodes, the second best, $\cdots$, the kth best. This problem is formulated using the functional equation technique of dynamic programming, and straightforward algorithms for its solution are described. These results are generalized to a situation where it is required to maximize a prescribed function of a final state of a system, where the states are specified by vectors. They are also generalized to a similar situation in which the results of making decisions are known only in that the probability of each result is known.

Mention is made of the need to know a kth best policy in the problem of searching for available telephone circuits. Here "best" is in the sense of "most likely to be available". It is also pointed out that knowledge of nearly best solutions can be helpful in assessing the sensitivity, or stability, of best solutions to certain problems.

T. E. Hull (Vancouver, B.C.)

13306:

Prabhu, N. U. A problem in optimum storage. Calcutta Statist. Assoc. Bull. **10** (1960), 35–40.

A procedure is given for determining an optimum policy from a very restrictive class of policies in a water storage problem. The procedure is based on a stationary analysis of the dam content for the special case in which the water input distribution is negative exponential.

H. Scarf (Stanford, Calif.)

13307:

Karlin, Samuel. Optimal policy for dynamic inventory process with stochastic demands subject to seasonal variations. J. Soc. Indust. Appl. Math. **8** (1960), 611–629.

If the number of state variables in an inventory problem is fairly small, optimal stockage policies may always be found by means of a recursive calculation or dynamic programming. There are some cases, however, in which more or less explicit analytic answers may be given in terms of the parameters of the problems. This can be done in a single period inventory model, or in an infinite period stationary model with suitable assumptions on the cost functions.

In the present paper, the author adds an important case to the two mentioned above. He develops an algorithm for the case in which the demand contributions are periodic. The algorithm involves the solution of at most $2n-1$ transcendental equations with unique solutions (n is the period of the demand), and at least for moderately small values of n, seems to be quite efficient.

H. Scarf (Stanford, Calif.)

13308:

Vorob'ev, N. N. Finite non-coalitional games. Uspehi Mat. Nauk **14** (1959), no. 4 (88), 21–56. (Russian)

A systematic exposition of the main features of the subject. § 1 studies n-person games in canonical form, culminating in the theorem of J. Nash [Ann. of Math. (2) **54** (1951), 286–295; MR **13**, 261] on the existence of equilibrium points. § 2 deals with zero-sum two-person games in canonical form. Among the material covered are the G. W. Brown and J. von Neumann [*Contributions to the theory of games I*, pp. 73–79, Princeton Univ. Press, Princeton, N.J., 1950; MR **12**, 514] method of solution by differential equations and the G. W. Brown and J. Robinson [Ann. of Math. (2) **54** (1951), 296–301; MR **13**, 261] iterative method, as well as (without proof) the result of D. Gale and S. Sherman [above *Contributions*, pp. 37–49; MR **12**, 513] on sets of optimal strategies. § 3 considers games in extensive form (called positional games) and proves the theorem [von Neumann and Morgenstern, *Theory of games and economic behavior*, Princeton Univ. Press, Princeton, N.J., 1944; MR **6**, 235; H. W. Kuhn,

Proc. Nat. Acad. Sci. U.S.A. **36** (1950), 570–576; *Contributions to the theory of games II*, pp. 193–216, Princeton Univ. Press, Princeton, N.J., 1953; MR **12**, 515; **14**, 999] on games of perfect information.

A. Dvoretzky (Jerusalem)

13309:

Vorobiev, N. N. Finite non-coalitional games. Acad. R. P. Romîne. An. Romîno-Soviet. Ser. Mat.-Fiz. (3) **14** (1960), no. 1 (32), 20–57. (Romanian)

Translation of a Russian original [#13308].

13310:

Dresher, Melvin. ★Games of strategy: Theory and applications. Prentice-Hall Applied Mathematics Series. Prentice-Hall, Inc., Englewood Cliffs, N.J., 1961. xii + 186 pp. $9.00.

There are eleven chapters in this book.

Chapter 1: Game, Strategy, and Saddle-Point. Chapter 2: The Fundamental Theorem. Chapter 3: Properties of Optimal Strategies. These three chapters lay the foundation of game theory on which the remainder of the book is based. They give a clear and concise statement of the basic rules and theorems. Here, as in the rest of the book, several illustrative examples serve to impress the subject matter upon the reader's mind.

Chapter 4: Games in Extensive Form. Chapter 5: Methods of Solving Games. The first of these two chapters is very short and deals essentially with the representation of games by topological trees. The other chapter discusses methods of obtaining optimal strategies.

The illustrative examples in the first five chapters are drawn from both parlor games and military games. In the remaining six chapters the emphasis is solely on the latter.

Chapter 6: Games with Infinite Number of Strategies. Chapter 7: Solution of Infinite Games. Chapter 8: Games with Convex Payoff Functions. Continuous games are introduced and discussed in these three chapters. Since a knowledge of Stieltjes integrals is necessary to the understanding of such games, a brief introduction to these integrals is given in Chapter 6.

Chapter 9: Games of Timing—Duels. Chapter 10: Tactical Air-War Games. The theory of continuous games developed in the preceding three chapters is applied to the above two general classes of war games. The chapter on duels is particularly comprehensive.

Chapter 11: Infinite Games with Separable Payoff Functions. Since the usual submatrix-inversion procedure for solving finite games is not applicable to infinite games, for the solution of a certain class of these latter (those with separable payoff functions), a method based on moment space theory is developed.

In an overall view, the distinguishing features which make this book a welcome addition to the game theorist's library, are the emphasis on the application to war games and the copious use of pertinent illustrative examples.

N. H. Choksy (Silver Spring, Md.)

13311:

Wu, Wen-tsün. A remark on the fundamental theorem in the theory of games. Sci. Record (N.S.) **3** (1959), 229–233.

In contrast to H. Weyl's purely algebraic proof of the fundamental minimax theorem of von Neumann, the author gives a purely topological statement and proof of a generalized minimax theorem which uses point set topology without recourse to fixed point theorems or to theorems about convex sets. The main theorem reads as follows: Let Y be a compact separable space while X is arcwise connected. If $f: X\times Y\to R$ is strongly connected in X, and f_x, f_y are all continuous for any $x\in X$, $y\in Y$, then $\inf_y \sup_x f(x,y)=\sup_x \inf_y f(x,y)$. Strong connectivity is defined in a topological manner and it is indicated that statements and proofs of the minimax theorem by Ville, Wald, Kneser, Nikaido, etc., are subsumed under the author's version.

H. Raiffa (Cambridge, Mass.)

13312:

Fox, Martin. Some zero sum two-person games with moves in the unit interval. Pacific J. Math. **10** (1960), 1235–1242.

In this generalization of games on the unit square, the two players alternately choose points t_i on $[0,1]$, $i=1,2,\cdots,n$. After the ith move, the information available to the next player is a function $\varphi_i(t_1,t_2,\cdots,t_i)$. The pay-off is a continuous real-valued function $f(t_1,t_2,\cdots,t_n)$. The pure strategies for players I and II are vectors $x=(x_1,x_2,\cdots,x_{[(n+1)/2]})$ and $y=(y_1,y_2,\cdots,y_{[n/2]})$ where the x_j, y_k are functions of the available φ's. Thus on the fourth move of the game, $t_4=y_2[\varphi_3(t_1,t_2,t_3)]$.

For the classical case $(n=2)$, it is shown, in agreement with Ville, that the game has a value. It is not necessary that $\varphi_1(t_1)$ should be measurable.

For $n=3$, a counter-example is presented in which the functions φ_i each take only a finite number of values and the game has no value.

For $n=4$, a counter-example with continuous φ_i is given; again the game has no value. The case $n=3$ with continuous φ_i is unsolved.

Finally it is shown that with arbitrary n the game has a value provided that the φ_i each take only a finite number of values and each is constant on sets which are finite unions of i-dimensional generalized intervals.

E. S. Keeping (Edmonton, Alta.)

13313:

Dubins, Lester E.; Savage, Leonard J. Optimal gambling systems. Proc. Nat. Acad. Sci. U.S.A. **46** (1960), 1597–1598.

This is an announcement of sample results of an extensive study of gambling systems. We quote one of the three theorems of the paper: Let $0<r<w<1$, and suppose a gambler whose initial fortune is positive can make a sequence of bets on any of which he can stake any amount then in his possession. On each bet the ratio of possible gain to possible loss is $1-r:r$ and the probabilities of win and loss are w and $1-w$, respectively. Assume a prescribed G is the gambler's goal and call a betting strategy optimal if it maximizes the probability of the gambler's fortune ever attaining G. Then the bold betting strategy (stake as much as possible consistent with the condition that the fortune never exceed G nor become negative, i.e., stake $\min(x, r(G-x)/(1-r))$ when the fortune $x\in(0,G)$) is optimal, but there exist also other optimal strategies.

A. Dvoretzky (Jerusalem)

13314:

Katz, Melvin. Infinitely repeatable games. Pacific J. Math. **10** (1960), 879–885.

The author considers sequences of plays of a finite 2-person game with vector payoffs. For each play the players choose a pair of pure strategies (i, j) and, depending on these choices, a number a is chosen from a finite set according to a probability distribution which depends on (i, j). Let a_k be the number chosen on the kth play. A strategy for player I consists of a sequence of functions $f=\{f_n\}$, where f_n prescribes the mixed strategy to be used in the $(n+1)$st play as a function of $(a_1, \cdots, a_n)$. This device allows player I to obtain information about the outcomes of the previous plays. A strategy for player II is a fixed sequence h of mixed strategies. The pair (f, h) defines a vector-valued stochastic process.

If Y_n is the vector outcome for the nth play and S is a closed set of vectors, the set S is approachable with the fixed strategy f^* if $\lim \delta_n=0$ with probability 1 uniformly for all (f^*, h), where δ_n is the distance between $\sum_{i=1}^n Y_i/n$ and S. The author gives some necessary and sufficient conditions and some sufficient conditions for a set S to be approachable.

Let $g(i, j)$ be the vector outcome corresponding to (i, j). For a given mixed strategy $P=(p_1, \cdots, p_r)$ for player I, let $R(P)$ be the convex hull of the points $\sum_i p_i g(i, j)$. Theorem 1: If a is independent of j in its dependence on (i, j), then a set S is approachable if and only if there exists a P such that $R(P) \subseteq S$. If player I is given complete information about player II's past choices but not about his own, then a set is approachable if for every $x \notin S$, there exists a P such that the plane through y, the closest point in S to x, perpendicular to the line segment xy separates x from $R(P)$. Theorems 3 and 4 are generalizations of theorems by Blackwell [same J. **6** (1956), 1–8; MR **18**, 450]. *E. D. Nering* (Tempe, Ariz.)

13315:

Isbell, J. R. A modification of Harsanyi's bargaining model. Bull. Amer. Math. Soc. **66** (1960), 70–73.

The author modifies Harsanyi's bargaining model [*Contributions to the theory of games*, Vol. IV, pp. 325–355, Princeton Univ. Press, Princeton, N.J., 1959; MR **21** #4062] and shows that the modified theory results in a unique imputation, called the bargaining value of the game, with the property that each player obtains at least as much as he could obtain if all other players formed a coalition against him. The author indicates by means of a counter-example that Harsanyi's solution does not satisfy the above property. *H. Raiffa* (Cambridge, Mass.)

13316:

Ingerman, Peter Z. A note on the calculation of interest. Comm. ACM **3** (1960), 542–543.

This note deals with a method for determining the periodic payment necessary to retire a loan when the interest rate on the loan varies as a (step-) function of the as-yet-unpaid principal. The author states that "the method, while requiring in some instances several iterations, converges rapidly and usually requires fewer iterations than a pure cut-and-dry-technique". A description of the process in the ALGOL programming language is given. {The reviewer believes that B_n should be changed to B_{n+1} in the inequality on page 542 and that "$+\cdots$" has been omitted from the left member of each equation in the last set of equations on page 542.} *T. N. E. Greville* (Kensington, Md.)

13317:

Sumitsuji, Osamu. Some elementary researches in the mathematics of life insurance. II. Mitt. Verein. Schweiz. Versich.-Math. **60** (1960), 201–213. (German, French and Italian summaries)

Fortsetzung der 1959 erschienenen Arbeit [selbe Mitt. **59** (1959), 163–198; MR **21** #3270]. Näherungsformeln für Prämie und Reserve der gemischten Versicherung bei erhöhter Sterblichkeit. *W. Saxer* (Zürich)

13318:

Iff, Paul. Zur Darstellung versicherungstechnischer Werte durch Reihen. Mitt. Verein. Schweiz. Versich.-Math. **60** (1960), 239–245. (English, French and Italian summaries)

An integral or a sum of a product of two functions is developed in a well-known series of repeated integrals or sums of one function only, the differential quotients of the other function delivering coefficients to the series. When used for actuarial functions the procedure gives some approximations from life insurance mathematics. *P. Johansen* (Copenhagen)

BIOLOGY AND SOCIOLOGY

See also 12681, 13382.

13319:

Verhagen, A. M. W. Growth curves and their functional form. Austral. J. Statist. **2** (1960), 122–127.

The paper discusses the justification for using the Gompertz and logistic curves as graduation formulae for growth data. The mathematics is elementary. Not all assumptions used in the proofs are stated in the formulation of the theorems. {In section 9 the statement "$F(\tau)$ has thus only the first derivatives" should be replaced by "$F(\tau)$ has all derivatives beyond the first one equal to zero".} *E. Seiden* (E. Lansing, Mich.)

13320:

Anderson, R. L. Uses of variance component analysis in the interpretation of biological experiments. Bull. Inst. Internat. Statist. **37** (1960), no. 3, 71–90. (French summary)

13321:

Ashford, J. R.; Smith, C. S.; Brown, Susannah. The quantal response analysis of a series of biological assays on the same subjects. Biometrika **47** (1960), 23–32.

The paper is an able exposition of the principles and practice of a succession of biological assays on the same subjects classified under several arbitrary groups which remain the same in all the assays. Methods of analysis of quantal responses in single assays for single poisons whose actions on a number of definite organisms are non-reversible, or for a number of such poisons exhibiting similar non-reversible joint action have been given and extended to cover the case of a series of assays.

The effect of errors of observation has received particular attention. An illustrative example giving results from researches conducted on coal miners, of the incidence of

pneumoconiosis due to inhalation of coal dust, as revealed through radiological surveys, has been used to thoroughly illustrate the theory and the straightforward calculations involved in the estimation of parameters by the method of maximum likelihood, the variance covariance matrix, tests of goodness of fit by the χ^2 method and the 95% fiducial limits for the ED 50's. Certain refinements such as weighting the observations which were considered unnecessary in the particular example were omitted also from the theoretical discussion.

The example dealing with two assays on the same groups of subjects in the presence of error could lead to four different responses, viz., $(--)$, $(-+)$, $(++)$ and $(+-)$; but the actual experiment did not show the result $(+-)$, which, indeed, would not appear, unless errors of observation or classification occurred, in the case of non-reversible similar action. The theory developed by the authors has been fully worked out taking into account the true quantitative responses y, and the observed quantitative responses, y', $y'=z+\varepsilon+\xi$, where z is a function only of the poisons and their doses, ε is a function only of the susceptibilities of the particular subjects and ξ is a random normal error with zero mean. It is interesting both for the final results obtained and for the many assumptions involved. Biological assays, more than other branches of statistics, require assumptions which are often very difficult to justify or to verify without extensive experimental study. Nevertheless, the exposition has been clear-cut and systematic. In cases of doubt or difficulty the reader may consult the references cited.

Q. M. Husain (Dacca)

13322:

Malecot, G. Les modeles stochastiques en génétique de population. Publ. Inst. Statist. Univ. Paris **8** (1959), 173–210.

Valuable survey article outlining the author's contributions to the theory of random processes in the mathematical theory of natural selection. Particular attention is paid to cases of a number of different populations, with different selection and mutation rates, and intermigration between them.

P. A. P. Moran (Canberra)

13323:

Castoldi, Luigi. Attorno a un problema di genetica delle popolazioni: la stabilità endemica dell'anemia di Cooley e della drepanocitemia. Rend. Sem. Fac. Sci. Univ. Cagliari **28** (1958), 161–168. (French summary)

Es seien A und a alle eines Gens, so daß der Genotyp aa mit der Wahrscheinlichkeit λ eine bestimmte Krankheit wie die im Titel genannten mit vorzeitig tödlichem Ausgang und Fortpflanzungsunfähigkeit verursacht. Um die aus einem Modell mit konstanten Häufigkeiten der Genotypen AA, Aa und aa innerhalb einer Generation nicht abzuleitende Stationarität dieser tödlichen Krankheitsform zu erklären, nimmt der Verfasser an, daß in jeder Generation mit einer gewissen positiven Wahrscheinlichkeit μ eine Mutation $A\to a$ stattfindet. Sein Modell führt im Fall $\lambda=1$ und bei kleinem μ darauf, daß die Häufigkeit der tödlichen Krankheitsform näherungsweise gleich μ wird.

K. Krickeberg (Heidelberg)

13324:

Castoldi, Luigi. A proposito di una recente ipotesi genetica. Rend. Sem. Fac. Sci. Univ. Cagliari **29** (1959), 218–219. (English summary)

In einer früheren Arbeit [Atti Accad. Ligure **13** (1957), 45–54] hatte der Verfasser zur Erklärung des Häufigkeitsverhältnisses der beiden menschlichen Geschlechter die Existenz von Maskulinen vom Typ YY postuliert, was der Referent dieser Arbeit in MR **19**, 988 als "biologically untenable" bezeichnete. Der Verfasser verteidigt nun seinen Standpunkt und erwartet baldige experimentelle Bestätigung.

K. Krickeberg (Heidelberg)

13325:

Thullen, Peter. Über die Eintritts- und Abnahmeintensitäten einer Bevölkerung und über das Verhalten der Bevölkerungsfunktion, insbesondere relativ stationärer Bevölkerungen. Mitt. Verein. Schweiz. Versich.-Math. **60** (1960), 187–200. (French, English and Italian summaries)

The size of a population is regarded as a function of the age of the individuals and the calendar time, together with the corresponding forces of increase (entrance) and decrease (elimination).

For a relatively stationary population, the existence of a stable structure is proved and this structure is discussed.

P. Johansen (Copenhagen)

13326:

Consael, R.; Lamens, A. Processus markoviens d'embranchement en démographie. Bull. Inst. Internat. Statist. **37** (1960), no. 3, 271–289. (English summary)

13327:

Leslie, P. H.; Gower, J. C. The properties of a stochastic model for the predator-prey type of interaction between two species. Biometrika **47** (1960), 219–234.

In P. H. Leslie, Biometrika **45** (1958), 16–31 [MR **19**, 1245], the deterministic model for prey and predator population sizes ($N_1(t)$, $N_2(t)$ respectively) at unit time intervals, given by

$$N_1(t+1) = \lambda_1 N_1(t)/[1+\alpha_1 N_1(t)+\beta_1 N_2(t)],$$
$$N_2(t+1) = \lambda_2 N_2(t)/[1+\alpha_2\{N_2(t)/N_1(t)\}],$$

was replaced by the following stochastic model: Given $N_1(t)$ and $N_2(t)$, $N_1(t+1)$ and $N_2(t+1)$ are normally distributed with the above means, with variances appropriate to constant birth-rates in birth and death processes and with negative values replaced by zero. In continuation of P. H. Leslie and J. C. Gower, Biometrika **45** (1958), 316–330 [MR **21** #1241], Monte Carlo runs were made to give insight into (i) fluctuations of $N_1(t)$, $N_2(t)$, (ii) the chances of random extinction, and (iii) the case when only a fraction of the prey population is exposed to the predator. However, insufficient runs were taken to permit very firm conclusions.

M. Stone (Princeton, N.J.)

13328:

Willner, Dorothy (Editor). ★Decisions, values and groups. Vol. 1. Reports from the First Interdisciplinary Conference in the Behavioral Science Division held at the University of New Mexico; sponsored by the Air Force Office of Scientific Research. With an introduction by

Anatol Rapoport. Pergamon Press, New York-Oxford-London-Paris, 1960. xxix+348 pp. $12.50.

The articles of mathematical interest will be reviewed separately.

13329:

Bush, Robert R.; Estes, William K. (Editors). ★Studies in mathematical learning theory. Stanford Mathematical Studies in the Social Sciences, III. Stanford University Press, Stanford, Calif., 1959. viii+432 pp. $11.50.

This collection of related papers concerns two families of models for animal learning behavior. The "linear" learning models concern processes in which the organism has, at each of a sequence of discrete moments, a certain state matrix of probabilities p_{ij} that the response R_j will occur as a result of the stimulus S_i. After each stimulus-response, the creature is "reinforced" by one of a variety of operators which modify the p_{ij}'s in a manner depending linearly on their previous values. The choice of the reinforcement operator is determined by some function of the previous stimulus, response, and reinforcement signal.

The second class of models consists of the "stimulus-element" learning models. Here one considers the creature to be in contact with the environment through a set of input channels, each of which is a Boolean function of the state of the environment and is called a "stimulus-element". At each moment, the creature randomly selects a subset of these, independently from trial to trial. An element so sampled becomes "conditioned" to the subsequent response; the responses are in turn determined by the distribution of the responses that have previously become conditioned to the elements chosen in the current sample. For large numbers of stimulus-elements, this "all-or-none" conditioning yields behavior which can be approximated by special cases of the linear models mentioned above, so that all the work here is closely related.

The book contains seven papers on each of these classes of models and six papers extending and comparing them. All are theoretical; some discuss experimental work as well. One paper, by Bush and Mosteller, compares eight different models with the same set of experimental data.

For each choice function and set of reinforcement operators, we have a different linear learning model. If one further chooses an experimental situation, i.e., an environment which interacts with the model to provide it with a sequence of stimuli and reinforcement events, one can then ask questions about the transient and asymptotic behavior of the state matrices. Many forms of these situations are examined, and theorems proven by a variety of difference-equation and stochastic-process methods. There is also a body of work concerning the statistical question of estimating the parameters for an assumed model, given a body of empirical data. There has been much work on these problems over the last decade, and this volume appears to contain all known results and to go considerably further. The results are too involved to be summarized here; the editors are to be complimented on collecting (and creating) in one place so much material on the subject. The fifteen authors include those primarily responsible for this body of theory, notably, W. K. Estes, R. R. Bush, F. Mosteller, and C. J. Burke.

There is no really general discussion to help the novice to place this rather special class of models in perspective with the general problems of learning in psychology and in the artificial intelligence area. This seems serious to the reviewer, because only a lengthy and often tedious exploration of the results will clearly reveal that the theory has not progressed to the point of handling problems in which the choice of reinforcement operator depends in a non-trivial way on the pattern of responses; this makes it difficult to apply the results to real problem-solving models. (There are no claims that this can be done, to be sure.) The specialized nature of these theories is further obscured by the practice of referring in the literature to the linear models as "Stochastic Learning Theory" and to the stimulus-element models as "Statistical Learning Theory"; this nomenclature should be withdrawn.

Reading of the text requires substantial familiarity with probability theory, e.g., at the level of Feller's text. Valuable preparation would be provided also by the text of Bush and Mosteller [*Stochastic models for learning*, Wiley, New York, 1955; MR **16**, 1136].

M. L. Minsky (Cambridge, Mass.)

13330a:

Rosenblatt, Frank. ★The perceptron: A theory of statistical separability in cognitive systems. Cornell Aeronautical Laboratory, Inc., Rep. No. VG-1196-G-1. U.S. Department of Commerce, Office of Technical Services, PB 151247, 1958. xii+262 pp. $4.00.

13330b:

Rosenblatt, Frank. ★Two theorems of statistical separability in the perceptron. Cornell Aeronautical Laboratory, Inc., Rep. No. VG-1196-G-2. U.S. Department of Commerce, Office of Technical Services, PB 151247 S, 1958. iii+42 pp. $1.25.

These papers discuss a family of learning devices composed of two layers of computing elements. Each element of the first layer, called an "A-unit", computes some Boolean function of the set of stimulus signals from the environment; the A-unit "fires" if the stimulus satisfies some condition associated with that unit. Each A-unit has outputs connected to one or more second-layer elements called R-units. When an A-unit fires it transmits a signal of a certain magnitude to each of the R-units to which it is connected; the R-units simply sum these signals, and are arranged to inhibit each other so that just that one with the largest sum is permitted to "fire". If an A-unit fires, and is connected to an R unit which fires, then the magnitude of that A-unit's signal is incremented. If the experimenter forces a given R-unit to fire, then just those A-units are incremented which (i) respond to stimuli that occur and (ii) are connected to the chosen R-unit; the experimenter can thus influence the connections between stimulus-detectors and responses, and learning can occur. This is similar to the "stimulus-sampling" models described by Estes [see review #13329 above] except that with each reinforcement every connection involved is modified slightly, while in the stimulus-element model there is a large change in each of a small sample of the connections involved. In order to make such a system stable, one requires a technique for depressing the magnitudes of unreinforced connections; several configurations are proposed and arguments advanced concerning biological plausibility. Systems of this class have been reviewed recently by Hawkins in

[Proc. IRE **49** (1961), 31–48]. The mathematical analysis given is too involved to discuss here, especially in the light of the author's apparent admission that some of the theorems are false; more recent publications of the author appear to set a more rigorous standard. Fortunately, computer simulation of these systems is feasible, and in view of the mathematical difficulties encountered in statistical learning theories, the reviewer feels that simulation is the only usable standard for evaluation of such systems. The reviewer doubts that with the simple, randomly chosen A-unit functions proposed by the author, any perceptron of plausible size will be able to recognize visual patterns of considerable complexity, independent of size and position, as the author seems to maintain, without having seen essentially a dense subset of instances of the pattern. As indicated in the reviewer's article [ibid. **49** (1961), 8–30; esp. p. 13] such a device could approximate a Bayes analysis of the input data, assuming that the A-units detect independent information.

A new report [Cornell Aeronautical Lab. Rep. No. VG-1196-G-8] contains a much more comprehensive and careful analysis of models of learning in the central nervous system. This new report presumably corrects the mathematical errors and appears to replace these original papers by a discussion which meets many of the criticisms of the reviewer and others.

M. L. Minsky (Cambridge, Mass.)

13331:

Rapoport, Anatol; Horvath, William J. The theoretical channel capacity of a single neuron as determined by various coding systems. Information and Control **3** (1960), 335–350.

Die informationstheoretische Kanalkapazität eines Neurons war von MacKay und McCulloch [Bull. Math. Biophys. **14** (1952), 127–135] mit Hilfe zweier diskreter Modelle berechnet geworden. Die Verfasser bedienen sich eines kontinuierlichen Modells und berechnen die Übertragungsgeschwindigkeit bei einer Poissonschen Quelle mit zwei verschiedenen Störungsverteilungen, einmal einer in einem Intervall gleichverteilten und das andere Mal mit einer Gaußschen. Das letztere Modell steht in guter Übereinstimmung mit gewissen Messungen.

K. Krickeberg (Heidelberg)

INFORMATION AND COMMUNICATION THEORY

See also 12673, 12675, 13331.

13332:

Gnedenko, B. V. Some Soviet work on information theory. Transactions of the first Prague conference on information theory, statistical decision functions, random processes held at Liblice near Prague from November 28 to 30, 1956, pp. 21–28. Publishing House of the Czechoslovak Academy of Sciences, Prague, 1957. 354 pp. Kčs 34.00. (Russian)

A brief exposition, with 12 Soviet references going back to Kotel'nikov (1933).

13333:

Stewart, John L. ★Fundamentals of signal theory. McGraw-Hill Electrical and Electronic Engineering Series. McGraw-Hill Book Co., Inc., New York-Toronto-London, 1960. xiii + 346 pp. $9.00.

According to the author's preface, "signal theory is comprised of the set of analysis techniques adaptable to physical systems that respond in some manner as a consequence of being excited." The book itself presents a proper subset of the above-mentioned set, all of it being devoted to a meticulous study of physical systems in the so-called frequency domain. Liberal use is made of vectors, phasors, integral transforms, complex variables, and other such interrelated tools of analysis. Only linear time-invariant systems are considered. The author has tried to make the book wholly self-contained within its context. The eleven chapter headings are: Systems and vectors; Systems equations; Poles and zeros; Additional pole-zero concepts; An introduction to approximation; Operational methods; The direct Laplace transform; Contour integration; Further aspects of complex variable theory; Fourier series and discrete spectra; Fourier integrals and continuous spectra. The really nice feature of the book is the abundance of thought-provoking problems. This undoubtedly makes it useful as a text at a senior or first-year graduate level. The book is remarkably free of typographical errors.

N. H. Choksy (Silver Spring, Md.)

13334:

McMillan, B. A descriptive introduction to the statistical theory of communication. Nuovo Cimento (10) **13** (1959), supplemento, 345–352.

An expository lecture dealing briefly with statistical inference, communication theory as related to measurement and control, and communication theory considered as a service.

A. J. Lohwater (Providence, R.I.)

13335:

Shannon, Claude E. Coding theorems for a discrete source with a fidelity criterion. Information and decision processes, pp. 93–126. McGraw-Hill, New York, 1960.

Let S be a discrete independent source with output letters $a_1, \cdots, a_N$ whose probabilities are $p_1, \cdots, p_N$ respectively. Suppose that all information concerning the output of S is to be conveyed by means of an alphabet $z_1, \cdots, z_M$, and that there is given a distortion measure (or cost function) $d_{ij} \geq 0$ ($i=1, \cdots, N$, $j=1, \cdots, M$) which respresents the distortion or cost involved in the occurrence of a_i being conveyed by z_j. Let $[A, B]$ be the range of values of $\sum_{i,j} p_{ij} d_{ij}$ under the conditions $p_{ij} \geq 0$ and $\sum_{j=1}^{M} p_{ij} = p_i$ ($i=1, \cdots, N$). For any $d > A$ it is shown that there is an $R(d) \geq 0$ such that any channel with capacity $C > R(d)$ may be used to connect S to the z-alphabet in such a way as to keep the average distortion per source letter $\leq d$, while no channel with capacity less than $R(d)$ will do. The function $R(d)$ is studied, and evaluated in certain special cases. The general theory is sketched for continuous sources, and then worked out in greater detail for a particular distortion measure. Finally, a more general class of distortion measures is defined, and it is shown that the earlier results may be carried through for an arbitrary discrete ergodic source. These results represent a detailed elaboration and extension of ideas presented in the author's basic 1948 papers.

A. Feinstein (Urbana, Ill.)

13336:

Perez, Albert. Information theory with an abstract alphabet. Generalized forms of McMillan's limit theorem for the case of discrete and continuous times. Theor. Probability Appl. **4** (1959), 99–102.

English translation of Teor. Veroyatnost. i Primenen. **4** (1959), 105–109 [MR **21** #4860].

13337:

Costa de Beauregard, Olivier. Sur l'équivalence entre information et entropie dans le rapport $1/k \ln 2$. C. R. Acad. Sci. Paris **251** (1960), 2898–2900.

Author's summary: "On oppose les caractères très fictifs des mecanismes proposés pour la transition information→néguentropie, et très convaincants des analyses de la transition néguentropie→information."

S. Kullback (Washington, D.C.)

13338:

Bennett, William R. ★Electrical noise. McGraw-Hill Book Co., Inc., New York-Toronto-London, 1960. viii+280 pp. $10.00.

This book is addressed to the practicing communications engineer with only an undergraduate mathematics and physics background. About half the book is devoted to a description of noise sources (for example, in semiconductors), the remainder to noise equipment, modulation systems, and related material. The book is carefully written, in a lucid style, and could be of interest to an applied mathematician interested in background material.

E. Reich (Aarhus)

13339:

Brown, William M. Time statistics of noise. IRE Trans. **IT-4** (1958), 137–144.

An engineering exposition of various concepts of probability theory, e.g., ergodicity, useful in connection with problems of noise modulation. *E. Reich* (Aarhus)

13340:

Stutt, C. A. A note on invariant relations for ambiguity and distance functions. IRE Trans. **IT-5** (1959), 164–167.

Let $u_1(t)$ and $u_2(t)$ be two complex square-integrable functions such that $\int_{-\infty}^{\infty} |u_i|^2 dt = 1$. If the cross-ambiguity of u_1 and u_2 is defined as

$$\chi_{12}(\tau, \Delta) = \int_{-\infty}^{\infty} u_1(t) u_2^*(t+\tau) e^{-2\pi i t \Delta}\, dt,$$

it is shown that $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} |\chi_{12}(\tau, \Delta)|^2 d\tau d\Delta = 1$. This generalizes a result of Woodward for the case $u_1 = u_2$. For the remainder, assume that the imaginary part of each u_i is the Hilbert transform of its real part. If $\chi_{12} = \zeta_{12} + \xi_{12}$, it is shown that $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \zeta_{12}^2(\tau, \Delta) dt d\Delta = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \xi_{12}^2(\tau, \Delta) d\tau d\Delta = \frac{1}{2}$. If the distance between u_1 and u_2 with time shift τ and frequency shift Δ is defined by

$$d_{12}^2(\tau, \Delta) = \int_{-\infty}^{\infty} |u_1(t) - u_2(t+\tau) e^{2\pi i (t+\tau)\Delta}|^2\, dt,$$

then it is further shown that

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (1 - \tfrac{1}{2} d_{12}^2(\tau, \Delta))^2\, d\tau\, d\Delta = \tfrac{1}{2}$$

and

$$\lim_{\substack{T \to \infty \\ W \to \infty}} (4TW)^{-1} \int_{-T}^{t} \int_{-W}^{W} d_{12}^2(\tau, \Delta)\, d\tau\, d\Delta = 2.$$

H. O. Pollak (Murray Hill, N.J.)

13341:

Price, Robert. A useful theorem for nonlinear devices having Gaussian inputs. IRE Trans. **IT-4** (1958), 69–72.

Cet article comprend quatre parties: l'énoncé du théorème; sa démonstration; l'étude d'un cas particulier intéressant; la déduction de la fonction d'autocorrélation dans plusieurs cas simples, où celle-ci a été calculée antérieurement par d'autres auteurs (limiteur à pente raide, détecteur linéaire, écrêteur et limiteur progressif).

Le théorème s'applique uniquement à des processus qui obéissent à la loi de Laplace-Gauss. Il donne l'expression des dérivées partielles successives de la fonction de corrélation d'ordre maximum à la sortie des appareils par rapport aux fonctions de corrélation à l'entrée de ceux-ci. Cette expression, simple et commode, fait naturellement intervenir les dérivées des fonctions $f_i(x)$ qui relient les variables d'entrée à celles de sortie.

La démonstration utilise la représentation des fonctions $f_i(x)$ précédentes au moyen de transformées de Laplace et la méthode de Rice de la fonction caractéristique.

Les cas particulier considéré dans la troisième partie est celui de deux appareils où interviennent les fonctions de corrélation propre et mutuelle classiques. L'auteur montre l'intérêt de son résultat et retrouve une propriété intéressante de la fonction de corrélation mutuelle entre l'entrée et la sortie d'un appareil non linéaire, propriété découverte par Bussgang. *L. Robin* (Paris)

13342:

Brown, J. L. Application of the theory of orthogonal polynomials in two variables to a multi-gain equivalent linearization problem. Proc. Inst. Elec. Engrs. C **108** (1961), 115–118.

The main contribution of this paper is a demonstration of how the theory of orthogonal polynomials can be applied in finding a best, in the mean square sense, linear approximation of the form $k_1 x(t) + k_2 y(t)$ to the function $f(\beta + x(t) + y(t))$, where $x(t)$, $y(t)$ are correlated zero-mean stationary random processes, β is a constant, and k_1 and k_2 are to depend upon the input statistics and the device in question. It is also shown that orthogonal polynomials can be used to prove that the zero delay cross-correlation between the input to a non-linear device and the error resulting from the use of a linear approximation to the output is zero.

B. Hazeltine (Providence, R.I.)

13343:

Janos, William A. Optimal filtering of periodic pulse-modulated time series. IRE Trans. **IT-5** (1959), 67–74.

From the author's summary: "The present study concerns the optimal filtering of a class of input time series in which the amplitude is modulated by uniformly-pulsed periodic functions. A uniform sampling of the output at a period equal to the pulsing period displays the property of time invariance. The consequent usage of bilateral Fourier-Laplace transformations and the separability of terms implicit in the time-invariant nature of the processing effectively inverts the Wiener-Hopf equation and solves the problem." *K. S. Miller* (New York)

13344:

Pugachev, V. S. The method of determining optimum systems using general Bayes criteria. IRE Trans. **CT-7** (1960), 491–505.

The general problem considered here is that of finding an algorithm of an optimum system for estimating a signal in the presence of noise. A random function $Z(t)=\varphi(t, U)+X(t)$ is observed in a certain interval T of the variable t, and it is desired to find the best estimate $W^*(s)$ of a signal $W(s)$ of the form $W(s)=\psi(s, U, Y(s))$. Here φ and ψ are given definite functions, U is a finite-dimensional random vector with a known probability density $f(u)$, and $X(t)$ and $Y(s)$ are normally distributed random vector functions independent of U. The argument t denotes the current time in the observation interval T and s the end of this interval. (s may also belong to some domain S. T and S may or may not be interrelated and may be discrete or continuous or may even be abstract spaces.)

To determine this problem completely it is necessary to establish some optimality criterion. The majority of practically useful criteria belong to the class of Bayes criteria and are of the form $M[l(W, W^*)]=\min$, where l is some given loss function or functional and $M[\ ]$ denotes mathematical expectation.

Two further conditions are required for the present method. (1) For each realization z of the input Z there exists a function $W^*(s)$ such that for any function $W(s)$ the inequality $M[l(W, W^*)|z]\leq M[l(W, W)|z]$ is valid, where $M[\ |\ ]$ denotes conditional expectation. (2) For all possible values u of the vector U, $\varphi(t, u)$ is representable by the series of eigenfunctions of the random function $X(t)$ in the domain T.

The author then obtains a formal solution of the problem (the details of which cannot be given here) and applies it to the case of linear dependence of the input on signal parameters, to the case of normally distributed signal and noise, to signal detection, to signal parameter estimation using a maximum likelihood criterion, and to three examples of signal parameter estimation using different loss functions.

The author points out here and there throughout the paper that the theory presented can also be used for systems which are in some sense or other more general than described above; for example, some of the components of the vector U may be nonrandom unknown parameters which may assume any value.

N. H. Choksy (Silver Spring, Md.)

13345:

Middleton, David. A note on the estimation of signal waveform. IRE Trans. **IT-5** (1959), 86–89.

The problem of estimating a signal waveform S when the signal process $S(t)$ is corrupted by an additive Gaussian noise $N(t)$ is considered from the viewpoint of decision theory in extension of the work of Middleton and Van Meter [J. Soc. Indust. Appl. Math. **3** (1955), 192–253; **4** (1956), 86–119; MR **18**, 180; **19**, 824]. The author represents $S(t)$ in the form $\sum_{k=1}^{\infty} a_k\phi_k(t)$, where $\{\phi_k\}$ is an appropriately chosen orthonormal set of real functions on the observation period $(0, T)$ and $\{a_k\}$ a set of real parameters. He discusses both optimum structure for the case of different cost functions and its physical realization.

E. Reich (Aarhus)

13346:

Rudnick, Philip. Likelihood detection of small signals in stationary noise. J. Appl. Phys. **32** (1961), 140–143.

Author's summary: "An approximation to the likelihood ratio which may be used in detecting a small signal in stationary noise is derived. The result contains only low-order moments of the signal and only stationary properties of the noise; hence it is applicable without change of form to any sufficiently long observation period. In the Gaussian case, with the signal also stationary and both signal and noise power spectra continuous, the result represents passage through a linear Eckart filter, followed by square-law detection and equal-weight smoothing."

S. P. Lloyd (Murray Hill, N.J.)

13347:

Bedrosian, Edward. Weighted pcm. IRE Trans. **IT-4** (1958), 45–49.

Author's summary: "A modified form of pulse-code modulation, called weighted pcm, is described in which the relative amplitudes of the pulses within the pulse-code groups are adjusted so as to minimize the noise power in the reconstructed signal due to errors in transmission. A performance analysis shows the knee of the output signal-to-noise ratio curve to be moved 1.4 db to the left for a weighted seven-digit pcm system. An information rate study reveals that the maximum improvement which can ever be achieved by any encoding process over a conventional seven-digit pcm system is only 8 db. The importance of selecting a suitable system worth criterion is emphasized by showing that weighting increases the information rate relative to an rms fidelity criterion but decreases it on a pure equivocation basis."

S. P. Lloyd (Murray Hill, N.J.)

13348:

Blum, Marvin. On the mean-square noise power of an optimum linear digital filter for correlated noise input. IRE Trans. **IT-5** (1959), 58–61.

Consider a discrete-time stochastic process $e(t)=P(t)+N(t)$ where $P(t)$ is a polynomial and $N(t)$ forms a stationary process with mean value zero. Using $e(t)$ as input to a linear filter with the output $e_m^*=\sum_0^m W_u e_{m-u}$, it is desired to approximate the desired value $S_m^*=\int k(u)\times P(m-u)du$, where $k(u)$ is a given function. The best (unbiased, minimum variance) weighting sequence W is derived and results in a certain mean square $\bar{\sigma}_m^2$. Do the same thing under the assumption that $N(t)$ is white noise with the same variance. We get then a mean square $\hat{\sigma}_m^2$. Then the author shows that the two filters are asymptotically equivalent in the sense that $\lim_{m\to\infty}\hat{\sigma}_m^2/\bar{\sigma}_m^2=1$.

U. Grenander (Stockholm)

13349:

Galejs, Janis. Enhancement of pulse train signals by comb filters. IRE Trans. **IT-4** (1958), 114–125.

Author's summary: "The relative performance of different types of comb filters is investigated in conjunction with signal and noise types similar to those expected in radar applications. The filter types considered are idealized filters with zero transmission stop bands between their pass bands, optimum filters maximizing the peak signal-to-rms-noise ratio, cascaded delay line filters, feedback type filters, and storage tube filters. The pulse train signals consist of rectangular or sin x/x pulses with rectangular or sin x/x pulse envelope

shapes. Power spectra of noise considered are rectangular and triangular. With a given number of signal pulses, the performances of the different filters vary from each other only by a few decibels in most cases analyzed. Storage tube filters exhibit lower signal-to-noise power ratios, but higher peak signal-to-rms-noise ratios, than the feedback type filters. Inaccurate delay times of filter delay lines are shown to decrease the peak signal output more than the signal power output and to affect the cascaded delay line filter less than the feedback type filter. Correlation techniques are compared with comb filters. The cross-correlator exhibits the same peak signal-to-rms-noise ratio as the optimum filter."

S. P. Lloyd (Murray Hill, N.J.)

13350:

Galejs, Janis. Signal-to-noise ratios in smooth limiters. IRE Trans. **IT-5** (1959), 79–85.

Author's summary: "Signal-to-noise ratios associated with smooth band-pass limiting and subsequent narrow-band filtering of a periodic signal and random noise are computed. Observed changes in signal-to-noise ratios may be used to estimate detectability losses. The error function is used to represent the limiter characteristic at various degrees of limiting. First-order corrections with an increasing input signal to the signal-to-noise ratios, which are based on the small signal theory, are computed for limiter input noise with sin x/x, Gaussian, and exponential correlation functions."

S. P. Lloyd (Murray Hill, N.J.)

13351:

Slepian, David. Some further theory of group codes. Bell System Tech. J. **39** (1960), 1219–1252.

Author's summary: "The notion of equivalence for group codes is explored in some detail. A dual for a code, and the sum and product of two or more codes, are defined. Properties of these constructs are investigated. Indecomposable codes are defined and are shown to be optimal in two different senses. Various classes of codes are enumerated."

S. P. Lloyd (Murray Hill, N.J.)

13352:

Levenštein, V. I. A class of systematic codes. Dokl. Akad. Nauk SSSR **131** (1960), 1011–1014 (Russian); translated as Soviet Math. Dokl. **1**, 368–371.

Using the techniques introduced by Siforov, a class $S_{n,d}$ of systematic codes is constructed. $S_{n,3}$ is the Hamming code.

R. W. Hamming (Stanford, Calif.)

13353:

Lee, C. Y. Some properties of nonbinary error-correcting codes. IRE Trans. **IT-4** (1958), 77–82.

Let $S_k(n)$ be the set of all n-tuples $(x_1, \cdots, x_n)$ where the values of x_i lie in $0, \cdots, k-1$. Define a metric over $S_k(n)$ by

$$\rho(x, y) = \sum_{i=1}^{n} [\min\{x_i - y_i, y_i - x_i\} \bmod k].$$

A subset S of $S_k(n)$ is called a d-code if $\rho(x, y) \geqq d$ for every pair $x \neq y$ in S; a full d-code if no other d-code has a greater number of elements; a group code if S is a group with respect to component-wise addition modulo k; a full group d-code if it is full among all group d-codes, and close-packed if it is a d-code with d odd and $S_k(n)$ is the union of all closed spheres with radius $(d-1)/2$ and centers in S. A number of results are given concerning the existence of close-packed group codes and the number of elements in them, as well as various remarks pertaining to general full and close-packed codes.

A. Feinstein (Urbana, Ill.)

13354:

Golay, Marcel J. E. Notes on the penny-weighing problem, lossless symbol coding with nonprimes, etc. IRE Trans. **IT-4** (1958), 103–109.

A method, presented in an earlier paper by the same author, for the construction of lossless (close-packed) single-error correcting codes when the symbol order is a prime p, is extended to cases when the symbol order is a prime power p^q, by a matrix iteration process. These "master iterating matrices" are given for the cases $2^2, 2^3, 2^4, 2^5, 2^6, 3^2, 3^3, 3^4, 3^5, 5^2, 5^3, 5^4, 7^2, 7^3$. In addition, it is shown that these matrices exist for codes with symbol order p^2 for all prime p. The code matrix for the case $p=3$, $q=1$ is applied to the penny-weighing problem in which it is desired to determine, in the minimum number of weighings on a balance, which of 13 pennies, if any, is lighter or heavier than the standard weight. Finally, a condition on error-correcting codes observed by Zaremba is used to show the non-existence of a close-packed, two-error correcting code of 90 symbols, and to describe some of the properties that the binary one-error correcting lossless code of symbol order six must possess if it exists.

J. J. Stiffler (Pasadena, Calif.)

13355:

Cocke, John. Lossless symbol coding with nonprimes. IRE Trans. **IT-5** (1959), 33–34.

In answer to a question posed in #13354, the author describes a method of construction of lossless one-error correcting codes with symbol order p^n where p is a prime. An algorithm for achieving these codes using the Galois field $GF(p^n)$, and a decoding method are presented. In this way, all of the codes whose existence was established by Zaremba can be obtained. The "master iteration matrix" is determined, thereby enabling one to decode using the multiplication table of the integers modulo p rather than that of the Galois field.

J. J. Stiffler (Pasadena, Calif.)

13356:

Sacks, Gerald E. Multiple error correction by means of parity checks. IRE Trans. **IT-4** (1958), 145–147.

The coding properties of an error correcting parity check code with n bits, r of which are redundant, can be characterized by a matrix whose rows are determined by the r parity check conditions. The columns of this matrix are called characteristics. The author shows that a code has minimum distance d if and only if there exist d characteristics which are linearly dependent modulo 2, but every set of less than d characteristics is linearly independent modulo 2. Moreover, he derives an upper bound for the number of redundant bits necessary to achieve a code with minimum distance d, by constructing explicitly d linearly independent characteristics. For small d, this bound is not too far from Hamming's lower bound and thus facilitates heuristic construction of parity check codes.

F. L. Bauer (Mainz)

13357:

Chien, Robert. On the characteristics of error-correcting codes. IRE Trans. IT-5 (1959), 91.

Remarks to the foregoing paper by Sacks [#13356]. Using linear independence of characteristics, conditions are given for a code to be lossless. *F. L. Bauer* (Mainz)

13358:

Meggitt, J. E. Error correcting codes for correcting bursts of errors. IBM J. Res. Develop. 4 (1960), 329–334.

It has been observed recently [#13356] that a certain class of error correcting codes $\{a_1, a_2, \cdots, a_n\}$ can be described in a Galois field of characteristic 2 by constraints of the form $\sum_1^n a_\mu T^{\mu-1}x=0$, where T is a matrix and x a suitable vector. The author shows that the error correcting properties of the code are determined only by the characteristic equation of T. The practical result is that by similarity transformations certain existing codes can be transformed such that the implementation of encoding and decoding devices is simplified. Some misprints. *F. L. Bauer* (Mainz)

13359:

Tylkin, M. E. On Hamming geometry of unitary cubes. Dokl. Akad. Nauk SSSR **134** (1960), 1037–1040 (Russian); translated as Soviet Physics. Dokl. **5** (1961), 940–943.

"The isometric mapping of finite metric spaces into unitary cubes with Hamming metric is considered. The problem can be restated in terms of the algebra of logic, or linear programming, or the theory of self-correcting codes, or the theory of graphs." *R. W. Hamming* (Stanford, Calif.)

13360:

Kim, Wan H.; Freiman, Charles V. Single error-correcting codes for asymmetric binary channels. IRE Trans. IT-5 (1959), 62–66.

Authors' summary: "In a highly-asymmetric binary channel it may be necessary to correct only those errors which result from incorrect transmission of one of the two code elements. Minimum weight-distance relationships and rules for generating single-error correcting codes in such situations are given. More code characters are generally obtained for a given character length than are obtained with codes designed for single-error correction in symmetric channels. Examples are given, including one which specifies the code which results in the highest average probability of correct transmission of equiprobable messages through a highly-asymmetric channel." *A. Feinstein* (Urbana, Ill.)

13361:

Plotkin, Morris. Binary codes with specified minimum distance. IRE Trans. IT-6 (1960), 445–450.

The author extends work of R. W. Hamming [Bell System Tech. J. **29** (1950), 147–160; MR **12**, 35]. Define $A(n, d)$ to be the size of the largest binary code (not necessarily a group code) of length n which can exist such that the minimum distance (number of disagreements) between code words is at least d ($\leq n$). The author obtains various bounds and relations for the $A(n, d)$, the most interesting of which is a result (due in essence, as the author points out, to R. E. A. C. Paley [J. Math. and Phys. **12** (1933), 311–320]) which states that $A(4m, 2m) = 8m$ at least if $4m-1$ is a prime. The author conjectures that this is true for arbitrary m. *C. Posner* (Claremont, Calif.)

13362:

Abramson, N. M. A note on single error correcting binary codes. IRE Trans. IT-6 (1960), 502–503.

The author discusses a class of codes for which the encoding (and decoding) can be done by linear shift registers. It should be noted that these codes are precisely the single error-correcting codes of Bose and Ray-Chaudhuri [Information and Control **3** (1960), 68–79; MR **22** #3619]. *N. Zierler* (Pasadena, Calif.)

13363:

Zierler, Neal. On decoding linear error-correcting codes. I. IRE Trans. IT-6 (1960), 450–459.

Author's summary: "A technique is described for finding simply computable numerical-valued functions of a received binary word whose value indicates where errors in transmission have occurred. Although it seems that a certain condition must usually be fulfilled for such functions to exist, or for our method to constitute an efficient procedure for finding them, there is, on the one hand, a strong tendency for 'good' codes to satisfy the condition, while, on the other, it appears to be straightforward to construct codes which are good for a specified channel and also fulfill the condition. An advantage of the resulting decoding procedure is that it corrects and detects all possible errors; more precisely, if a word u is received and the coset $\bar{u}$ to which u belongs has a unique leader e, the procedure concludes that $u+e$ was sent, while if $\bar{u}$ has no unique leader, that fact, along with the weight of $\bar{u}$ (and sometimes a little more), can be indicated. The ideas and techniques are illustrated by the construction of decoding procedures for the perfect (23, 12) three-error-correcting code." *A. Feinstein* (Urbana, Ill.)

13364:

Gilbert, E. N. A problem in binary encoding. Proc. Sympos. Appl. Math., Vol. 10, pp. 291–297. American Mathematical Society, Providence, R.I., 1960.

Let n and E be two given positive integers with $2E<n$. Then an error-correcting encoding is a list of n-tuples of binary digits such that every pair of n-tuples in the list differ in at least $2E+1$ of the n places. Only errors which group together in bursts extending over at most E digits (which start with 1 and end with 1) and such that each pair of bursts is separated by at least D correct digits are considered in this paper. An n-tuple of this form where runs of 0's of length $<D$ may occur at the beginning and at the end of the n-tuple will be called a burst n-tuple; and it is always assumed that $D \geq E-1$ to guarantee the uniqueness of the decomposition of the burst n-tuple into burst sequences and runs of 0's. A burst-correcting encoding is defined to be a list of n-tuples $a, b, \cdots, w$ such that, for every binary n-tuple x, at most one of the n-tuples $a+x, b+x, \cdots, w+x$ is a burst n-tuple.

The author proves that a necessary and sufficient condition for a list $a, b, \cdots, w$ of n-tuples to be a burst-correcting encoding is that no pair i, j of distinct n-tuples in the list have a sum $i+j$ which equals a sum of two burst n-tuples. Furthermore, he proves that no burst-correcting encoding can contain more than $[2^n/F(n)]$ n-tuples, where

$F(n)$ is the number of distinct burst n-tuples. A burst-correcting encoding is called full if no additional n-tuple can be adjoined to it to make an enlarged list which is also a burst-correcting encoding. Then, it is proved that any full encoding contains at least $2^n/G(n)$ n-tuples where $G(n)$ is the number of distinct n-tuples which can be expressed as sums of two burst n-tuples.

In the last section of the paper special interesting burst-correcting encodings are constructed in such a way that their structures are systematic enough to permit machines to perform the encoding and decoding computations without great difficulty.

A. L. Duquette (Pasadena, Calif.)

13365:

Belevitch, V. On the statistical laws of linguistic distributions. Ann. Soc. Sci. Bruxelles. Sér. I **73** (1959), 310–326.

This paper offers a reinterpretation of the much discussed rank-frequency relation of words in natural languages as distribution curves of the cumulative probability of types in the catalogue versus the probability of tokens in the text. The derived formulas are applied to actual data from a number of languages, both to letter and phoneme distributions and to word distributions. On the basis of these experimental data, the author concludes that the truncated normal distribution with $\sigma \cong 2.8$ bits may be accepted as the general law for words, while in the case of letter and phoneme distributions, the standard deviation appears to have the universal value $\sigma \cong 1.4$ bits. *H. Kucera* (Providence, R.I.)

13366:

Mandelbrot, B. Statistical macro-linguistics. Nuovo Cimento (10) **13** (1959), supplemento, 518–520.

The basic relationship, familiar in information theory, $\beta C = -\log_e p$ (where p is the probability of a signal in a message, C is the "cost" of transmitting this signal, and β is a factor which depends upon the scale chosen for C), is applied to words of natural languages. The author assumes that a word is any sequence of letters between successive occurrences of space, and interprets the cost of coding a word as being equal to the number of proper letters plus the cost of the improper letter "space". In this manner he derives a relationship which appears to explain also Zipf's data on the distribution of words other than the most frequent ones.

H. Kucera (Providence, R.I.)

13367:

Yngve, Victor H. A model and an hypothesis for language structure. Proc. Amer. Philos. Soc. **104** (1960), 444–466.

This paper presents a model of language production which incorporates the hypothesis of a depth limitation. The model assumes that language is produced by a finite-state device and by an essentially left-to-right process; the actual sentence generation is viewed as a successive expansion of the basic constituents of the sentence through application of the rules of the language. In producing a sentence, the device has to remember what it is committed to do next according to the rules of grammar. In expanding the left-most constituent, for example, it must keep in a temporary memory, at each step in the expansion, the sentence constituents which still have to be dealt with. The temporary memory of the device is assumed to be limited to more-or-less seven items; this specific limitation is justified on the basis both of psychological experiments dealing with human memory and of the analysis of English sentences. Because of the limitations on temporary memory, which the author expects to be similar in languages other than English, grammars contain restrictions and devices which retain them within the capabilities of the finite-state device. Thus, in the author's hypothesis, only about seven steps are possible in the expansion of a constituent of a sentence before the basic left-to-right direction of sentence production is resumed.

H. Kucera (Providence, R.I.)

13368:

Èpšteĭn, V. L. Algorithm for the statistical selection of highly probable symbols in a self-adjusting system with limited memory faculty. Radiotehn. i Èlektron. **4** (1959), 1563–1565 (Russian); translated as Radio Engrg. and Electronics **4**, no. 9, 237–241.

An algorithm is presented for establishing the relative frequencies of the M most frequent distinct symbols in a corpus when the total number of distinct symbols G in the alphabet is much greater than M, but the available computer memory capacity is only of order M. While most of the essentials of the algorithm are presented in the paper, the author points out: "In order to define the numbers R and T, in conjunction with the given number M, it is necessary to have some a priori probability judgments which we won't discuss here. It is obvious that these numbers can be determined experimentally." It is not at all obvious whether or not this determination requires the use of question-begging techniques.

A. G. Oettinger (Cambridge, Mass.)

SERVOMECHANISMS AND CONTROL

See also A12054, 13072, 13281.

13369:

Ku, Y. H. Theory of nonlinear control. J. Franklin Inst. **271** (1961), 108–144.

This is the complete version of the author's report at the First International Congress of IFAC (Moscow, 1960). The task of explaining in 28 small pages the contents of 215 research papers and books listed at the end of this report is obviously a very difficult one. At any rate, the reviewer was unable to understand most of it.

J. L. Massera (Montevideo)

13370:

Solodownikow, W. W. ★Grundlagen der selbsttätigen Regelung. Bd. I: Allgemeine Grundlagen der Theorie linearisierter selbsttätiger Regelungssysteme. Deutsche Bearbeitung unter Heinrich Kindler. R. Oldenbourg Verlag, München; VEB Verlag Technik, Berlin; 1959. xvi + 727 pp. (3 inserts) DM 65.00.

This and the companion volume [see #13371] are translations of *Osnovy avtomatičeskogo regulirovaniya* [Mašgiz, Moscow, 1954].

The author is a well-known Soviet worker in the field of automatic control and a leading proponent of

automation in industry. Drawing on his experience, he has produced a book whose outstanding characteristic is a close integration of the theory and applications of automatic control, with many illustrative examples drawn from actual practice.

The amount of material covered is prodigious. The major subdivisions of the book are: an introductory section describing various examples of control systems encountered in practice, a section on the fundamentals of linear systems, a section on the stability of linear and nonlinear systems, a long section on the more specialized techniques of analysis of linear systems, a section on sampled-data systems, and a final section on the optimal design of control systems subjected to random inputs. The treatment of these topics is very thorough and, on the whole, quite up-to-date. The viewpoint, however, is conventional and one hardly gets an inkling of the important developments in control-system theory which have taken place during the past three or four years. Nonetheless, this work is a definitive treatise on the theory of linear control systems, and is a unique source of practical data pertaining to the design of such systems.

L. A. Zadeh (Berkeley, Calif.)

13371:

Solodownikow, W. W. ★Grundlagen der selbsttätigen Regelung. Bd. 2: Einige Probleme aus der Theorie der nichtlinearen Regelungssysteme. Deutsche Bearbeitung unter H. Kindler. R. Oldenbourg Verlag, München; VEB Verlag Technik, Berlin; 1959. viii+452 pp. DM 52.00.

The second volume of this two-volume work [see review above] is concerned in its entirety with the analysis and design of nonlinear systems. Following an introductory descriptive account of various types of nonlinearities occurring in practice, there are two chapters on special types of second-order systems, profusely illustrated by examples drawn from practice. Chapter 3 presents a discussion of one of the basic problems of optimal control, namely, that of finding an input which takes the system from its initial state to a specified state in the shortest possible time. The material is based largely on earlier work of Fel'dbaum [Avtomat. i Telemeh. **14** (1953), 712–728] and Lerner [ibid. **13** (1952), 134–144] and does not include the recent contributions of Pontryagin and his students [Boltyanskiĭ, Gamkrelidze and Pontryagin, Dokl. Akad. Nauk SSSR **110** (1956), 7–10; MR **18**, 859] nor of N. N. Krasovskiĭ [Prikl. Mat. Meh. **21** (1957), 670–677; MR **20** #4453]. These are followed by extensive discussions of the techniques of equivalent linearization and harmonic balance as well as asymptotic methods. The exposition of the theory of relay systems occupies close to one hundred and fifty pages and covers much of the theory given in Cypkin's text [*Teoriya releĭnyh sistem avtomatičeskogo regulirovaniya*, Gosudarstv. Izdat. Tehn.-Teor. Lit., Moscow, 1955; MR **18**, 709]. The book closes with a very informative chapter on the numerical solution of nonlinear differential equations.

It is of interest to note that there is relatively little emphasis on the theory of stability of nonlinear systems in this work, with most of the general theory incorporated in Volume 1 rather than Volume 2. This probably reflects the abundance of specialized texts on stability theory in Russian, which makes it unnecessary to discuss at length the theory of stability in a general text on control systems.

L. A. Zadeh (Berkeley, Calif.)

13372:

Tschauner, Johann. ★Einführung in die Theorie der Abtastsysteme. R. Oldenbourg, Munich, 1960. 185 pp. DM 32.00.

The author of this book expects that the reader is not familiar with the theory of difference equations. Therefore he considers sampled-data systems as systems with piecewise continuous inputs. The physical meaning of the mathematical operations is well explained and many figures support the text.

In the first chapter open-loop systems are treated. Simple examples are given. Frequency response and stability are discussed. In the second chapter closed-loop systems are treated in a similar manner, and in the third and final chapter those mathematical methods (difference methods) are explained which are most suitable for the investigation of sampled-data systems.

I. Flügge-Lotz (Stanford, Calif.)

13373:

Filippov, A. F. On some questions in the theory of optimal regulation: existence of a solution of the problem of optimal regulation in the class of bounded measurable functions. Vestnik Moskov. Univ. Ser. Mat. Meh. Astr. Fiz. Him. **1959**, no. 2, 25–32. (Russian)

Consider the vector equation $dx/dt=f(x, y, t)$, $x(0)=c$, where the policy vector y is to be chosen subject to constraints so as to minimize the time required to reach the state $x=b$. The author establishes the existence of an optimal policy under suitable conditions on f, and discusses a particular example. Particular versions of this problem (the "bang-bang" control problem) are discussed in R. Bellman, I. Glicksberg, and O. Gross, Rand Corporation, Rep. No. R-313, Santa Monica, Calif., 1958 [MR **20** #800], where references to earlier work of LaSalle and later work by Pontryagin and others will be found.

R. E. Bellman (Santa Monica, Calif.)

13374:

Doležal, Václav. Über die Anwendung von Operatoren in der Theorie der linearen dynamischen Systeme. Apl. Mat. **6** (1961), 36–67. (Czech and Russian summaries)

The paper is, essentially, a careful study of the properties of impulsive response functions of systems characterized by linear differential equations with smoothly varying coefficients. To avoid the use of delta-functions, the author employs the apparatus of generalized functions in the sense of Gel'fand and Šilov.

L. A. Zadeh (Berkeley, Calif.)

13375:

Hájek, J. Über die Äquivalenz der drei Stabilitätskriterien. Z. Angew. Math. Mech. **39** (1959), 332–333.

13376:

Potapov, Yu. G.; Yablonskiĭ, S. V. On the synthesis of self-correcting relay circuits. Dokl. Akad. Nauk SSSR **134** (1960), 544–547 (Russian); translated as Soviet Physics. Dokl. **5** (1961), 932–935.

A study is made of the complexity of relay circuits (complexity measured by the number of relay contacts) for single error correcting circuits, although the notation and general background introduced are much more general.

R. W. Hamming (Stanford, Calif.)

13377:

Higonnet, René A.; Gréa, René A. Some aspects of switching algebra. Proc. Internat. Sympos. Switching Theory 1957, Part II, 281–284. Harvard Univ. Press, Cambridge, Mass., 1959.

The authors would like to see some research on the classification of sequential circuits which corresponds to, say, E. F. Moore's exhaustive tabulation of combinatorial switching functions of four variables that appears as an appendix in R. A. Higonnet and R. A. Gréa, *Logical design of electric circuits* [McGraw-Hill, New York, 1958].

A. A. Mullin (Urbana, Ill.)

13378:

Minnick, Robert C. Linear-input logic. IRE Trans. **EC-10** (1961), 6–16.

The problem considered is the realization of switching functions by means of threshold devices. Three main results are obtained. First, it is shown how functions represented in a (fully expanded) canonical form can be realized. Second, a method of realizing symmetric functions is presented. This method results in more economical circuits than those circuits obtained using previously reported methods. Third, a linear programming formulation of the problem is presented. A table giving economical realizations of switching functions of four variables derived using the linear programming scheme followed by "hand" simplification is included.

B. Hazeltine (Providence, R.I.)

13379:

Low, P. R.; Maley, G. A. Flow table logic. Proc. IRE **49** (1961), 221–228.

The authors are concerned with interconnection problems in large switching systems using micro-miniature elements, such that a high packing density of elements may be attained. They describe a technique for synthesizing a sequential circuit directly from the flow table of D. A. Huffman. The resulting logical circuit appears to be very similar to that which would be obtained by Huffman's "One-Relay-Per-Row" realization applied to primitive flow tables. The authors, however, propose a particular latch and transfer element and a convenient row and column layout of these elements which mimics the flow table configuration. This structure seems well suited for some types of logic, such as the neon-photoconductor logic illustrated in the paper.

H. H. Goldstine (Yorktown Heights, N.Y.)

13380:

Steinberg, Leon. The backboard wiring problem: a placement algorithm. SIAM Rev. **3** (1961), 37–50.

Given a set of n positions (lattice points) in E^3 with the usual metric or with the absolute value of the difference metric, and given a set of $k<n$ elements (e.g., electronic components) each pair of which is connected by a number (possibly zero) of edges (e.g., wires). The author studies an iterative algorithm for placing the elements in a subset of positions in an attempt to minimize the total length of the connecting edges according to the metric used. The elements are placed in the positions arbitrarily; an independent (pairwise unconnected) subset is then reassigned positions using the assignment problem of linear programming. The process is then repeated with another pairwise unconnected subset, etc. The algorithm depends on the initial placement, the unconnected subsets and the order in which they are used. An illustration is given.

T. L. Saaty (Silver Springs, Md.)

13381:

Birnbaum, Z. W.; Esary, J. D.; Saunders, S. C. Multi-component systems and structures and their reliability. Technometrics **3** (1961), 55–77.

Atemporal theory in Boolean algebraic spirit of reliability of structures of dichotomous components with special reference to the effects of redundancy [Antecedents: E. F. Moore and C. E. Shannon, J. Franklin Inst. **262** (1956), 191–208; MR **18**, 549; J. von Neumann, *Probabilistic logics and the synthesis of reliable organisms from unreliable components*, Princeton Univ. Press, Princeton, N.J., 1956; MR **17**, 1048; Hisashi Mine, IRE Trans. **IT-5** (1959), supplement May 1959, 138–150].

L. J. Savage (Ann Arbor, Mich.)

13382:

Reiss, Richard F. The digital simulation of neuromuscular organisms. Behavioral Sci. **5** (1960), 343–358.

The author describes a model of an organism consisting of a neural network and a means (muscle) to permit interaction (feedback) between the network and an environment. The model is set up to permit simulation of its operation on a large digital computer. Half of the paper is devoted to a description of the model and half to a description of the computer program used to carry out the simulation. In constructing his simulation model, the author attempts to improve on earlier models which were based on the notion that the nervous system can be represented as a synchronous logical network. Physiological evidence appears to refute this. The author postulates a neural net in which frequency of firing is the main variable. The synapse parameters are the firing threshold, leakage period and refractory period. The synapse is visualized as a "place where energy is stored" until it reaches the firing threshold or leaks away. The synapses fire asynchronously.

Another criticism of older simulations made by the author is their failure to include interaction with an environment, which, it is claimed, plays a crucial role. The present simulation has a closed-loop feedback circuit which include environments of a fairly wide class.

The computer program which simulates the behavior of this model consists of two parts, one to simulate the network and the other to simulate the interaction with the environment. The network simulating program uses associative lists. Networks of approximately 300 elements connected in an arbitrary way can be handled.

The author points out the difficulty of working with simulations of this order of complexity, in which perhaps thousands of parameters are involved, each conceivably playing a significant individual role so that statistical analyses are ruled out. He suggests that to reduce these simulation experiments to manageable and comprehensible proportions they will have to be controlled by an automaton; i.e., another computer program. The reviewer feels that statistical analyses cannot be so summarily dismissed, and in fact could be incorporated into the model to reduce its complexity or into the proposed supervisory program.

E. K. Blum (Pacific Palisades, Calif.)

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